Project no. TREN/07/FP6AE/S07.71574/037180 IFLY

iFly

Safety, Complexity and Responsibility based design and validation of highly automated Air Traffic Management

Specific Targeted Research Projects (STREP)

Thematic Priority 1.3.1.4.g Aeronautics and Space

iFly Deliverable D3.1
Complexity metrics applicable to autonomous aircraft

Version: Final (1.1)

Authors: M. Prandini (PoliMi)**, L. Piroddi** (PoliMi)**,**
S. Puechmorel (ENAC), **S.L. Brázdilová** (HNWL)

Due date of deliverable: 22 February 2008
Actual submission date: 14 January 2009

Start date of project: 22 May 2007 Duration: 39 months

Project co-funded by the European Commission within the Sixth Framework Programme (2002-2006)		
Dissemination Level		
PU	Public	X
PP	Restricted to other programme participants (including the Commission Services)	
RE	Restricted to a group specified by the consortium (including the Commission Services)	
CO	Confidential, only for members of the consortium (including the Commission Services)	

DOCUMENT CONTROL SHEET

Title of document:	*Complexity metrics applicable to autonomous aircraft*
Authors of document:	*Maria Prandini, Luigi Piroddi, Stehane Puechmorel, Silvie Luisa Brázdilová*
Deliverable number:	*D3.1*
Project acronym:	*iFly*
Project title:	*Safety, Complexity and Responsibility based design and validation of highly automated Air Traffic Management*
Project no.:	*TREN/07/FP6AE/S07.71574/037180 IFLY*
Instrument:	*Specific Targeted Research Projects (STREP)*
Thematic Priority:	*1.3.1.4.g Aeronautics and Space*

DOCUMENT CHANGE LOG

Version #	Issue Date	Sections affected	Relevant information
0.1	20.02.2008	all	First Draft
0.2	26.03.2008	all	Second Draft
0.3	01.04.2008	Chapters 4 and 5	Third Draft (added chapter 4)
1.0	19.06.2008	all	Final
1.1	14.01.2009	Chapter 1	Change in iFly

Version 1.1		Organisation	Signature/Date
Authors	Maria Prandini	PoliMi	
	Luigi Piroddi	PoliMi	
	Stephane Puechmorel	ENAC	
	Silvie Luisa Brázdilová	HNWL	
Internal reviewers	Henk Blom	NLR	
External reviewers	Prof. Dr. Uwe Voelckers	EC	

iFly, Work Package 3, D3.1
Complexity metrics applicable to autonomous aircraft

14 January, 2009

Abstract

This is the first deliverable of work package 3 of the iFly project. The objective of work package 3 is to study and develop methods for timely predicting potentially complex air traffic conditions that may be over-demanding to the autonomous aircraft design. The characterization of encounter situations that appear safe from the individual aircraft perspective, but are actually safety-critical from a global perspective can provide useful information for the trajectory management and conflict resolution operations, and can also help in identifying the potential ground support needs within the autonomous aircraft Air Traffic Management (ATM) concept developed in the iFly project.

Deliverable 3.1 consists in a comparative study of the different metrics proposed in the literature for complexity characterization and prediction in ATM. Most of the metrics address ground-based ATM and are conceived so as to assess the impact of a given air traffic configuration on the workload of the air traffic controllers in charge of safely handling it. We review these metrics in view of a possible application to advanced autonomous aircraft ATM systems.

Contents

List of Figures

1 Introduction

An Air Traffic Management (ATM) system is a multi-agent system, where many aircraft are competing for a common, congestible resource, represented by airspace and runways space, while trying to optimize their own cost (travel distance, fuel consumption, passenger comfort, etc.). Coordination between different aircraft is needed to avoid conflicts where two or more aircraft get too close one to the other. In principle, this can be achieved via a decentralized control scheme where each aircraft evaluates the criticality of forthcoming encounters based on the information on the current position and intended destination of neighboring aircraft, and eventually coordinates with them to avoid that a conflict actually occurs. Notice that in a decentralized approach to conflict detection each aircraft employs local information only, and evaluates the criticality of the situation based on a partial viewpoint. A high-level coordination layer can possibly be required to avoid safety-critical encounters corresponding to a level of risk that is considered low by the aircraft involved, but is actually high for the overall multi-aircraft system.

1.1 iFly work package 3

The objective of work package 3 is to study and develop methods for the timely prediction of air traffic conditions that may be over-demanding to the autonomous aircraft design. This is a crucial task for avoiding encounters that appear safe from the individual aircraft perspective, but are actually safety-critical from a global perspective. The characterization of globally safety-critical encounters can provide useful information for the trajectory management and conflict resolution operations, and can also help in identifying the potential ground support needs within the autonomous aircraft ATM concept developed in work package 1 of the iFly project.

Work package 3 is structured in the following two sub-work packages:

WP3.1: Comparative study of complexity metrics. In this sub-work package, we shall carry out a critical survey of different metrics proposed in the literature for complexity modelling and prediction in ATM. Most of the current complexity metrics address ground-based ATM. Though this is reasonable within the current centralized ATM system, where aircraft follow predefined routes according to some prescribed 4D flight plan, it becomes restrictive within advanced autonomous aircraft ATM systems.

WP3.2: Timely predicting complex conditions. In this sub-work package, we shall study the problem of predicting complex conditions for autonomous aircraft and developing an appropriate complexity metric. Aspects that need to be addressed are the sensitivity to the prediction time, and various other conditions. For work package 3 studies no specific choice is made regarding where to use the novel method, airborne and/or on the ground.

Work package 3 will receive input from work package 1, in terms of the Autonomous Aircraft Advanced Concept of Operations (A^3 ConOps). WP3 will provide input to work package 8 as for ground support needs for the A^3 ConOps, thus contributing to the refinement of the A^3 ConOps. Appropriate interaction with work packages 1 and 8 is required to identify potential areas of usage of complexity metrics and clarify the requirements for the developed metrics.

The specification of the requirements on complexity metrics for the A^3 ConOps is one of the first activities planned under WP3.2, based on the precise formalization of the A^3 ConOps in Deliverable 1.3. As for the output to work package 8, a phase of tuning with WP8.1 is planned for the last stage of WP3.2. The objective of WP8.1 is in fact integrating within the A^3 ConOps the innovative methods for complexity prediction and for multi-agent situation awareness inconsistency identification and conflict resolution, developed within WP3, WP4, and WP5, respectively.

1.2 Objectives of Deliverable 3.1

Deliverable 3.1 is the outcome of the sub-work package 3.1 and consists in a comparative study of the different approaches proposed in the literature for air traffic complexity modelling and prediction. Most of the current air traffic complexity studies relates to ground-based ATM, where the airspace is divided into sectors and Air Traffic Controllers (ATCs) are in charge of guaranteeing safety in air travel within their sector. In Deliverable 3.1, we revise these studies in view of a possible application to advanced autonomous aircraft ATM systems, where part of the responsibility in maintaining the appropriate separation between aircraft is delegated to the pilots. In particular, pilots will take over the ATC tasks for separation assurance in self-separation enroute airspace, and they will rely for this purpose on advanced tools enabled by advanced technologies for sensing, communicating, and decision making. Centralized control will assume a new role consisting in a higher level, possibly automated, supervisory function as opposed to lower level human-based control, which should allow an increase in the airspace capacity without compromising safety.

Complexity measures that have been to some extent successful within the current human-based centralized ATM system may actually be inappropriate within the foreseen automated self-separation airspace.

1.3 Organization of the document

Deliverable 3.1 is structured as follows. In Chapter 2 we illustrate the notion of air traffic complexity within the current ground-based ATM. As pointed out in Section 2.1, studying air traffic complexity is fundamental in this context to evaluate the impact on the ATC workload of possible modifications of the ATM system introduced to adapt its capacity to the increased air traffic demand. For this reason, most of the measures of complexity proposed in the literature try to incorporate the difficulty perceived by ATCs in handling different air traffic situations. In order to use a complexity metric as a traffic management tool, it is necessary to predict its future behavior. In Section 2.2 we describe some of the approaches for air traffic complexity modelling and prediction, and in Section 2.3 we compare them in view of a possible application to advanced autonomous aircraft ATM design. The complexity-related concepts of trajectory flexibility and aircraft clustering are described in Section 2.4. In Chapter 3, we focus on an approach developed by ENAC that provides a measure of the intrinsic complexity of air traffic, independent of the ATCs perceived difficulty in accomplishing their task, and looks promising for application to advanced autonomous aircraft ATM. Finally, in Chapter 4 we draw some final conclusions on this survey on air traffic complexity, outlining possible directions of research under WP3.2 on complexity characterization for enroute autonomous aircraft ATM.

2 Air traffic complexity studies in the literature

2.1 Introduction

The growth in air traffic demand is pushing to its limit the current ground-based ATM system. As reported in [28], in 2006 the average daily traffic above Europe was 26286 flights per day, with an increase of 4.1% over 2005, whereas the total delay increased by 4.6%, much more than expected based on the 4.1% of air traffic growth.

In Figure 1 managed airspace is represented as a control system where ATM acts as feedback controller of the airspace. The thick lines connecting the ATM and airspace blocks represent multi-dimensional signals (airspace measurements and ATM actions). The objective of the ATM controller is to guarantee safety and efficiency in air travel, despite the airspace system time-variability due, for instance, to temporary structural modifications when the access to some areas is forbidden because of military missions or bad weather conditions, and to disturbances like aircraft entering/leaving the airspace because departing/landing at airports.

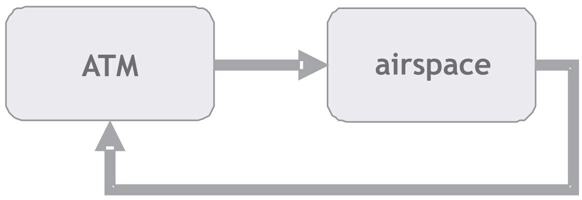

Figure 1: Managed airspace control scheme.

The main components of the current ATM system are the Air Traffic Control and Traffic Flow Management functions. The former is in charge for maintaining the appropriate separation between aircraft within the airspace, whereas the latter has to define the flow patterns so as to ensure a smooth and efficient organization of the air traffic. These two functions operate on different time scales, namely on a mid-term and on a long-term time horizon, as detailed in Deliverable 5.1 of the iFly project entitled "Comparative Study of Conflict Resolution Methods" where resolution methods are distinguished based the reference time-horizon. The airspace is structured into Air Traffic Control Centers (ATCCs) partitioned into sectors, each controlled by a team of 1 to 3 Air Traffic Controllers (ATCs). Sectors are designed so that the nominal flow of traffic through each sector can be safely handled by the ATCs that are in charge of that sector. The ATCC capacity is limited by the sector with the minimum capacity.

The basic control unit in mid-term control is plotted in Figure 2. In the feedback control scheme, the controlled system is not the whole airspace but only a sector, and the feedback controller is represented by the ATC which interacts with the controlled system through sensing and actuating interfaces. The exogenous input to the control system represents aircraft entering/exiting the sector under consideration and models the interactions with neighboring sectors. Information on the controlled system behavior and on the exogenous input are provided to the ATC through "sensors" (radar, software equipment, and radio connections), whereas the control strategy is implemented issuing commands (speed, altitude, heading changes) to the pilots via radio connections (the "actuators").

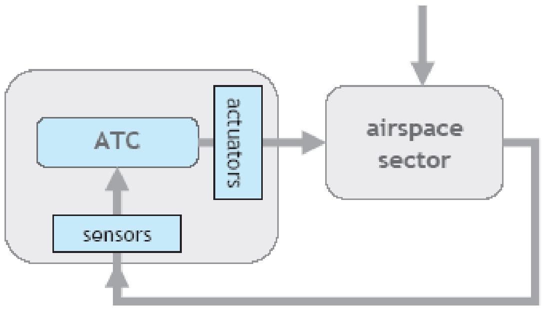

Figure 2: Mid-term enroute control in ground-based ATM.

A method to accommodate air traffic growth within the current ATM system is to adapt its capacity to the increased demand by appropriately redesigning the airspace, reconfiguring sectors, modifying traffic patterns, and also reassigning staff, [41, 83]. The most common operational means of increasing capacity is to partition a sector into smaller ones, each with an independent controller team. These modifications are currently adopted temporarily in presence of extraordinary events, as particular weather conditions or constraints on the airspace usage due, e.g., to military missions. The purpose of such permanent or temporary modifications is to avoid an increase of the ATC workload, which could eventually compromise air travel safety and efficiency.

Generally speaking, ATC workload can be regarded as the mental and physical effort involved in handling air traffic. In [66], workload is defined as "... a function of three elements, firstly, the geometrical nature of the air traffic; secondly, the operational procedures and practices used to handle the traffic and thirdly, the characteristics and behavior of individual controllers (experience, orderliness, etc.) ...". The configuration of sectors, in particular, is recognized as a main factor affecting workload, [31], and workload is held responsible for limiting sector capacity [15, 59, 67, 61]. In [30, 85], a macroscopic workload model is studied to assess sector capacity.

Air traffic complexity, intended as a "... measure of the difficulty that a particular traffic situation will present to an air traffic controller ...", [66], is commonly thought as responsible for generating workload.

