Robust Multiplexed Model Predictive Control for Agent-based Conflict Resolution

Elilini Siva, Jan Maciejowski and Keck-Voon Ling

Abstract— **Multiplexed Model Predictive Control (MMPC) was originally developed for a multi-input system as a strategy to reduce online computation. In this paper, we demonstrate how distributed control of a system of agents can be posed as a Multiplexed Model Predictive Control problem. Specifically, we consider using robust multiplexed MPC for controlling a system of agents in the presence of coupling constraints in the form of a collision avoidance requirement. The system is subject to persistent unknown, but bounded disturbances. The contribution of this paper is the extension of the original robust multiplexed MPC algorithm to include a disturbance feedback policy in between updates. This facilitates finding feasible solutions and inherits the property of rapid response to disturbances from multiplexing the control. In addition, it is observed that the computational time of the proposed MMPC schemes scales favourably with the number of agents.**

I. INTRODUCTION

Model Predictive Control (MPC) is a control formulation which solves an open loop optimal control problem at each time step [9], with explicit inclusion of operating constraints in the online optimisation. Multiplexed Model Predictive Control was developed initially by [5] for multi-input systems of a general structure, as a strategy for reducing online computation, and was developed further in [13] and [6]. The original MPC problem is divided temporally into a collection of controllers optimising in sequence on the same processor. In this paper we demonstrate its application as a distributed MPC scheme for performing cooperative control of a system of dynamically decoupled agents with coupled constraint sets.

Performing a centralised optimisation for an entire system of agents can be computationally expensive, and a considerable body of work exists on Distributed MPC (DMPC) seeking to address this. A scheme for dynamically coupled agents with separable cost but decoupled constraints is presented in [2], and is proved to be nominally stable, subject to the requirement of move suppression. Schemes of [18] involve simultaneous optimisation of agents, with iteration until convergence, but the case of coupling in the constraints is not considered. Similarly, in the formulation of [4], agents optimise in parallel, with consideration given to neighbouring agents, but no global coupled constraint guarantees exist. The challenge posed by the existence of coupled constraints when optimising in a distributed fashion is clearly that of ensuring consistency between agents. Methods addressing this include iterative schemes such as dual decomposition techniques [16] and Nash bargaining schemes [3], which can suffer from slow convergence and do not necessarily achieve optimality in the presence of nonconvex constraints.

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The presence of disturbances presents further challenges to DMPC. Robust feasibility and stability guarantees are obtained in the scheme of [11] which adopts a 'leader-follower' architecture. Agents optimise their plans according to a sequence, and constraint satisfaction is enabled via the tightening of constraints on each agent to accommodate both disturbances, and the plans of agents yet to optimise in the sequence. Our proposed formulation improves on this by permitting only a single agent to update at a time, thus reducing the communication requirements and computation time per timestep, permitting improved disturbance rejection. This single update feature is also common to the distributed tube MPC method in [17]. Our robust MMPC algorithm however employs constraint tightening to achieve robustness, which is less conservative than the tube approach.

Direct application of the robust MMPC algorithm as presented in [13] for unstable systems of decoupled structure would result in infeasibility in most cases, due to the resulting unstable disturbance rejection feedback policies and unbounded disturbance invariant sets. To overcome this, we adapt the formulation so that nonupdating agents apply a feedback policy, determined offline, in between updates, thus enabling feasibility. The resulting scheme then is a means of distributing computation without iteration, with guarantees of robust constraint satisfaction obtained by constraint tightening. Agents exchange plans and their experienced disturbances, and optimise for new plans using nominal dynamics. We describe two approaches; the first is equivalent to single update DMPC, and the second is akin to single update with move-blocking. Both schemes are subsumed by the MMPC framework.

Some of the notation used in this paper is outlined here. The state, control and change in control inputs are denoted x, u and Δu respectively. We use $x_{j,k}$ to depict the *actual* measured state of agent j at time k. At the time instant k, the *prediction* of the state i steps in the future is denoted $x_{j,k+i|k}$. We use $\sim$ to denote Pontryagin set difference.

The structure of the paper now follows. We begin by formulating the distributed control problem in consideration in section II. Robust Multiplexed MMPC is then reviewed, and the two novel formulations developed for our specific problem are detailed. In section III we include simulations performed on collision avoidance scenarios in a realistic air traffic control setting, and concluding remarks are made in section IV.