Since about forty years ago, air traffic complexity models have been studied to relate airspace configuration and traffic to the workload of ATCs, introducing indicators of the workload level that are based on airspace and air traffic measurements. The work [18] is perhaps the first one to systematically examine the relationship between air traffic complexity and controller workload.

It is generally recognized that a measure of the difficulty experienced by ATCs in controlling air traffic is fundamental to evaluate how the current ground-based ATM system is operated, and that it could also provide guidelines on how to obtain more manageable sectors by reconfiguring the airspace and by modifying traffic patterns. In [29], it is suggested that the capacity of an ATCC could be increased by a timely prediction of the traffic complexity bottleneck areas and a reconfiguration of the traffic patterns so as to evenly balance traffic complexity between sectors. A "complexity resolution" algorithm is introduced for dynamically modifying flight profiles to reduce the predicted complexity of more critical sectors and balance the complexity of adjacent sectors. In [86], a methodology for optimal design of airspace sectors is proposed. The airspace is partitioned into hexagonal cells and each cell is assigned a workload measure. Then, the airspace sectors are constructed by clustering algorithms using optimization methods. In [65], indicators of sector workload are studied, that could be operationally useful to Traffic Management Coordinators (TMCs) taking decisions affecting how much traffic ATCs will have to handle as well as traffic complexity.

In the current practice, complexity of air traffic is commonly evaluated in terms of number of aircraft and on a per-sector basis, [83]. Many researchers have found that air traffic indicators other than the number of aircraft are relevant to ATC workload. For instance, depending on the air traffic structure, ATC perceives situations with the same number of aircraft in the sector as different. A list of "complexity factors" is provided in the literature review [37]. Most researchers agree that complexity depends on both structural and flow characteristics of air traffic, [67, 37]. The former are fixed for a sector and given by spatial and physical attributes such as terrain configuration, number of airways, airway crossings and navigation aids (*static air traffic characteristics*). The latter vary as a function of time and depend on features such as number of aircraft, weather, aircraft separation, closing rates, aircraft speeds, mix of aircraft and flow restrictions (*dynamic air traffic characteristics*). These static and dynamic factors interact in a nonlinear complex way to produce air traffic complexity, [4, 59, 13]. Nevertheless, according to the literature review [37], most of the complexity metrics developed to date depend heavily on the traffic density alone.

Besides planning and redesign, improved measures of complexity could be of use for the evaluation of air traffic management productivity, and the assessment of the impact of new tools and procedures, [37].

It is perhaps worth mentioning that air traffic complexity has been studied in relation not only with controller workload, but also with such different issues as:

- the occurrence of operational errors (events where two or more aircraft violate the separation standard and the cause is attribute to ATC) or incidents, [67, 36, 11, 76, 77, 84, 70];
- controller decision making, [68];
- the design of decision support and flight planning tools [19, 64, 78];
- conflict risk [3, 48, 80].

2.2 Approaches to air traffic complexity modelling and prediction within ground-based ATM

Most studies on air traffic complexity in the literature have been developed with reference to ground-based ATM. In this section, we review selected approaches to air traffic complexity. For a more extensive overview, we suggest the comprehensive literature reviews [67, 37].

Strengths and weaknesses of each approach are presented within its description. A classification of the approaches based on characteristics that are relevant to autonomous aircraft ATM is postponed to Section 2.3.

2.2.1 Aircraft density

The number of aircraft in a sector is the air traffic characteristic that has been most cited, studied, and evaluated in terms of its influence on workload. It is, at the same time, considered as the best available indicator of complexity and criticized for not being able to appropriately characterize what controllers find complex. Currently, it is the complexity measure most adopted in practice, possibly because it is easier to interpret than other complexity measures: if the aircraft number exceeds the operationally-defined threshold by four aircraft, then the situation is classified to be of high-workload, and it can be resolved by removing four aircraft.

Sectors are designed so that the controllers are able to handle the usual flow of traffic. In the event of increased demand or re-routing required due to weather conditions or special use airspace constraints, Traffic Flow Management (TFM) techniques such as staff reallocation and alternative airspace configurations are used for maintaining the ATCs workload constant so as not to compromise safety and efficiency levels. In the United States, the peak aircraft count (the largest number of aircraft in a sector during any minute of a 15 minutes time interval) is compared with an acceptable peak traffic count value determined by traffic flow managers based on practical experience, and adopted for operational TFM decisions like re-routing flights out of an overloaded sector, [65]. The drawbacks of this measure are that it is insensitive to the duration of a high workload period, it is very sensitive to the entry and exit times of a few flights which do not change the amount of sustained workload. In addition, this measure does not take into account such factors as the traffic pattern, traffic mix, weather, etc., that may greatly influence the actual workload levels experienced in practice. Finally, it is important to remark that operational errors are more likely to occur after rather than during a peak in traffic count, as suggested in [76], a simulation study involving a human-in-the-loop.

The European Flow Management Positions (FMP) staff determines the airspace configuration schedule (successive aircraft configuration during the day) by splitting or merging sectors based on the number of ATCs on duty and the traffic load assessed by means of flight counts and sector capacities. In [32], it is suggested that more realistic airspace configurations could be obtained by adopting more appropriate complexity metrics.

The Enhanced Traffic Management System (ETMS) is a decision support system for traffic management whose monitor/alert function is based on a comparison of the prediction of traffic volume in the sector against some established threshold volume representing the maximum number of aircraft that the ATCs are willing to accept in that sector. Threshold volume, however, does not adequately represent the actual ATC workload since, in certain circumstances, controllers accept traffic beyond the threshold, whereas, in other circumstances, they reject it although the number of aircraft is well below the threshold. The level of organization of the traffic should be considered jointly with the air traffic volume, since, depending on the air traffic structure, ATC perceives situations with the same number of aircraft in the sector as different. It is recognized by the Radio Technical Commission on Aeronautics (RTCA) that this monitor/alert function should be integrated with more precise measures of sector complexity and controller workload to adequately represent the level of difficulty experienced by the controllers under different traffic conditions, [1]. Also, the behavior in time of the traffic volume affects workload: a traffic volume that highly fluctuates over time is more likely to generate conflicts and appears more complex to the controller than a uniform traffic flow, [27]. Air traffic controllers adapt their strategy to regulate the workload as the traffic volume increases, sacrificing secondary objectives in order to maintain their principal objectives, [81, 82]. For example, in a low traffic situation, they take into account performance objectives when solving conflicts, whereas, as the traffic level increased, they are only concerned with guaranteeing the appropriate separation between aircraft. Moreover, controllers select operating procedures based on economy, i.e., as air traffic density increases, they use less costly procedures to avoid overload.

2.2.2 Dynamic density

Dynamic density, [46, 56, 83, 49, 50, 64, 65], is a metric for assessing the controller activity level in a sector that was introduced within a research program in U.S.

involving FAA, NASA, MITRE, and Wyndemere Corporation as main participants. Laudeman et al from NASA, [56] defined dynamic density as "a measure of control-related workload that is a function of the number of aircraft and the complexity of traffic patterns in a volume of airspace".

Dynamic density is a single aggregate indicator obtained as a linear combination of traffic density and other controller workload contributors (the number of aircraft undergoing trajectory change and requiring close monitoring due to reduced separation) identified through interviews to several qualified air traffic controllers.

More precisely, dynamic density is the weighted sum of the number of aircraft and the following aggregate indicators of the aircraft changing geometries during a one-minute sample time interval:

- the number of aircraft with heading change > 15 degrees,
- the number of aircraft with speed change > 0.02 Mach,
- the number of aircraft with altitude change > 750 ft,
- the number of aircraft with 3-D Euclidean distance between 0-5 nautical miles excluding violations,
- the number of aircraft with 3-D Euclidean distance between 5-10 nautical miles excluding violations,
- the number of aircraft with lateral distance between 0-25 nautical miles and vertical separation $< 2000/1000$ feet above/below 29000 ft,
- the number of aircraft with lateral distance between 25-40 nautical miles and vertical separation $< 2000/1000$ feet above/below 29000 ft,
- the number of aircraft with lateral distance between 40-70 nautical miles and vertical separation $< 2000/1000$ feet above/below 29000 ft.

The weights were determined both by subjective ratings obtained showing different traffic scenarios to the interviewed controllers and by regression analysis of controllers activity data. The dynamic density measure with subjective weights was validated in an operational environment and showed to be highly correlated with observed controllers activity, more than the traffic volume.

Note that the complexity indicators entering the dynamic density metric depend on the aircraft trajectories during a one-minute sample time interval and do not include observed metrics of the ATCs physical work such as data entry and radio communications. By using trajectory prediction tools, it is then possible to project the dynamic density measure over a suitable time horizon so as to forecast future workload levels and use this information for traffic management purposes.

In [83], the future values of the dynamic density in a sector are computed based on the aircraft positions and speeds predicted by the Center-TRACON Automation System (CTAS, [26]) using aircraft dynamic models, flight plans, radar tracks within the Air Route Traffic Control Center (ARTCC), and weather data.

Apparently, dynamic density can be accurately predicted 5 minutes in advance (short-term prediction). The long-term prediction over a 20 minutes time horizon is affected by errors that can be reduced by integrating in CTAS inter-Center data on aircraft entering the ARTCC. The performance obtained in the 20 minute range prediction suggests that there is further room for improvement, both in the trajectory prediction tool and in the dynamic structure of the model. Sources of prediction errors are aircraft departures within the considered sector, wind prediction and radar tracker. Also, prediction does not take into account the ATCs action (open loop prediction).

There are some important weaknesses regarding dynamic density to be aware of. First of all, the computed weights are extremely variable from sector to sector and therefore need to be re-estimated and re-validated for each sector (and possibly periodically retuned). Also, the proposed dynamic density model for air traffic complexity is actually a static model, that does not incorporate explicitly neither future predicted aircraft topology, neither past state information. The predictions of the dynamic density future behavior is calculated with the same equation using the predicted values of the complexity factors involved as provided by the adopted trajectory prediction tool. Therefore, the prediction capabilities allegedly attributed to the dynamic density indicator are in fact a merit of the prediction tool. Furthermore, it is difficult for decision makers as Traffic Management Coordinators (TMCs) to understand from a single aggregate measure how to solve a high-workload situation. Information on which complexity factor has caused the problem is in fact lost. On the other hand, having too many complexity factors to analyze may slow down the decision process due to overload in information processing. Potentially nonlinear relations between complexity indicators are missed, [38]. The results of the dynamic density work could be improved by adopting non-linear techniques, including neural networks, genetic algorithms, and non-linear regression, [50]. Finally, the adopted indicator of the controller workload used to determine the weights is critical. Behavioral measurements miss the cognitive aspects of controller activity. Subjective ratings are often subject to biases.

Dynamic density is used in a variety of contexts in the literature and does not correspond to a single metric. Overall, there is a significant consensus over 20-30 dynamic density metrics. Most of the factors used in dynamic density models are dynamic traffic characteristics that are generally useful for realtime decision support. Various linear and nonlinear methods are used to perform the correlation.

A human-in-the-loop simulation study with controllers actively controlling traffic in a real-time simulation environment was performed in [52] with the two-fold objective of introducing a new dynamic density complexity model and of validating dynamic density versus aircraft count. In [51], it was also shown that the measurement of complexity using the dynamic density is better than a simple aircraft count for both the instantaneous and the predicted complexity measures.

In [65], the use of dynamic density as an operationally useful sector workload measure for enabling TFM personnel to prevent overloads is investigated. The dynamic density metric is defined as the weighted sum of multiple sector workload factors. 12 complexity factors out of a set of 41 were selected based on their correlation to the subjective measure of the workload experienced by ATCs and their predictability, avoiding redundancy. The study of the dependence of the resulting dynamic density metric on the considered airspace reveals that different factors contribute to the perceived difficulty in different centers.

The instantaneous positions and speeds of the traffic itself do not appear to be enough to describe the total complexity associated with an airspace. Efforts to define dynamic density have identified the importance of a wide range of potential complexity factors, including structural considerations. A few previous studies have attempted to include structural consideration in complexity metrics, but have done so only to a restricted degree. The importance of including structural consideration has been explicitly identified in work at Eurocontrol. In a study to identify complexity factors using judgement analysis, Airspace Design was identified as the second most important factor behind traffic volume [54]. Histon et al. [39, 40] investigated how this structure can be used to support structure-based abstractions that controllers appear to use to simplify traffic situations (cognitive aspect affecting workload). Within the dynamic density research program, the Wyndemere Corporation proposed a dynamic density metric that included a term based on the relationship between aircraft headings and the dominant geometric axis in a sector [46]. Also, specific emphasis on the traffic and airspace characteristics that impact the cognitive and physical demands placed on the controller was given. An attempt was made to include the level of knowledge about the intent of the aircraft.

Many approaches to air traffic complexity characterization by a dynamic density model similar to that in [56] are proposed in the literature (see, e.g., the survey [37]), where various sector and aircraft status indicators are correlated to the perceived workload on a relevant dataset of ATC activity recording. In [39], sector workload factors to be combined in a single workload metric are classified into airspace design factors, dynamic traffic characteristics, and operational factors, and a list per category is provided. The importance of representing complexity in a way that can help TMCs deciding on actions affecting the ATCs workload is pointed out in [65].

2.2.3 Interval complexity

Recently, the interval complexity of a sector was introduced as an estimate of the ATC workload in that sector, [29]. The interval complexity of a sector is defined as the average over a time window of the linear combination of the following complexity factors: number of aircraft flying within the sector, number of aircraft flying on nonlevel segments, and number of aircraft flying close to the border. Nonlevel flights and flights close to the boundary of a sector in fact require special attention and procedures to be followed by the ATC. The weights in the linear combination depend on the specific sector. Interval complexity can be considered as a smoothed version of a dynamic density-like complexity measure. The prediction of this measure of complexity over a time horizon of 20 to 90 minutes is used for selecting appropriate "complexity resolution" actions minimizing and balancing traffic complexities between adjacent sectors of a certain airspace region.

2.2.4 Fractal dimension

A characterization of traffic structure based on the fractal dimension of the traffic pattern has been proposed in [69]. Fractal dimension is a metric suggested for comparing traffic configurations resulting from various operational concepts. It allows in particular to decouple the complexity due to airspace partitioning in sectors from the complexity due to traffic flow features.
The dimension of (compact) geometrical figures is well-known: a curve is of dimension 1, a surface of dimension 2, and so on. It is quite simple to derive those integers from a covering measure since the minimal number of balls of radius ϵ needed to cover the object will evolve roughly as $\left(\frac{1}{\epsilon}\right)^d$ as $\epsilon \to 0$, d being the dimension. Fractal dimension is simply the extension of this concept to more complicated figures, whose dimension may not be an integer. The block count approach is a practical way of computing fractal dimensions. It consists in considering a rectangular grid of size ϵ and counting the number N of blocks of linear dimension ϵ covering the given geometrical entity. Then, the fractal dimension of the geometric entity is defined as:

$$d = \lim_{\epsilon \to 0} \frac{\log N}{\log(\frac{1}{\epsilon})}.$$

The application of this concept to air route analysis consists in computing the fractal dimension of the geometrical figure composed of existing air routes. Currently, aircraft cruise on linear routes at specified altitudes, corresponding to a geometrical dimension of 1. In the future, it is expected that flights will be allowed to move from these linear routes. If all of the airspace was covered by routes, the fractal dimension of the future route structure would be 3. However, there will still be preferred routes (due to the position of connected airports, or to wind currents, etc.), thereby decreasing the actual dimension of the route structure.

An analogy of air traffic with gas dynamics then shows a relation between fractal dimension and conflict rate (number of conflicts per hour for a given aircraft). Fractal dimension also provides information on the number of degrees of freedom used in the airspace: a higher fractal dimension indicates more degrees of freedom. This information is independent of sectorization and does not scale with traffic volume. Fractal dimension is thought to be an aggregate metric for measuring the geometrical complexity of a traffic pattern, and is an example of complexity metric indepenedent of workload aspects. The important point about fractal dimension is that it is a long term structural complexity metric: fractal dimension must be thought as a geometrical feature of a limit shape obtained by observing trajectories on an infinite time period. The fact that timing information is a main limitation of fractal dimension as a complexity measure.

2.2.5 Input-output model

In [57, 58], air traffic complexity is defined in terms of the control effort needed to avoid the occurrence of conflicts, i.e., of those situations where the relative aircraft distance gets lower than a given safe distance, when an additional aircraft enters the airspace. For this purpose the authors introduce an input-output system consisting of the air traffic within the region of the airspace under consideration and a feedback controller, similarly to Figure 2, with the ATC replaced by an automatic solver and the airspace sector by an airspace region. The input to the closed-loop system is represented by the (fictitious) aircraft entering the airspace region, whereas the output is given by the deviation from their original flight plans issued by the feedback controller to the aircraft already present in the airspace so as to avoid conflicts. The deviation imposed by the controller is taken as measure of the air traffic complexity.

Each aircraft i is described by a very simple 2D kinematic model:

$$\dot{x}_i = V_i \cos\theta_i \quad \dot{y}_i = V_i \sin\theta_i$$

where (x_i, y_i) denotes the aircraft position at some fixed altitude, V_i the speed, and θ_i the heading. For the computation of the air traffic complexity it is assumed that a conflict solver is available as controller and that every aircraft can instantly change the heading θ_i but has to keep the speed V_i constant. Based on these assumptions, complexity is computed as follows: introduce an additional aircraft in the traffic at a given point with an arbitrary bearing, launch the conflict solver for the obtained air traffic situation, and count the overall number of manoeuvres needed to recover a conflict-free condition.

A solver based on mixed integer programming is used. This solver determines the conflict resolution maneuvers that minimize the total heading change. Complexity can then be measured as the total change in heading summed over all aircraft, and a "complexity map" as a function of the entering aircraft position and bearing can be built. A scalar measure of air traffic complexity, e.g., the “worst-case” value for the control activity, can be extracted from the complexity map.

Note that different measures of the control effort and different solvers could be used, and that the choice of the conflict solver has a large impact on complexity evaluation.

2.2.6 Intrinsic complexity measures

Some researchers were not so inclined to acknowledge a direct cause-effect relation between complexity and workload, and also that the relationship between the two can be adequately expressed mathematically. This has led to a radically different view of the complexity issue, which aims at building metrics of the “intrinsic” complexity of the air traffic distribution in the airspace, without incorporating any measure of the ATC workload, [21]. According to this viewpoint, complexity metrics should capture the level of disorder as well the organization structure of the air traffic distribution, irrespectively of its effect on the ATC workload.

Two classes of intrinsic complexity metrics are presented in [21], both based on the (objective) measurements of the aircraft velocities and positions. The first class consists in a geometrical approach where complexity is a function of the relative position vectors and relative velocity vectors of the aircraft. The second class describes traffic flow organization using the topological Kolmogorov entropy of a dynamical system modelling air traffic.

The approach based on topological entropy was further developed in later works, [20, 23, 22], where the authors explore both linear and nonlinear system modelling of air traffic to derive topological entropy measures for air traffic complexity characterization. The limitations of the linear modelling-based approach is that it provides only a measure of the global tendency of the traffic, and that it does not fit exactly with all traffic situations. Then, nonlinear extension can be used to produce maps of local complexity thus allowing for the identification of critical air traffic areas. This approach is discussed in detail in Chapter 3.

Inspired by [21, 23, 22], in [47] an interpolating velocity vector field is determined based on a snapshot of the air traffic, with each aircraft represented by a point at a certain position and with a certain velocity. The interpolating vector field should satisfy some constraints related to maneuvers feasibility (minimum and maximum speed, and continuity to limit acceleration and turn rate). If a smooth vector field is found, aircraft can follow non intersecting trajectories and the introduction of an additional aircraft causes a marginal increase in complexity. If no smooth solution is found, then the continuity constraint is relaxed, which leads to the introduction of a separation boundary where the vector field loses continuity. Complexity can be evaluated based on the representation of the resulting vector field. The location of the separation boundary corresponds to critical areas. The main challenge of the approach is computing the separation boundary in real-time.

2.3 Discussion on the approaches to air traffic complexity

2.3.1 The workload issue

The literature on air traffic complexity refers mainly to the current ground-based ATM system, and studies air traffic complexity as a means to quantify the ATC workload, intended in general terms as the difficulty perceived by ATCs in safely handling air traffic. Given a certain air traffic situation, a measure of the air traffic complexity should be computed based on the available information on the air traffic characteristics so as to provide an indicator of the expected ATC workload.

A main issue that makes the problem difficult (and actually not so well-posed) is that a clear and globally accepted definition of ATC workload is actually not available in the literature, as pointed out in [60] stating that "controller workload is a confusing term and with a multitude of definitions, its measurement is not uniform". Workload depends both on the difficulty and demands of a task (task load) and on the effort in terms of physical and mental activities required to accomplish the task, [36]. ATCs use spatial and temporal traffic patterns seen on the display along with their domain knowledge for controlling air traffic. As shown in Figure 3, the relationship between air traffic complexity and ATC workload is an indirect one that is highly mediated by the influence of cognitive strategies and individual variables of the controller, and quality of the equipment, [67]:

- The amount of workload experienced by ATCs is modulated by the information processing and decision-making strategies adopted, [81, 55, 17, 37]. There are very few constraints on how controllers should handle traffic beyond that of maintaining adequate separation between aircraft. Thus the space of possible solutions is large and can accommodate a variety of resolution strategies. Controllers use more economical procedures to address more difficult tasks, and the knowledge of appropriate procedures depends on training and job experience. As traffic volume increases, ATCs adapt their information processing and decision-making strategies in an attempt to regulate workload. Controllers use more economical control procedures and more standard strategies to control air traffic at higher traffic densities, often resorting to heuristics developed with the experience. They react to task load fluctuations with compensatory strategies such as shedding or deferring tasks, prioritizing tasks, or becoming more cautious in bad weather, [53]. In this way, they preserve their cognitive resources available for the task.

- Workload results from the task/operator interaction, which is largely variant depending on the specific task and the specific operator. Even for a given task/operator pair the workload may vary due to individual factors such as age, skill, experience, and anxiety level, and contingent factors such as time pressure, noise, stress, distraction. The workload history affects the perceived complexity as a long period of heavy load tends to reduce the ATC efficiency.

- The ATC software equipment also influences the workload. The way information is displayed, in particular, affects the ATC capabilities of processing information and abstracting the underlying structure of a traffic pattern, which is essential to understanding and simplifying it, [41]. The way information is presented to ATCs and the availability of automation tools may influence the workload.

Figure 3: Mediating factors affecting the impact of air traffic complexity on workload.

According to the model discussed in [37], the ATC activity consists of four elements: monitoring, evaluating, formulating decisions, and implementing decisions, [71], which maps into the Input, Processing, and Response (I, P, and R) phases of human information processing. The P-phase of the ATC activity can eventually involve the support of automated tools. Workload is the response to the I-P-R effort mediated by Performance Shaping Factors (PSFs) such as skill, fatigue, age, training, proneness to anxiety, etc. The ATC workload is a function of both the task load (determined not only by the complexity of the air traffic situation but also by the available interfaces for sensing the air traffic situation and for implementing the control strategy) and the internal and subjective response to task load.

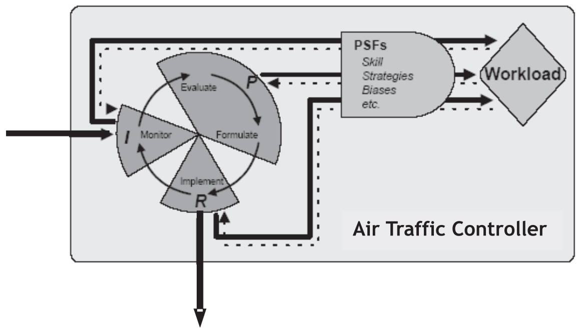

Figure 4: Model of the Air Traffic Controller, [37].

In many approaches, by empirically correlating the measured workload to the available measurements of airspace configuration and traffic patterns, workload and air traffic measurements are incorporated within a single aggregate complexity indicator for the purpose of describing the ATC perceived complexity (see Figure 5).

While nearly all of the studies found a statistically significant correlation between air traffic factors and workload, not all airspace factors were related to the same measure of workload, which makes difficult to compare the different approaches to air traffic complexity. Clearly, the choice of the variable that measures the workload is crucial to determine how well complexity is actually evaluated.

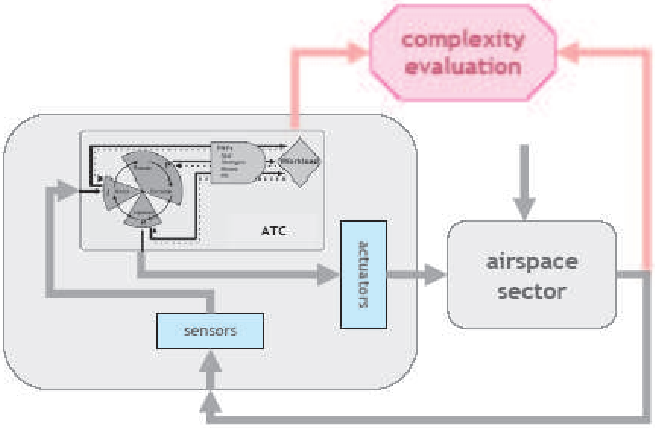

Figure 5: Complexity evaluation within ground-based ATM.

Workload can be measured using direct subjective indicators (such as self-report measures and controller's rating obtained through some questionnaire) or indirect indicators including behavioral/physical activity (e.g., key strokes, slew ball entries, number of control actions, communication time, decision and action frequency) and physiological (e.g., EEG/EMG/EOG, blood pressure, heart rate measures, eye blink rate, respiration, biochemical activity, pupil diameter) indicators. Various data collection methods are listed in [37].

Workload is often indirectly and partially evaluated by measuring task performance such as time to perform discrete ATC tasks, [14]. In this way, the cognitive process (including planning, decision making, strategy, and factors such as skill, training, experience, fatigue, etc.) of the controller is masked. A list of "workload indicators" is provided in the literature review [37]. The issue of evaluating ATC performance and workload based on data collected from operational and simulated air traffic control is discussed in [75], where an extensive taxonomy of air traffic control measurements adopted in the literature is provided.