II. Problem formulation and background

We consider the problem of driving a system of agents to their respective target regions in minimum time, with minimum control effort, in a 2-D setting. We consider linear dynamics for each of the m point mass agents with state $x(t)$, comprising position and velocity, subject to control input accelerations $u(t)$ and disturbance inputs $w(t)$. Time discretisation of the dynamics with zero order hold yields:

$$x_{j,k+1} = Ax_{j,k} + Bu_{j,k} + w_{j,k}. \quad (1)$$

The state $x_{j,k}$ of agent j comprises its position and velocity so that

$$x_{j,k} = [r_{j,x} \ r_{j,y} \ v_{j,x} \ v_{j,y}]^T.$$

The disturbances act on the state and are assumed to be unknown but bounded, so that

$$w_{j,k} \in \mathcal{W} \ \forall \ j,k \quad (2)$$

with

$$\mathcal{W} = \{w_k : \|w_k\|_\infty \leq W_{\max}\}. \quad (3)$$

It is assumed that the states are perfectly measured at each time step. Each agent is subject to local constraints on speed and input accelerations

$$[0 \ I_2]x_{j,k} \in \mathbb{Y} \ \forall \ k, \ j \in N_m := \{1,\dots,m\}, \quad (4)$$

in addition to state constraints coupled across all the agents. We can represent the individual agents' states as a single combined state, by stacking the state vectors $x_{j,k} \in \mathbb{R}^4$ to obtain $x_k \in \mathbb{R}^{4m}$,

$$x_k = \begin{bmatrix} x_{1,k} \\ \vdots \\ x_{m,k} \end{bmatrix}$$

so that the constraints coupling the agents can be expressed

$$x_k \in \mathbb{X}. \quad (5)$$

Given a minimum separation distance R, we have the following positional constraints:

$$\mathbb{X} := \{x_k \in \mathbb{R}^{4m} : \|[I_2 \ 0](x_{p,k} - x_{q,k})\|_2 > R\} \quad \forall \ k \forall p,q \in N_m, p \neq q \quad (6)$$

which we approximate using polygons [15].

The cost function to be minimised by each agent j is a combination of the time taken to reach its target region T_f and the weighted one norm in control input

$$V(k) = \sum_{i=0}^{T_f} (\gamma |u_{j,k+i|k}| + 1) \quad (7)$$

It is shown in [12], [14] how minimisation of (7) is achieved by minimisation of a hybrid objective

$$V(k) = \sum_{i=0}^{N-1} (\gamma |u_{j,k+i|k}| + i.t_{j,k+i|k}) \quad (8)$$

where N, identified by the user, denotes the maximal horizon time and provides an upper bound on the arrival time T_f. When the binary variable $t_{j,k} \in \{0,1\}$ takes the value 1, agent j is required to enter its target region P_j at the next time $k+1$, so that

$$x_{j,k+1} \in P_j \quad (9)$$

This constraint is enforced using a *big-M* [1] formulation.The target regions, chosen by the user, are assumed to be polygonal.

We impose the terminal constraint that the target set is reached by the end of the prediction horizon,

$$\sum_{i=0}^{N-1} t_{j,k+i|k} = 1. \quad (10)$$

A. Robust Multiplexed MPC

This section reviews multiplexed MPC, introduced by [5] and [13], in the context of the problem statement presented. In multiplexed MPC, the agents' predictions and control moves are updated in a sequential and cyclic manner, with only one agent updating at any one time. Without loss of generality, we use the indexing function $\sigma(k) = (k \bmod m) + 1$ to identify the agent updating at time instant k. We define $\tilde{A} = I_m \otimes A, \tilde{B} = I_m \otimes A, \tilde{K} = I_m \otimes A$,

$$B_{\sigma(k)} = \begin{bmatrix} 0 \\ \vdots \\ B \\ 0 \\ \vdots \\ 0 \end{bmatrix} \text{ and } K_{\sigma(k)} = \begin{bmatrix} 0 \\ \vdots \\ K^T \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T,$$

where A and B are one-step matrices and $B_{\sigma(k)}$ and $K_{\sigma(k)}$ are periodic matrices whose $\sigma(k)$th entries are B and K, and 0 everywhere else. Note that whilst the formulation permits distinct dynamics for each subsystem, we consider only homogeneous agents. The dynamics of the joint system according to the original multiplexed algorithm can be expressed as

$$x_{k+1} = \tilde{A}x_k + B_{\sigma(k)}u_k + w_k \quad (11)$$

where $w_k = [w_{1,k}^T, \dots w_{j,k}^T, \dots w_{m,k}^T]^T, \quad w_k \in \mathbb{R}^{4m}$. In our proposed modified formulation, agents employ an affine disturbance rejection policy, in which the feedforward term is updated sequentially, and the feedback gains are calculated offline. The control input corresponding to agent j is given by

$$u_{j,k|k} = u_{j,k|k-l} + \sum_{i=0}^{l-1} P_i w_{j,k-l+i} \quad (12)$$

where the feedback gains P_i assume the form $P_i = K(A + BK)^i$, and l is the number of steps since agent j last performed an optimisation, with $0 \leq l \leq m$. We can then express the dynamics as:

$$x_{k+1} = \tilde{A}x_k + B_{\sigma(k)}u_k + \sum_{n \in N_m \setminus \sigma(k)} B_n(u_{n,k|k-1} + K_n w_{n,k-1}) + w_k$$

In the next section we review how the constraint sets are constructed to ensure feasibility of (12).