Subjective measures suffer from several drawbacks, such as memory effects, unwillingness to report damaging information, etc (however, it should be noted that workload is –after all– a subjective reaction to traffic complexity). In [62, 63], the relevance of measures of the controller activity for realtime decision support is evaluated. Communications measures are found to be correlated with subjective workload, but not to provide any incremental benefit when used for prediction of its future behavior. Certain physical parameters related to the interaction with the workstation (route displays and strips requests) are found to be not well correlated with controller performance and mental workload, whereas others (such as data entries) as significantly correlated.

With respect to behavioral measures it should be noticed that controller activity is not necessarily related to effort, since controllers may employ strategies to maintain specific traffic patterns. Also, most workload is actually unobservable mental activity. The monitoring, evaluating, and planning activities involved in the ATC task generate mental effort, but only the implementing action is directly observable (see Figure 4). In any case, both direct and indirect workload measurements are very expensive to collect, since they require the active participation of controllers.

Notice also that there does not exist any such thing as a "standard" or "average" controller, to use for standardized workload evaluation, and it seems hazardous to maintain that a complexity measure is well correlated with workload without taking into account the experience and skill of the operators involved in the workload assessment.

In [79] and [2], an attempt is made to model human-machine interaction for ATC according to a system engineering approach. Queuing theory is used in [79] to analyze ATC workload and the resulting mathematical model is validated on ATC operational data to predict average delay and server occupancy as a function of demand. A control theory-based approach is considered in [2] to describe the ATC system. Apparently, the proposed model involving different functions (planning, controlling, communicating, and data management) has not been empirically evaluated.

Finally, as in [32, 33, 34], the airspace configuration, with sectors possibly split or merged, is suggested as a measurable variable of the ATC workload to determine those factors that are relevant to complexity evaluation through a correlation analysis based on neural networks.

2.3.2 Classification of the revised approaches

It is difficult to compare the results from all the different studies on air traffic complexity because of the wide variety of indicators used to assess it. What is more, indicators of air traffic complexity are often mentioned "at a high level" without precisely defining them and specifying how they should be measured, especially in non-empirical works, [15].

Here, we focus on the approaches reviewed in Section 2.2 and classify them with respect to some characteristics that are relevant to the autonomous aircraft application.

As discussed before, those complexity metrics where workload and air traffic measurements are incorporated within a single aggregate indicator for the purpose of describing the ATC perceived complexity depend on the specific notion and measure of workload adopted, and inherently incorporate various human factors aspects. Also, such workload-oriented metrics are sector-based (in [29], reference is even directly made to complexity of a sector as an estimate of the ATC workload of that sector), and often show structural dependence on the sector characteristics, which further limits their applicability to a sector-free context such as autonomous aircraft ATM.

The various dynamic density-like metrics and the interval complexity are clearly workload-oriented and sector-based measures of complexity. The same holds for the aircraft density, since complexity is evaluated by comparing the number of aircraft in a sector with a threshold determined based on the capabilities of ATCs to safely handle air traffic in that sector.

The difficulty in obtaining reliable workload measures has been one of the strongest motivations for investigating complexity metrics independent of the ATC workload in the context of ground-based ATM. These metrics can be control-dependent or control-independent, in that the evaluation of complexity can explicitly account for the controller in place or not. The input-output model is a control-dependent metric, since complexity is evaluated in terms of control effort, whereas fractal dimension and intrinsic complexity measures are control-independent. All workload-oriented metrics are obviously control-dependent.

Control-independent metrics do not require the knowledge of the controller in place, which is indirectly accounted for through the effect of its action on the air traffic organization. This makes them better suited for the airborne self-separation framework.

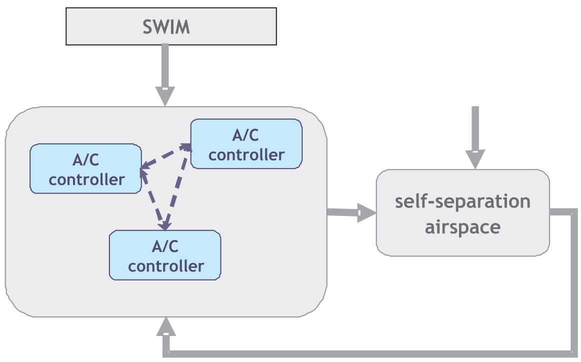

Figure 6: Control scheme in autonomous aircraft ATM.

In autonomous aircraft ATM, control is delegated to the aircraft according to a partially decentralized control scheme where aircraft eventually communicate one with the other (air-to-air communication) and with the ground (air-to-ground communication) to get information on the other aircraft intent through the System Wide Information Management (SWIM) system, as schematically represented in Figure 6. As a result, the controller has a decentralized time-varying structure, difficult to characterize for the purpose of control effort evaluation, and possibly involving the pilots in the decisions on trajectory management and conflict resolution. A notion of complexity explicitly accounting for a human-in-the-loop component would present the drawbacks that are common to all system engineering approaches applied to model human-machine interaction, that is:

i) Decision making or planning are generally unobservable processes and measurements are largely unreliable (due, e.g., to time varying human behavior and to differences between individuals).

ii) Humans typically do not "optimize" but rather try to "satisfy" the requirements, choosing a solution that is satisfactory though not necessarily the best one. As a consequence, prediction of decision making processes is hard.

iii) Analytic modeling of human behavior is strictly related to the specific context (pervasiveness of task environment).

Thus, a control-independent measure of complexity appears to be the better suited in view of the introduction of an airborne self-separation ATM system.

For those complexity measures that are computed based on the air traffic state rather than the whole aircraft trajectories, complexity prediction can be obtained by projecting in the future the air traffic state and recomputing the complexity measure, whereas those measures that take into account the aircraft trajectories are naturally evaluating complexity over a prediction time horizon. In any case, the reliability of the resulting complexity prediction depends on that of the aircraft trajectories prediction. Surprisingly, it seems that uncertainty in the trajectory prediction is not accounted for in any of the (deterministic) approaches proposed in the literature.

Depending on the reference time horizon, complexity metrics could be useful for conflict resolution or for trajectory planning (tactical and strategic decisions). In this respect, those approaches providing a spatial complexity map could help isolating critical areas and support trajectory management. The practical impact of those approaches providing a scalar aggregate indicator of complexity is instead not so clear in most cases.

A further aspect that is interesting to point out is if and how the metric takes into account the organization of traffic, which is recognized as an important factor in the assessment of complexity.

Table 1 is a schematic classification of the approaches reviewed in Section 2.2 based on the aspects mentioned above, that is: the data required for complexity evaluation; the output provided; the reference time horizon; dependence on the controller, on the sector, and on the traffic organization. A rough evaluation of the computational load is also given, which is obviously an important information for the design of a time critical task such as air traffic management.

Table 1: Classification of the approaches to air traffic complexity.

metric	**required data**	**output type**	**time horizon**	**control-dependent**	**sector-based**	**dependent on air traffic organization**	**computational load**
aircraft density	number of aircraft in the sector	aggregate indicator (time dependent)	short term (extendable with trajectory prediction)	yes (through the threshold on the number of aircraft)	yes	no	small
dynamic density	number of aircraft and other aggregate indicators of traffic distribution and aircraft changing geometries, sampled over 1 minute	aggregate indicator (time dependent)	short term (extendable with trajectory prediction)	yes (the relative weights of the complexity factors are workload dependent)	yes (through the complexity factors and their relative weights)	partially	significant when defining the relative weights, small in the on-line usage
interval complexity	number of aircraft and other aggregate indicators of traffic distribution, sampled over 20–90 minutes	aggregate indicator (time dependent)	medium/long term	yes (the relative weights of the complexity factors are workload dependent)	yes (through the complexity factors and their relative weights)	slightly	significant when defining the relative weights, medium in on-line usage (trajectory prediction is needed)
fractal dimension	aircraft trajectories	aggregate indicator	long term	no	no	yes	significant
input-output model	aircraft trajectories	complexity map, representing the control effort to accommodate a new aircraft as a function of its initial conditions	short/medium term	yes	no	yes, indirectly	high
intrinsic metric based on Lyapunov exponents	aircraft trajectories	complexity map representing the largest Lyapunov exponent as a function of space	short/medium/long term	no	no	yes	high

2.4 Concepts related to air traffic complexity

2.4.1 Trajectory flexibility

A concept that is intimately related to air traffic complexity is that of flexibility of the aircraft trajectory, i.e., the extent to which a trajectory can be modified without causing a conflict with neighboring aircraft or entering a forbidden airspace area.

The use of flexibility measures in an airborne self separation framework is the object of an ongoing research activity by NASA [44, 45, 43]. In the short/medium term flexibility is used as a criterion to rate different conflict resolution maneuvers, so that the adopted solution is the easiest to adapt to unexpected behavior by intruder traffic. In the long term horizon, a flexibility preservation function is adopted to plan the aircraft trajectory by minimizing its exposure to disturbances such as weather cells and dense traffic areas.

Flexibility is evaluated in terms of two different characteristics, namely robustness and adaptability to disturbances. Robustness is defined as the ability of the aircraft to keep its planned trajectory unchanged in response to the occurrence of a disturbance, and is measured as the fraction of operationally feasible trajectories that keep feasible. Adaptability is defined as the ability of the aircraft to change its planned trajectory in response to the occurrence of a disturbance that makes the current planned trajectory infeasible. Adaptability is measured as the amount of feasible trajectories that avoid the conflict while respecting all the navigational constraints.

The work [43] discusses computational aspects related to the actual computation of such metrics in a probabilistic setting. Simplified scenarios are explored, where a 2D space is considered and only 1 degree of freedom (e.g., speed or path stretch) is analyzed at a time for trajectory modification. The concepts, however, are extendable to a more general setting.

2.4.2 Aircraft clustering

Aircraft clustering consists in identifying groups of closely spaced aircraft and was originally studied in connection with conflict resolution. With reference to the conflict resolution problem, clustering should isolate all the aircraft involved in multiple conflicts close in time and ensure that solving conflicts within each cluster separately will not generate any new inter-cluster conflict.

Assuming that aircraft density is a relevant factor for complexity characterization, [37, 54], cluster identification can complement and accelerate complexity assessment by isolating those airspace areas where concentrating the attention. Once a high density area is identified by aircraft clustering, the following step is to evaluate its complexity, so as to eventually determine those critical airspace areas that should be avoided.

Clusters play within the advanced autonomous ATM system a role similar to sectors within the centralized human-operated ATM system. Under current operations, airspace congestion refers to a sector that cannot accept additional aircraft due to ATC workload limitations. In view of automated separation assurance, which is independent of the airspace geometry, a methodology for identifying aircraft clusters is suggested in [7, 6] as a first step towards obtaining a sector-independent evaluation of the airspace congestion: aircraft clusters are isolated first and then congestion is evaluated based on cluster complexity.

We next briefly illustrate the main contributions in the literature on aircraft clustering, making the description uniform by adopting a common mathematical framework. No evaluation of which approach is best suited to support complexity evaluation in the autonomous aircraft case is given at this stage.

Let A be a set of aircraft in the airspace $\mathcal{S} \subset \mathbb{R}^3$ of interest (in our case the self-separation enroute airspace), and $T \subset \mathbb{R}^+$ some reference time interval. The flight of aircraft $a \in A$ can be described through some function $p_a = (x_a, y_a, z_a) : T \to \mathcal{S}$, where $p_a(t) = (x_a(t), y_a(t), z_a(t))$ represents the position of aircraft a at time $t \in T$. We denote by $C_T^{h,v} \subseteq A \times A$ the collection of aircraft pairs that get closer than some distance specified in terms of horizontal and vertical separation, during the time interval T, i.e.

$$c_{ab} := (a,b) \in C_T^{h,v}$$
$$\Updownarrow$$
$$\exists t_s, t_f \in T : d_e\big((x_a(t), y_a(t)), (x_b(t), y_b(t))\big) < h \wedge d_e(z_a(t), z_b(t)) < v, \, \forall t \in [t_s, t_f]$$

where $d_e(w_1, w_2)$ denotes the Euclidean distance between vectors w_1 and w_2 and h, v are some positive constants.

Note that $C_T^{h,v}$ is a relation over A that is symmetric (if $c_{ab} \in C_T^{h,v}$ then $c_{ba} \in C_T^{h,v}$, $a, b \in A$) and reflexive ($c_{aa} \in C_T^{h,v}$, $\forall a \in A$), but not transitive. The transitive closure of $C_T^{h,v}$ allows to partition set A in equivalence classes, each class, say $E = \{a, b, c\}$, corresponding to a cluster of aircraft that get close one to the other directly ($c_{ab} \in C_T^{h,v}$) or indirectly (c_{ac}, $c_{cb} \in C_T^{h,v}$ but $c_{ab} \notin C_T^{h,v}$), [35]. When clustering is used for the purpose of conflict resolution, h and v can be set equal to the minimum safe horizontal and vertical distances, and clustering constitutes a preliminary step towards a global rather than a pairwise approach to conflict resolution, which may turn out to be the only way to guarantee safety.

The aim of [35] is to study cluster structure, compare clusters in traffic with direct routes to the one with standard airways, and study the sensitivity of cluster size to simulated uncertainties on trajectories forecast. In [35], clusters are represented by graphs where aircraft are nodes and conflicts are edges. Several ways of assessing the structure of clusters are introduced:

- the number of nodes;
- the number of edges;
- the diameter of the graph of the cluster;
- the graphic sequence

The graphic sequence is defined as a sequence of numbers that can be the degree sequence of a graph (a degree of a node is the number of attached edges).
The paper first presents how structure variability (in terms of graphic sequences) grows with the number of aircraft in a cluster. Then the two scenarios (traffic with direct routes and traffic with standard airways) are simulated with the conclusion that direct routes traffic results in less conflicts. On the other hand, the cluster diameter as a function of the number of aircraft is slightly larger for direct routes scenario. Experiments with added uncertainty showed that more clusters are detected, therefore a good trajectory prediction/knowledge is needed in order not to overwhelm the conflict resolution module.

The disadvantage of this approach is that if aircraft a and b are close up to time $t_f(c_{ab})$ and then b and c start being close at $t_s(c_{bc})$, all three aircraft belong to the same cluster, even if $|t_s(c_{bc}) - t_f(c_{ab})|$ is relatively big. To solve this issue, a new relation ρ is introduced in [16] for expressing the temporal proximity of conflicts defined by the relation $C_T^{h,v}$:

$$c_{ab}\,\rho\,c_{bc} \Leftrightarrow \min_{t' \in [t_s(c_{ab}),t_f(c_{ab})],t'' \in [t_s(c_{bc}),t_f(c_{bc})]} |t' - t''| < \Delta$$

where $\Delta \in \mathbb{R}$ represents a time threshold.
Each cluster of aircraft corresponds to one equivalence class of the transitive closure of the proximity relation ρ. A cluster corresponding to the equivalence class with representative c_{ab}, denoted as $[c_{ab}]_\rho$, is built by the union of all aircraft in it, i.e. as

$$\bigcup_{(r,s)\in[c_{ab}]_\rho} \{r, s\}$$

Note that in this case clusters do not have to be disjoint, and time specification is therefore needed, e.g., an aircraft can be part of a given cluster only within a specified time interval.

In [8], clusters are built as equivalence classes of aircraft where the equivalence is the transitive closure of the relation $C_T^{h,v}$ for $T = \{t^*\}$, for each fixed discretized time instant t^* in some look-ahead time horizon. Clusters with less than 5 aircraft are not taken into account, and such aircraft are put in so-called background traffic.

Project MAICA (Modelling and Analysis of the Impact of the Changes in ATM – briefly described in [73]) aimed at evaluating the impact of several changes in ATM (including autonomous aircraft) to ATM performance.

The definition of cluster in this project is rather vague (unfortunately, reports are not available, and all information is extracted from other reviews and references). Similarly to [16], aircraft clusters are built based on a proximity relation, which, however, considers both the temporal and spatial proximity of aircraft pairs defined by relation $C_T^{h,v}$. The idea is that aircraft a, b and aircraft c, d with $c_{ab}, c_{cd} \in C_T^{h,v}$ should belong to the same aircraft cluster only if the event where a and b get close one to the other (which is the reason why $c_{ab} \in C_T^{h,v}$) is both spatially and temporarily close to the event where c and d get close one to the other. Neither in this case the aircraft clusters have to be disjoint, therefore a time specification of the inclusion of an aircraft into a given cluster may be needed.

It is worth also mentioning the fact that innovative cluster definitions are proposed in the Master thesis of G. Aigoin, which is not available, but whose contribution is briefly described in [23].

3 A dynamical approach to intrinsic air traffic complexity characterization

As discussed in Chapter 2, air traffic complexity has been typically studied with the objective of quantifying the workload of air traffic controllers in handling air traffic. The workload of a controller is determined by two aspects: an intrinsic complexity related to the air traffic structure, and a subjective component related to the controller himself/herself (cognitive strategies and individual characteristics). Most complexity metrics proposed in the literature aim at capturing both these aspects within a single aggregate indicator of "perceived complexity". A measure of the intrinsic air traffic complexity aspect only is instead needed within a highly automated ATM system.

Site visits to air traffic facilities and a review of previously identified complexity factors suggests that organization in the distribution of aircraft positions and speeds can have an important effect on the perceived complexity of the traffic situation [39]. In particular, situations where the relative distances between aircraft do not change over time are more predictable and easier to control. These situations are classified as fully organized traffic. On the other hand, quasi-random situations are difficult to handle and are thus associated with high complexity.

In this section we describe a dynamical approach to complexity characterization: a complexity indicator which quantifies the level of organization of the air traffic is introduced by using tools borrowed from the theory of dynamical systems.

3.1 The underlying principle

In order to capture the complexity associated to a lack of organization, an air traffic situation can be modeled by an evolution equation, with the aircraft trajectories interpreted as integral lines of some dynamical system. The Lyapunov exponents of the dynamical system provide an indicator of the air traffic complexity, allowing for the identification of different organizational structures of the aircraft speed vectors such as translation, rotation, divergence, convergence, or a mix of them. Lyapunov exponents are a standard complexity measure adopted in dynamical systems theory. They are the natural generalization to time dependent linear differential equations of the eigenvalues for autonomous linear systems and characterize the growth rates of the solution. For systems described by nonlinear differential equations, they measure the rate of exponential convergence or divergence of nearby trajectories, and can be taken as indicators of the level of order/disorder of a system. Quantitatively, two trajectories with initial relative position δx_0 diverge as $e^{\lambda t}\|\delta x_0\|$, where λ is a Lyapunov exponent. The rate of separation can be different for different orientations of the initial separation vector δx_0. Thus, there is a whole spectrum of Lyapunov exponents. The number of Lyapunov exponents is upper bounded by the dimension of the state space of the dynamical system. The value taken by the Lyapunov exponents at a certain position represents the local contraction/expansion rate of the field. The larger is a positive Lyapunov exponent, the higher is the rate at which one loses the ability to predict the system response. Those areas where the traffic is predictable are then easily identified by plotting the *air traffic complexity map* that is the largest Lyapunov exponent as a function of the airspace position.

The claim is that high air traffic complexity is associated with high Lyapunov exponents.

The part which follows is largely taken from the draft paper [74] by S. Puechmorel under the iFly project. For more details on Lyapunov exponents, the reader is referred to [5].

3.2 Lyapunov exponents as complexity indicator

3.2.1 Definition and properties

We start by recalling some basic notions of dynamical system theory.

Definition 1 *Let $\mathcal{M} \subseteq \Re^n$ be a smooth manifold. A flow on $\mathcal{M}$ is a mapping $\psi : \Re \times \mathcal{M} \to \mathcal{M}$ such that:*

- $\psi(0, x) = x$, $\forall x \in \mathcal{M}$;
- $\psi(t + s, x) = \psi(t, \psi(s, x))$, $\forall x \in \mathcal{M}$, $\forall s, t \in \Re$.

The pair $(\mathcal{M}, \psi)$ constitutes a continuous time dynamical system with state space $\mathcal{M}$, and $x(\cdot) = \psi(\cdot, x_0) : \Re \to \mathcal{M}$ is the trajectory associated with $x_0 \in \mathcal{M}$, i.e., such that $x(0) = x_0$.

Remark 1 *We restrict here our attention to continuous time systems described by flows, which are the objects of interest for our complexity application; however, it is possible to extend the discussion to discrete time systems resulting in iterated maps. Most of the time extra assumptions are made on the flow. For example, measurability (resp. continuity) with respect to the couple (t, x) yields measurable (resp. continuous or topological) flows. Smoothness assumptions are generally made with respect to the state space variable x only: a flow is of class C^k if the mapping $\psi(t, \cdot) : \mathcal{M} \to \mathcal{M}$ is differentiable up to order k and the resulting derivative $\psi^{(k)}$ is continuous as a mapping $\Re \times \mathcal{M} \to \mathcal{M}$. The two previous properties are often called cocycle properties.*

According to Definition 1, flows are two-sided in time, i.e., it is possible to set arbitrary values for time t. Very often however, flows are implicitly defined by a differential equation and fail to be defined at every time or everywhere on $\mathcal{M}$. To cope with this situation, the relaxed notion of local flow is introduced. The only important modification to the definition is that ψ is a mapping from $\mathcal{D} \subset \Re \times \mathcal{M} \to \mathcal{M}$, satisfying the cocycle conditions and with the domain $\mathcal{D}$ such that:

- $\mathcal{D}$ is open, non void;
- $\mathcal{D}_x = \{t \in \Re | (t, x) \in \mathcal{D}\}$ is an open interval containing 0, for any $x \in \mathcal{M}$;
- $t \in \mathcal{D}_{\psi(s,x)}$ if and only if $t + s \in \mathcal{D}_x$.

With the previous notation, $\mathcal{D}_x = \{\tau^-(x), \tau^+(x)\}$, where $\tau^+(x)$ ($\tau^-(x)$) is the forward (backward) explosion time of the trajectory starting at x at time $t = 0$. A dynamical system will be defined in the following by a local flow. The domain $\mathcal{D}$ is assumed to be implicit from the differential equation generating the flow unless otherwise noted.

Consider a system governed by the linear differential equation $\dot{x}(t) = Ax(t)$ in $\Re^n$. The behavior of $x(t)$ as time grows to infinity can be derived from the eigenvalues of A. However, even by relaxing only the fact that the system is autonomous (i.e. $\dot{x}(t) = A(t)x(t)$) this simple approach breaks down and knowing the eigenvalues of $A(t)$ is of little help to understand the asymptotic behavior of the system. It turns out that the right definition is a kind of local shear factor called the Lyapunov exponents. We shall start by introducing Lyapunov exponents for the linear system described by $\dot{x}(t) = A(t)x(t)$. Lyapunov exponents for $\dot{x}(t) = A(t)x(t)$ play the same role in asymptotic stability analysis as the real parts of the eigenvalues of A for $\dot{x}(t) = Ax(t)$, and in fact they are given by the real parts of the eigenvalues of A in this case. The considerations developed in the linear time-varying setting will be useful when dealing with nonlinear systems.

Let $x(t) = \phi(t)x$ be the solution of the non autonomous linear differential equation $\dot{x}(t) = A(t)x(t)$ initialized with $x(0) = x$. The (forward) Lyapunov exponent associated with $x \in \Re^n$ is defined as:

$$\lambda(x) = \limsup_{t \to +\infty} \frac{1}{t} \log \|\phi(t)x\|$$

One can define similarly the backward Lyapunov exponent by considering $\phi(-t)x$ instead of $\phi(t)x$. Conventionally, $\lambda(0) = -\infty$.

Remark 2 *In numerical analysis, Lyapunov exponents are studied to quantify the sensitivity to the initial condition of the solution to a differential equation. Indeed, the Lyapunov exponent $\lambda(x)$ measures the mean (exponential) rate of convergence/divergence of two trajectories with initial relative position $\delta x(0) = x$. Note in fact that $\delta x(t) = \phi(t)x$ represents the evolution in time of the distance between the two trajectories, so that $\lambda(x) = \limsup_{t \to +\infty} \frac{1}{t} \log \|\delta x(t)\| = \limsup_{t \to +\infty} \frac{1}{t} \log \frac{\|\delta x(t)\|}{\|x\|}$ is such that $\|\delta x(t)\| \simeq e^{\lambda(x)t} \|x\|$.*

Proposition 1 *Let $x, y \in \Re^n$ be such that $\lambda(x) \neq \lambda(y)$. Then, x and y are linearly independent.*

Proof: The proposition can be easily established by first showing two simple properties of Lyapunov exponent. First, let $a \neq 0$ be a real number. Then:

$$\begin{aligned}\lambda(ax) &= \limsup_{t\to+\infty} \frac{1}{t} \log \|\phi(t)ax\| = \limsup_{t\to+\infty} \frac{1}{t} \big(\log \|\phi(t)x\| + \log(|a|) \big) \\ &= \limsup_{t\to+\infty} \frac{1}{t} \log \|\phi(t)x\| = \lambda(x)\end{aligned}$$

Now, consider two vectors x and y. Since $\|\phi(t)(x+y)\| \leq 2\max(\|\phi(t)x\|, \|\phi(t)y\|)$, we have

$$\lambda(x+y) = \limsup_{t\to+\infty} \frac{1}{t} \log \|\phi(t)(x+y)\| \leq \max(\lambda(x), \lambda(y))$$

From this it can be deduced that if $\lambda(x) > \lambda(y)$, then

$$\lambda(x+y) \leq \lambda(x) \leq \max(\lambda(x+y), \lambda(-y))$$

and since $\lambda(x) > \lambda(y)$ the max in the right hand side must be $\lambda(x+y)$ so that finally if $\lambda(x) \neq \lambda(y)$, then

$$\lambda(x+y) = \max(\lambda(x), \lambda(y))$$

Now we are ready to prove the proposition since if x and y are two non zero vectors such that $\lambda(x) \neq \lambda(y)$, then if $ax + by = 0$, $\lambda(0) = -\infty = \max(\lambda(x), \lambda(y))$, which in turn implies that $a = b = 0$. ∎

It is interesting to note that the previous result is obtained by using only the following three fundamental properties of a Lyapunov exponent function $\lambda : \Re^n \to \Re \cup \{-\infty\}$

- $\lambda(ax) = \lambda(x)$, for any $x \in \Re^n$ and $a \in \Re \setminus \{0\}$;
- $\lambda(x+y) \leq \max(\lambda(x), \lambda(y))$, for any $x, y \in \Re^n$, with equality holding if $\lambda(x) \neq \lambda(y)$
- $\lambda(0) = -\infty$ (normalization condition)

Functions satisfying these requirements are called characteristic exponents and share most of the salient features of Lyapunov exponents. From the linear independence of vectors with different Lyapunov exponents, it is clear that there exists only a finite number $p \leq n$ of Lyapunov exponents $\lambda_1, \lambda_2, \ldots, \lambda_p$. Assuming that the exponents are ordered $\lambda_1 < \lambda_2 < \cdots < \lambda_p$, we can construct a filtration $\{0\} = V_0 \subset V_1 \subset V_2 \subset \cdots \subset V_p = \Re^n$ of $\Re^n$ such that $\lambda(x) = \lambda_i$ for $x \in V_i \setminus V_{i-1}$. The multiplicity of λ_i is the number $k_i = \dim V_i - \dim V_{i-1}$. A basis $\{e_1, e_2, \ldots, e_n\}$ of $\Re^n$ is normal (ordered) with respect to the filtration if there exists an increasing sequence n_i,, $i = 1, 2, \ldots, p$, such that $\{e_1, e_2, \ldots, e_{n_i}\}$ form a basis of V_i for all $i = 1, 2, \ldots, p$. A normal basis $\{e_1, e_2, \ldots, e_n\}$ is characterized by the fact that:

$$\sum_{i=1}^{n} \lambda(e_i) = \inf \Big\{ \sum_{i=1}^{n} \lambda(f_i), \{f_1, f_2, \ldots, f_n\} \text{ basis of } \Re^n \Big\}$$

For Lyapunov exponent computations, we need some extra work on orthogonality relations.

Let $\Re^n$ be considered as it own dual with pairing $\langle \cdot, \cdot \rangle$. Let $\{e_1, e_2, \ldots, e_n\}$ and $\{f_1, f_2, \ldots, f_n\}$ be two dual bases, that is $\langle e_j, f_i \rangle = \delta_{i,j}$, $\forall i, j = 1, 2, \ldots, n$. Consider now a characteristic exponent function $\gamma : \Re^n \to \Re \cup \{-\infty\}$ defined by the three properties mentioned before. Let $\Gamma = \{\gamma_i, i = 1, \ldots, n\}$ be the characteristic exponents associated to $\{e_1, e_2, \ldots, e_n\}$ (i.e. $\gamma_i = \gamma(e_i)$) and $\Gamma' = \{\gamma_i', i = 1, \ldots, n\}$ be the characteristic exponents associated to $\{f_1, f_2, \ldots, f_n\}$ (i.e. $\gamma_i' = \gamma(f_i)$).

Definition 2 *The sets of characteristic exponents* Γ, Γ' *are said to be dual if* $\gamma_i + \gamma_i' \geq 0$, $\forall i = 1, 2, \ldots, n$.

For dual characteristic exponents, we can define the regularity coefficient

Definition 3 *The regularity coefficient* $\kappa(\Lambda, \Lambda')$ *of the dual characteristic exponents* Γ, Γ' *is*

$$\kappa(\Gamma, \Gamma') = \min \max\{\gamma_i + \gamma_i',\ i = 1, \ldots, n\}$$

where the minimum is computed over all possible pairs of basis in duality.

One can check that dual Lyapunov exponents have a positive regularity coefficient.

Definition 4 *Dual characteristic exponents* Γ, Γ' *are said to be regular if* $\kappa(\Gamma, \Gamma') = 0$.

Proposition 2 *Let* Γ, Γ' *be regular exponents. The filtrations associated with* Γ *and* Γ' *are mutually orthogonal.*

If the filtration $\{0\} = V_0 \subset V_1 \subset V_2 \subset \cdots \subset V_p = \Re^n$ is associated with Γ and $\{0\} = W_0 \subset W_1 \subset W_2 \subset \cdots \subset W_q = \Re^n$ with Γ', orthogonality between the two filtrations means that

- $p = q$
- $\dim V_i + \dim W_{p-i} = n$, $\forall i = 0, 1, \ldots, p$
- $V_i \perp W_{p-i}$, $\forall i = 0, 1, \ldots, p$

The dual equation of the non autonomous linear differential equation $\dot{x}(t) = A(t)x(t)$ is given by

$$\dot{y}(t) = -A^H(t)y(t)$$

where $A^H(t)$ is the Hermitian conjugate of $A(t)$. By simple computations:

$$\begin{aligned}\frac{d}{dt}\langle x(t), y(t)\rangle &= \langle A(t)x(t), y(t)\rangle + \langle x(t), -A^H(t)y(t)\rangle \\ &= \langle A(t)x(t), y(t)\rangle - \langle A(t)x(t), y(t)\rangle = 0\end{aligned}$$

so that orthogonality is preserved if $x(0)$ and $y(0)$ are orthogonal. Taking dual bases of $\Re^n$, it can be proven that the Lyapunov exponents Λ associated with the original equation and Λ' associated with the dual equation are dual. The original system is said to be regular if Λ and Λ' are regular.
Now we formulate the key theorem for the computation of the Lyapunov exponents under the assumption of regularity of the system.

Theorem 1 *If the system described by $\dot{x}(t) = A(t)x(t)$ is regular, then, for any normal ordered basis $\{e_1, e_2, \ldots, e_n\}$ and any $k = 1, 2, \ldots, n$:*

$$\lim_{t\to+\infty} \frac{1}{t} \log|\det G_k(t)| = 2\sum_{i=1}^{k} \lambda(e_i) \tag{1}$$

where $G_k(t)$ is the Gram matrix formed from the inner products: $[G_k(t)]_{i,j} = \langle e_i(t), e_j(t)\rangle$, $i, j = 1, \ldots, k$, and $e_i(t)$ the solution of the differential equation with initial condition $e_i(0) = e_i$.

The Gram matrix $G_k(t)$ can be obtained from the normal fundamental matrix solution to $\dot{x} = A(t)x$, that is the matrix $E(t)$ satisfying the differential equation $\dot{E}(t) = A(t)E(t)$ with initial condition $E(0)$ given by the matrix with the vectors $\{e_1, e_2, \ldots, e_n\}$ of the normal basis on the columns. Lyapunov showed that a normal basis always exists and how it can be constructed from a fundamental matrix solution, [25].

It is worth noting that in equation (1) we have a standard limit instead of the lim sup as in the definition of Lyapunov exponents, and that the determinant of the Gram matrix is the square of the volume of the parallelepiped spanned by the vectors $e_1(t), \ldots, e_k(t)$, so that Lyapunov exponents can be viewed as local rates of growth.

Lemma 1 *Let $Q : \Re \to GL_n$ be a smooth mapping with value in the linear group of dimension n. The change of variable $y(t) = Q^{-1}(t)x(t)$ results in the equation:*

$$\dot{y}(t) = M(t)y(t)$$

with $M(t) = Q^{-1}(t)(A(t)Q(t) - \dot{Q}(t))$

The proof of this lemma is straightforward.

Lemma 2 *There exists a smooth change of variables Q with values in the unitary matrices such that matrix $M(t) = Q^{-1}(t)(A(t)Q(t) - \dot{Q}(t))$ in the previous equation is upper triangular.*

Proof: Let $\{e_1, \ldots, e_n\}$ be a basis of $\Re^n$ and let $e_i(t)$, $i = 1, \ldots, n$, be the solution of the equation $\dot{x}(t) = A(t)x(t)$ with initial condition $e_i(0) = e_i$. Let $Q(t)R(t)$ be the QR-decomposition of the matrix $E(t)$ with columns $\{e_1(t), \ldots, e_n(t)\}$. Since $\dot{E}(t) = A(t)E(t)$, i.e., $E(t)$ is a fundamental solution matrix for $\dot{x} = A(t)x$, we have

$$\dot{R}(t) = (Q^H(t)A(t)Q(t) - Q^H(t)\dot{Q}(t))R(t) = M(t)R(t)$$

Given that the solution $R(t)$ to this differential equation has to be upper triangular, then, matrix $M(t)$ must be upper triangular as claimed. ∎

This lemma from Perron is the key ingredient in the proof of the main theorem. Moreover, it is interesting in its own right and thus has been included. Perron's lemma is in fact adopted in nearly all numerical algorithms for computing the k largest Lyapunov exponents by temporal integration. Lyapunov exponents are invariant with respect to the smooth change of coordinates by the unitary matrices $Q(t)$ (since it satisfies the norm-preserving property), [24]. Moreover, regularity is preserved under such transformation, [24].

Consider the nonlinear system described by the differential equation:

$$\dot{x}(t) = X(x(t))$$

and denote the corresponding flow by ψ. Consider the trajectory $x(t) = \psi(t, x_0)$ associated with $x(0) = x_0$. The Lyapunov exponents are a characterization of the asymptotic properties of the solution $\psi(t, x_0)$ via analysis of the linearized problem. The linear approximation of the system around $\psi(t, x_0)$ is given by

$$\dot{\delta}x(t) = A(t, x_0)\delta x(t)$$

where $A_{x_0}(t) := D_x X(x)|_{x=\psi(t,x_0)}$ with D_x denoting the derivative with respect to the x variable. Matrix $A(t, x_0)$ of the linear approximation describes the instantaneous rate of shearing of the infinitesimal neighborhood of $x(t) = \psi(t, x_0)$.

From the Taylor expansion of the flow:

$$\delta x(t) := \psi(t, x_0 + \delta x_0) - \psi(t, x_0) = D_x(\psi(t, x))|_{x=x_0}\delta x_0 + \ldots$$

it follows that the deformation of an infinitesimal neighborhood of the initial condition after the time interval t is given by $D_x(\psi(t, x_0)) := D_x(\psi(t, x))|_{x=x_0}$, which represents the shearing after a finite time t. Its eigenvalues and eigenvectors describe deformation of an initial infinitesimal sphere of neighboring trajectories into an ellipsoid time t later. Nearby trajectories separate exponentially along the unstable directions and approach each other along the stable directions.

The Lyapunov exponent at x_0 associated with x is defined as

$$\lambda_{x_0}(x) = \limsup_{t\to+\infty} \frac{1}{t} \log \|D_x(\psi(t, x_0))x\| \tag{2}$$

and describes the mean exponential rate of divergence/convergence of the trajectory obtained by a perturbation $\delta x_0 = x$ of the initial state x_0 ($\limsup_{t\to+\infty} \frac{1}{t} \log \frac{\|\delta x(t)\|}{\|\delta x_0\|}$) computed based on the linear approximation.

Under standard regularity assumptions on X:

$$\frac{d}{dt} D_x(\psi(t, x_0)) = A_{x_0}(t) D_x(\psi(t, x_0))$$

with $D_x(\psi(0, x_0)) = I$. Thus $D_x(\psi(t, x_0))$ can be viewed as a fundamental solution matrix for the linear differential equation $\dot{y}(t) = A_{x_0}(t)y(t)$, which makes the discussion on the linear case useful also in the nonlinear setting of interest.

It is worth noting that $D_x\psi(t, x_0)$ defines a linear cocycle over the flow ψ.

Definition 5 *Let $(\mathcal{M}, \psi)$ be a dynamical system where ψ is a local flow with domain $\mathcal{D}$. A linear (invertible) cocycle over ψ is a mapping $S : \mathcal{D} \to GL(\Re)$ such that:*

- *$S(0, x) = I$, $\forall x \in \mathcal{M}$;*
- *$S(t + s, x) = S(s, \psi(t, x))S(t, x)$, $\forall x \in \mathcal{M}$, $\forall t, s \in \Re$.*

For a linear cocycle $S(t, x)$, the associated Lyapunov exponents are defined as:

$$\lambda_x = \limsup_{t\to+\infty} \frac{1}{t} \log \|S(t, x)\| \tag{3}$$

where $\|S(t, x)\|$ is the norm of matrix $S(t, x)$ induced by the 2-norm for vectors (largest singular value) or, equivalently, the square root of the maximum eigenvalue of the positive definite symmetric matrix $S(t, x)^T S(t, x)$.

According to the definition (2) of Lyapunov exponents of the dynamical system $\dot{x}(t) = X(x(t))$ at x_0, equation (3) with $x = x_0$ provides the largest Lyapunov exponent of the dynamical system at x_0 when applied to the linear cocycle $S(t, x_0) = D_x\psi(t, x_0)$.

3.2.2 Computational aspects

Numerical pitfalls. Computing Lyapunov exponents amounts to integrate a differential equation, giving the linear cocycle $S(t, x_0) = D_x\psi(t, x_0)$. Nearly all standard algorithms can be used for that purpose but one quickly realize that the problem is far from being well conditioned. In fact, most of the time it is easy to obtain the flow $\psi(t, x)$ (needed for computing matrix $A_{x_0}(t)$) with good accuracy but by construction $S(t, x_0)$ tends to grow exponentially fast in some directions (corresponding to positive Lyapunov exponents) and to decay exponentially fast in others (corresponding to negative Lyapunov exponents), thus giving a condition number increasing again exponentially. For this reason, it would be extremely inaccurate to compute Lyapunov exponents by merely integrating the linear cocycle, and some kind of rescaling is needed to recover good numerical properties. There is abundant literature on the subject, however all methods fall into one of two categories, i.e. spatial integration or temporal integration. Direct application of the definition gives the second approach while the ergodic theorem gives the first one. Both have advantages and drawbacks:

- Spatial integration is efficient and free from the slow convergence phenomenon occurring sometimes in temporal integration. However, in order to use the ergodic theorem, one must find an invariant measure. Most of the time, it has to be done by covering algorithms.
- Temporal integration can suffer from slow convergence. Moreover, it requires periodic rescaling to avoid numerical problems.

In the complexity application, only the temporal approach has been tested.

Algorithms based on differential geometry. All the machinery used in this part is borrowed from the journal paper [12] by T.J. Bridges and S. Reich. Basically, the trick is to use Lemma 2 and Theorem 1 to compute the k largest Lyapunov exponents. The problem reduces then to continuously update the QR factorization (or a polar decomposition which is very similar except that the right hand side matrix is symmetric instead of being upper triangular). The differential equation satisfied by Q can be established by noticing first that the matrix $M(t) = Q^H(t)A(t)Q(t) - Q^H(t)\dot{Q}(t)$ is upper triangular, so that adding its conjugate yields a symmetric matrix with the same coefficients except on the diagonal where they are doubled. However,

$$M^H(t) + M(t) = Q^H(t)(A(t) + A^H(t))Q(t)$$

since $Q^H(t)\dot{Q}(t)+\dot{Q}^H(t)Q(t) = 0$ ($Q(t)$ is unitary so the derivative of $Q^H(t)Q(t) = I$ vanishes). This implies that $M(t)$ can be obtained without the knowledge of $\dot{Q}(t)$. Now, simply use the relation:

$$\dot{Q}(t) = A(t)Q(t) - Q(t)M(t)$$

This differential equation can be solved readily by standard Runge-Kutta integration. However, the orthogonality of Q is hard to preserve, so that efficient implementations require specific algorithms. A natural approach is to use integrators working on the Stiefel manifold on which Q lives. We refer to the previously mentioned paper for details. It seems possible (although not implemented yet) to use the Levi-Civita connection on the flag manifold to obtain closely related algorithm, but enforcing the fact that there is filtration adapted to Lyapunov exponents. The currently used algorithm for computing Lyapunov exponents is the continuous polar decomposition updating.

3.3 Modelling air traffic as a dynamical system

Observed aircraft trajectories constitute only a finite set of the possible integral lines of the system, hence some extra assumptions are needed to determine a dynamical system describing the air traffic situation under consideration. Appropriate smoothness conditions are required to ensure that the solution to the problem is unique. More precisely, given the aircraft velocities v_i, $i = 1, 2, \ldots, N$, at the measurement points x_i, $i = 1, 2, \ldots, N$, we determine the autonomous nonlinear dynamical system described by

$$\dot{x}(t) = X(x(t)),$$

by using vector spline interpolation to fit the vector field $X : \Re^3 \to \Re^3$ to the available interpolation data $\{(x_i, v_i), i = 1, 2, \ldots, N\}$. Vector field X must satisfy the interpolation constraints $X(x_i) = v_i$, $i = 1, 2, \ldots, N$, while minimizing the smoothness *div-curl* energy functional. If X is a vector field of class at least C^2, then, the *div-curl* energy of X in the domain $U \subset \Re^3$ is given by:

$$\mathcal{E}(X) = \int_U \alpha \|\nabla \mathbf{div} X(x)\|^2 + \beta \|\nabla \mathbf{curl} X(x)\|^2 dx$$

where $\mathbf{div}X$ and $\mathbf{curl}X$ respectively denote the divergence field and the rotational field of the vector field X, ∇ is the gradient operator, and α and β are positive real numbers controlling the relative weight of smoothness imposed on variations of the divergence field and the rotational field, respectively. It is a well known result in spline theory that the vector field X minimizing $\mathcal{E}(X)$ under the constraints $X(x_i) = v_i$, $i = 1, 2, \ldots, N$ is of the form :

$$X(x) = \sum_{i=1}^{N} c_i \psi(x - x_i) + \theta(x)$$

where $\theta(x) = Ax + b$ is an element of the kernel of the differential operator considered in the energy functional and ψ is the elementary solution of the operator P:

$$P = \alpha\, \mathbf{div}^T \nabla^T \nabla \mathbf{div} + \beta\, \mathbf{curl}^T \nabla^T \nabla \mathbf{curl}$$

If α is much greater than β, the optimal vector field X will tend to have a nearly constant divergence, while if β is much greater than α, the curl will be nearly

constant. An interesting special case occurs when $\alpha = \beta$.
When $\alpha = \beta$, P reduces to Δ^2 and ψ is the biharmonic kernel on all three coordinates which becomes independent. The corresponding splines are the well known thin-plate splines. Thus in the case of $\alpha = \beta$, the vector spline problem is converted into three separated scalar thin-plate spline problems, which causes a dramatic decrease in computation. The drawback is that the thin-plate spline deals with the components of a vector independently, hence the interpolating vector field does not preserve possible significant correlations between them. The procedure described above can be extended to time-dependent vector fields with a slight modification of energy functional. In a case of short time periods (say around 10 sampled or forecasted positions on each trajectory) it is possible to compute an interpolating vector field for 5000 aircraft within 10 minutes.

4 Conclusions and future work

Based on the review of the studies in the literature on air traffic complexity, we can conclude that

- most complexity metrics are strictly related to the current ground-based ATM system, where ATCs are in charge of guaranteeing safety in air traffic, and cannot be extended to the autonomous aircraft context because they incorporate a measure of workload and often strongly dependent on the sector-based airspace structure;
- those metrics that do not incorporate the measured workload but depend explicitly on the controller in place are difficult to apply to the autonomous aircraft context because of the decentralized time-varying structure of the control in the self-separation airspace, possibly involving a human in the loop component;
- those approaches providing a single aggregate indicator of complexity often lack of an operational interpretation;
- approaches providing local information through a complexity map may be useful for trajectory management by isolating critical areas that should be avoided;
- in view of the above considerations, the approaches in the literature that appear more portable to the autonomous aircraft context are those providing a measure of intrinsic complexity of air traffic;
- the time dependence aspect should be better focused, introducing approaches to air traffic complexity evaluation specific for the long term (trajectory management) and the short/mid term (conflict detection and resolution), eventually accounting for uncertainty in the aircraft trajectory prediction.

Our plan for the future work under WP3.2 is

i) specify the requirements on complexity metrics for the iFly A^3 Concept of Operations defined in work package 1;

ii) further develop the dynamic approach to intrinsic air traffic complexity characterization described in Chapter 3;

iii) study alternative complexity metrics tailored to the short/medium term and to the long term.

Regarding the development of alternative complexity metrics, some preliminary ideas are as follows.

As for the long term, the goal is to allow for the identification of critical areas that should be possibly avoided. Aircraft density will be a relevant factor jointly with its evolution in time. Evaluation of complexity will be based on the intended aircraft trajectories, with the understanding that each aircraft should generally conform to its current intent information. Complexity will be recomputed from time to time to take care of possible modifications of the aircraft intent. Unexpected deviations at a finer time scale will be accounted for by the short/medium term complexity metric. Since complexity will be evaluated based on the intent information and current state of all aircraft in the self-separation airspace, it may be more convenient to perform computations on the ground and distribute only the (compact) description of the critical areas to aircraft, rather than performing computations on-board of all aircraft. Indications on how to set a generally applicable threshold to distinguish between low and high complexity areas will be provided. This is actually a critical aspect for the intrinsic complexity measure described in Chapter 3.

As for the short/medium term, the goal is to identify air traffic encounters that could generate situations of conflict difficult to solve based on air traffic measurements only, without directly referring to the adopted resolution methodology. A way of achieving this is to evaluate complexity in terms of availability of feasible resolution maneuvers for accommodating an additional aircraft entering the airspace region or for solving conflicts due to deviations of the aircraft from their planned trajectory: the larger is the solution space, the lower is complexity. Most likely a probabilistic approach will be taken.
In contrast with the input-output model in Section 2.2.5 where complexity is measured as the control effort required for implementing a specific optimal conflict resolution strategy, the idea is that complexity should be measured as the effort required for determining a feasible, not necessarily optimal, resolution maneuver. This makes complexity evaluation independent of the adopted optimality criterion. The research on trajectory flexibility, related to the aircraft manoeuvrability and to its capability of accommodating unexpected disturbances, and that on probabilistic conflict/collision prediction [72, 42, 10, 9] might turn out to be useful for developing this approach.

References

[1] RTCA Task Force 3. Free flight implementation. Technical report, RTCA Inc., October 1995.

[2] V.R. Hunt ad A. Zellweger. Strategies for future air traffic control systems. *Computer*, pages 19–32, 1987.

[3] B.A. Arad. The controller load and sector design. *Journal of Air Traffic Control*, pages 12–31, 1964.

[4] S. Athénes, P. Averty, S. Puechmorel, D. Delahaye, and C. Collet. Complexity and controller workload: Trying to bridge the gap. In *International Conference on Human-Computer Interaction in Aeronautics, HCI-Aero*, Cambridge (MA), USA, 2002.

[5] Luis Barreira and Yakov B. Pesin. *Lyapunov Exponents and Smooth Ergodic Theory*. Number 23 in University Lecture Series. American Mathematical Society, 2002.

[6] K. Bilimoria and M. Jastrzebski. Properties of aircraft clusters in the national airspace system. In *AIAA Aviation Technology, Integration and Operations Conference (ATIO)*, number AIAA 2006-7801, Wichita, Kansas, September 2006.

[7] K. Bilimoria and H. Lee. Analysis of aircraft clusters to measure sector-independent airspace congestion. In *AIAA Aviation Technology, Integration and Operations Conference (ATIO)*, number AIAA 2005-7455, Arlington, VA, September 2005.

[8] K.D. Bilimoria and M. Jastrzebski. Aircraft clustering based on airspace complexity. *7th AIAA Aviation Technology, Integration and Operations Conference*, 2007.

[9] H.A.P. Blom, G.J. Bakker, and J. Krystul. Probabilistic reachability analysis for large scale stochastic hybrid systems. In *46th IEEE Conf. on Decision and Control*, 2007.

[10] H.A.P. Blom, J. Krystul, and G.J. Bakker. A particle system for safety verification of free flight in air traffic. In *45th IEEE Conf. on Decision and Control*, 2006.

[11] A.L. Breitler, M. Lesko, and M. Kirk. Effects of sector complexity and controller experience on probability of operational errors in air route traffic. Technical Report DTFA01-95-C-00002, FAA, 1996.

[12] T.J. Bridges and S. Reich. Computing Lyapunov exponents on a Stiefel manifold. *Physica D*, 156(3):219–238, August 2001.

[13] E.P. Buckley, B.D. DeBaryshe, N. Hitchner, and P. Kohn. Methods and measurements in real-time air traffic control system simulation. Technical Report DOT/FAA/CT83/26, Federal Aviation Administration, Atlantic City, NJ, 1983.

[14] K.M. Cardosi and E.D. Murphy. Human factors in the design and evaluation of air traffic control systems. Technical Report DOT/FAA/RD-95/3, U.S. Department of Transportation, Volpe Research Center, Cambridge (MA), 1995.

[15] R. Christien, A. Benkouar, T. Chaboud, and P. Loubieres. Air traffic complexity indicators & ATC sectors classification. In *ATM 2003*, 2003.

[16] A. Cloerec, K. Zeghal, and E. Hoffman. Traffic complexity analysis to evaluate the potential for limited delegation of separation assurance to the cockpit. In *18th Digital Avionics Systems Conference*, 1999.

[17] J.F. Coeterier. Individual strategies in ATC freedom and choice. *Ergonomics*, 14(5):579–584, 1971.

[18] C.G. Davis, J.W. Danaher, and M.A. Fischl. The influence of selected sector characteristics upon ARTCC controller activities. Technical report, The Matrix Corporation, Arlington, VA, 1963. Contract No. FAA/BRD-301.

[19] P. Leal de Matos. *The Development of Decision Support Models for European Air Traffic Flow Management.* PhD thesis, University of Warwick, UK, 1998.

[20] D. Delahaye, P. Paimblanc, S. Puechmorel, J.M. Histon, and R.J. Hansman. A new air traffic complexity metric based on dynamical system modelization. In *21st Digital Avionics Systems Conference*, 2002.

[21] D. Delahaye and S. Puechmorel. Air traffic complexity: Towards intrinsic metrics. In *Proc. of the 3rd FAA/Eurocontrol ATM R&D Seminar*, Napoli, Italy, June 2000.

[22] D. Delahaye and S. Puechmorel. Air traffic complexity map based on nonlinear dynamical system. In *4th Eurocontrol Innovative Research Workshop & Exhibition*, 2005.

[23] D. Delahaye, S. Puechmorel, R.J. Hansman, and J.M. Histon. Air traffic complexity based on nonlinear dynamical systems. In *5th USA/Europe Air Traffic Management R&D Seminar*, Budapest, Hungary, June 2003.

[24] L. Dieci, R.D. Russel, and E.S. Van Vleck. On the computation of Lyapunov exponents for continuous dynamical systems. *SIAM J. Numer. Anal.*, 34(1):402–423, February 1997.

[25] L. Dieci and E.S. Van Vleck. Lyapunov and other spectra: a survey. In D. Estep and S. Tavener, editors, *Preservation of Stability under Discretization*. SIAM, 2002.

[26] H. Erzberger, T.J. Davis, and S. Green. Design of Center-TRACON automation system. In *AGARD Guidance and Control Symposium on Machine Intelligence in Air Traffic Management*, May 1993.

[27] Chaboud et al. Air traffic complexity: Potential impacts on workload and cost. Technical Report EEC note 11/00, Eurocontrol, 2000.

[28] Eurocontrol. ATFCM and capacity report 2006. Technical report, Eurocontrol, 2007.

[29] P. Flener, J. Pearson, M. Agren, C. Garcia-Avello1, M. Celiktin, and S. Dissing. Air-traffic complexity resolution in multi-sector planning using constraint programming. In *Air Traffic Management R&D Seminar*, 2007.

[30] G.M. Flynn, A. Benkouar, and R. Christien. Adaptation of workload model by optimization algorithms and sector capacity assessment. Technical Report EEC Note No. 07/05, Eurocontrol, 2005.

[31] General Accounting Office (GAO). Aviation safety (status of the air traffic control work force). Publication No. 99-64. U.S. Government Printing Office, Washington, DC, 1986.

[32] D. Gianazza. Airspace configuration using air traffic complexity metrics. In *ATM*, 2007.

[33] D. Gianazza and K. Guittet. Evaluation of air traffic complexity metrics using neural networks and sector status. In *2nd International Conference on Research in Air Transportation, ICRAT*, 2006.

[34] D. Gianazza and K. Guittet. Selection and evaluation of air traffic complexity metrics. In *25th Digital Avionics Systems Conference, DASC*, 2006.

[35] G. Granger and N. Durand. A traffic complexity approach through cluster analysis. *5th USA/Europe Air Traffic Management R&D Seminar*, 2003.

[36] M. Grossberg. Relation of sector complexity to operational errors. In *Quaterly Report of the FAA Office of Air Traffic Evaluations and Analysis*, Washington, DC: Federal Avaition Administration, 1989.

[37] B. Hilburn. Cognitive complexity in air traffic control: A literature review. Technical Report 04/04, EUROCONTROL Experimental Centre, 2004.

[38] B. Hilburn and G. Flynn. Toward a non-linear approach to modeling air traffic complexity. In *2nd Human Performance Situation Awareness and Automation Conference*, 2004.

[39] J. Histon, G. Aigoin, D. Delahaye, R.J. Hansman, and S. Puechmorel. Introducing structural considerations into complexity metrics. In Eurocontrol-FAA, editor, *Fourth USA/EUROPE ATM R& D Seminar*, 2001.

[40] J. Histon, R.J. Hansman, G. Gottlieb, H. Kleinwaks, S. Yenson, D. Delahaye, and S. Puechmorel. Structural considerations and cognitive complexity in air traffic control. In IEEE-AIAA, editor, *21st Air Traffic Management for Commercial and Military Systems*, 2002.

[41] V.D. Hopkin. Human factors in air traffic control. Technical Report AGARDograph No. 275, AGARD, Neuilly-Sur-Seine, France, 1982.

[42] J. Hu, M. Prandini, and S. Sastry. Aircraft conflict prediction in the presence of a spatially correlated wind field. *IEEE Transactions on Intelligent Transportation Systems*, 3:326–340, 2005.

[43] H. Idris, R. Vivona, and J-L Garcia-Chico. Trajectory-oriented approach to managing traffic complexity – operational concept and preliminary metrics definition. Ames Contract NNA07BB26C NASA/CR-2008-215121, National Aeronautics and Space Administration, Langley Research Center, Hampton, Virginia, 2008.

[44] H. Idris, R. Vivona, J-L Garcia-Chico, and D. Wing. Distributed traffic complexity management by preserving trajectory flexibility. *26th IEEE/AIAA Digital Avionics Systems Conference*, 2007.

[45] H. Idris, D. Wing, R. Vivona, and J-L Garcia-Chico. A distributed trajectory-oriented approach to managing traffic complexity. In *7th AIAA Aviation Technology, Integration and Operations Conference (ATIO)*, number AIAA 2007-7731, Belfast, Northern Ireland, September 2007.

[46] Wyndemere Inc. An evaluation of air traffic control complexity. Technical Report Contract NAS2-14284, October 1996. Final Report.

[47] M.A. Ishutkina, E. Feron, and K.D. Bilimoria. Describing air traffic complexity using mathematical programming. In *AIAA 5th Aviation, Technology, Integration, and Operations Conference (ATIO)*, 2005.

[48] W. Knecht, K. Smith, and P. Hancock. A dynamic conflict probe and index of collision risk. In *Human Factors and Ergonomics Society 40th Annual Meeting*, 1996.

[49] P. Kopardekar. Dynamic density: A review of proposed variables. Technical report, FAA, 2000. FAA NAS Advanced Concepts Branch ACT-540.

[50] P. Kopardekar and S. Magyarits. Dynamic density: Measuring and predicting sector complexity. In *21st Digital Avionics Systems Conference (DASC)*, Irvine, California, 2002.

[51] P. Kopardekar and S. Magyarits. Measurement and prediction of dynamic density. In *5th USA/Europe Air Traffic Management R&D Seminar*, Budapest, Hungary, June 2003.

[52] P. Kopardekar, A. Schwartz, S. Magyarits, and J. Rhodes. Airspace complexity measurement: an air traffic control simulation analysis. In *Air Traffic Management R&D Seminar*, 2007.

[53] A. Koros, P.S. Della Rocco, G. Panjwani, V. Ingurgio, and J.F. DArcy. Complexity in air traffic control towers: A field study. part 1: Complexity factors. Technical Report DOT/FAA/CT-TN03/14, FAA, Atlantic City, New Jersey, 2003.

[54] B. Kriwan, R. Scaife, and R. Kennedy. Investigating complexity factors in UK air traffic management. In *Human Factors and Aerospace Safety*, 2001.

[55] J.P. Krol. Variation in ATC workload as a function of variation in cockpit workload. *Ergonomics*, 14(5):585–590, 1971.

[56] I.V. Laudeman, S.G. Shelden, R. Branstrom, and C.L. Brasil. Dynamic density: An air traffic management metric. Technical Report TM-1998-112226, NASA, 1998.

[57] K. Lee, E. Feron, and A. Pritchett. Air traffic complexity: An input-output approach. In *Air Traffic Management R&D Seminar*, 2007.

[58] K. Lee, E. Feron, and A. Pritchett. Air traffic complexity: An input-output approach. In *American Control Conference*, New York City, USA, July 2007.

[59] A. Majumdar and W.Y. Ochieng. The factors affecting air traffic controller workload: a multivariate analysis based upon simulation modelling of controller workload. Technical report, Centre for Transport Studies, Imperial College, London, 2000.

[60] A. Majumdar and Y.W. Ochieng. The factors affecting air traffic controller workload: A multivariate analysis based upon simulation modeling of controller workload. In *81st Transportation Research Board (TRB) Annual Conference*, Washington, DC, January 2002.

[61] A. Majumdar and J. Polak. Estimating the capacity of europes airspace using a simulation model of air traffic controller workload. In *80th Annual Meeting of the Transportation Research Board*, 2001.

[62] C. Manning, S. Mills, C. Fox, E. Pfeiderer, and H. Mogilka. Investigating the validity of Performance and Objective Workload Evaluation Research (POWER). In *3rd USA/EUROPE Air Traffic Management R& D Seminar*, 2000.

[63] C. Manning, S. Mills, C. Fox, E. Pfeiderer, and H. Mogilka. The relationship between air traffic control communication events and measures of controller taskload and workload. In *4th USA/EUROPE Air Traffic Management R& D Seminar*, 2001.

[64] A. Masalonis, M. Callaham, Y. Figueroa, and C. Wanke. Indicators of airspace complexity for traffic flow management decision support. In *12th International Symposium on Aviation Psychology*, Dayton, Ohio, 2003.

[65] A.J. Masalonis, M.B. Callaham, and C.R. Wanke. Dynamic density and complexity metrics for realtime traffic flow management. In *5th USA/Europe Air Traffic Management Seminar*, Budapest, Hungary, 2003.

[66] C. Meckiff, R. Chone, and J-P. Nicolaon. The tactical load smoother for multi-sector planning. In *2nd FAA/Eurocontrol ATM R&D Seminar*, Orlando, Florida, December 1998.

[67] R.H. Mogford, J.A. Guttman, S.L. Morrow, and P. Kopardekar. The complexity construct in air traffic control: A review and synthesis of the literature. Technical Report DOT/FAA/-CT TN95/22, FAA, Atlantic City, NJ, 1995.

[68] R.H. Mogford, E.D. Murphy, R.J. Roske-Hofstrand, G. Yastrop, and J.A. Guttman. Research techniques for documenting cognitive processes in air traffic control: sector complexity and decision making. Technical Report DOT/FAA/CT-TN94/3, FAA, CTA Incorporated, Pleasantville, New Jersey, 1994.

[69] S. Mondoloni and D. Liang. Airspace fractal dimension and applications. In Eurocontrol-FAA, editor, *Fourth USA/EUROPE Air Traffic Management R& D Seminar*, 2001.

[70] L. Murphy and K. Smith. Heart-rate variabilityy is a robus measure of response to task demand: A study of operational errors in air traffic control. In *Human Factors and Ergonomics Society 45th Annual Meeting*, Minneapolis, MN, 2001.

[71] W.S. Pawlak, C.S. Brinton, K. Crouch, and K.M. Lancaster. A framework for the evaluation of air traffic control complexity. In *AIAA national Conference*, 1996.

[72] M. Prandini, J. Hu, J. Lygeros, and S. Sastry. A probabilistic approach to aircraft conflict detection. *IEEE Transactions on Intelligent Transportation Systems*, 1(4):199–220, 2000.

[73] INTENT project. Deliverable 2-1 – capacity. http://www.intentproject.org/, 2002.

[74] S. Puechmorel. A short introduction to complexity computation. Draf paper, July 2007.

[75] E. Rantanen. Development and validation of objective performance and workload measures in air traffic control. FAA Cooperative Agreement No. 02-G-019 AHFD-04-19/FAA-04-7, Institute of Aviation, Aviation Human Factors Division - University of Illinois at Urbana-Champaign, 2004.

[76] M.D. Rodgers, R.H. Mogford, and L.S. Mogford. The relationship of sector characteristics to operational errors. Technical Report DOT/FAA/AM-98/14, FAA, Washington DC, 1998.

[77] M.D. Rodgers and L.G. Nye. Factors associated with the severity of operational errors at air route traffic control centers. Technical Report An Examination of the Operational Error Database for Air Route Traffic Control Centers, M.D. Rogers (Ed.), DOT/FAA/AM-93/22, FAA, 1993.

[78] D. Schaefer, C. Meckiff, A. Magill, B. Pirard, and F. Aligne. Air traffic complexity as a key concept for multi-sector planning. In *Digital Avionics Systems Conference (DASC) meeting*, Daytona Beach, Florida, USA, 2001.

[79] D.K. Schmidt. A queuing analysis of air traffic controllers' workload. *IEEE Transactions on Systems, Man and Cybernetics*, 8(6):492–298, 1978.

[80] K. Smith, S.F. Scallen, W. Knecht, and P.A. Hancock. An index of dynamic density. *Human Factors*, 40(1):69–78, 1998.

[81] J.C. Sperandio. Variation of operator's strategies and regulating effects on workload. *Ergonomics*, 14(5):571–577, 1971.

[82] J.C. Sperandio. The regulation of working methods as a function of workload among air traffic controllers. *Ergonomics*, 21(3):195–202, 1978.

[83] B. Sridhar, K.S. Sheth, and S. Grabbe. Airspace complexity and its application in air traffic management. In *Proc. of the 2nd USA/Europe Air Traffic Management R & D Seminar*, Orlando, USA, December 1998.

[84] E.S. Stein. Air traffic controller workload: An examination of workload probe. Technical Report FAA/CTTN90/60, FAA, Atlantic City, New Jersey, 1985.

[85] J.D. Welch, J.W. Andrews, B.D. Martin, and B. Sridhar. Macroscopic workload model for estimating en route sector capacity. In *Air Traffic Management R&D Seminar*, 2007.

[86] A. Yousefi and G.L. Donohue. Temporal and spatial distribution of airspace complexity for air traffic controller workload-based sectorization. In *4th AIAA Aviation Technology, Integration and Operations Forum*, Chicago, IL, September 2004.