B. Robust MMPC with Constraint Tightening

In [13] a method of guaranteeing feasibility despite the presence of unknown disturbances is shown. Robust feasibility is achieved when, given an initially feasible input sequence, subsequent optimisations are guaranteed to be feasible, despite the presence of uncertainty in the dynamics. To achieve robust feasibility, the state constraints $x_k \in \mathbb{X}$ and input constraints $u_k \in \mathbb{U}_k$ are tightened using a recursion to compensate for the uncertain effects of future disturbances in the prediction horizon, as shown in the following

$$\mathcal{X}_{i+1,\sigma(k)} = \mathcal{X}_{i,\sigma(k+1)} \sim L_{i,\sigma(k+1)}\mathcal{W} \tag{13}$$

$$\mathcal{U}_{i,\sigma(k)} = \mathcal{U}_{i-1,\sigma(k+1)} \sim M_{(i-1),\sigma(k+1)}\mathcal{W}. \tag{14}$$

where

$$L_{0,\sigma(k)} = I \tag{15}$$

$$L_{i+1,\sigma(k)} = AL_{i,\sigma(k)} + B_{\sigma(k+i)}M_{i,\sigma(k)}. \tag{16}$$

The disturbance rejection matrices $M_{i,\sigma(k)}$ are designed offline. In view of the relations (15) and (16), our specific choice of feedback policy P_i defined earlier yields the following parameterisation in terms of the state transition matrix $L_{i,\sigma(k)}$

$$M_{i,\sigma(k)} = K_{\sigma(k+i)}L_{i,\sigma(k)} \tag{17}$$

so that

$$L_{i+1,\sigma(k)} = (\tilde{A} + B_{\sigma(k+i)}K_{\sigma(k+i)})L_{i,\sigma(k)}. \tag{18}$$

At time k, agent $\sigma(k) = j$ minimises

$$V(k) = \sum_{i=0}^{N-1} (\gamma|u_{j,k+i|k}| + it_{j,k+i|k}) \tag{19}$$

with respect to inputs $u_{k+i|k}$ and binary inputs $t_{j,k+i|k}$ for all $k + i = j$, subject to the nominal prediction model and tightened constraints:

$$x_{k|k} = x_k \tag{20}$$

$$x_{k+i|k} \in \mathcal{X}_{i,\sigma(k)} \tag{21}$$

$$t_{j,k+i|k} \in \{0, 1\} \tag{22}$$

$$x_{k+N|k} \in \mathcal{T}_{\sigma(k)} \tag{23}$$

$$\mathcal{T}_{\sigma(k)} = \mathcal{T}_f \ \forall k, \text{ with } \mathcal{T}_f := \{x \in \mathbb{R}^{4m} : t_j = 0 \ \forall j \in N_m\}$$

where the last constraint (23) is the terminal constraint in (10). A centralised solution which optimises over all agents' inputs is required to initialise the distributed scheme, and is a common requirement in distributed schemes [17], [12].

Centralised Objective for MMPC:

$$V(k) = \sum_{j=0}^{m}\sum_{i=0}^{N-1} (\gamma|u_{k+i|k}| + it_{j,k+i|k}) \tag{24}$$

The nonconvex collision avoidance and speed constraints in (4) and (6) are enforced using a big-M formulation [1]. The use of mixed integer programming for obstacle avoidance has been employed for instance in [15]. Details of the implementation of the variable horizon formulation with constraint tightening as a mixed integer linear program (MILP) are presented in [12] for the single agent case. It is shown how the target set constraints are tightened as a function of the horizon in the form of margins. Once an agent enters its target set, its operational constraints are relaxed using a big-M formulation. In our multi-agent setting, we are also required to relax all avoidance constraints involving completed agents. Relaxation of the constraints on reaching the terminal set is crucial for establishing robust completion guarantees, as noted in [12].

Algorithm 2.1: (Robust Multiplexed MPC with constraint tightening)

1) Initialise by minimising (24) over all variables $u_{j,k+i|k}, t_{j,k+i|k}$ subject to (20)-(23).
2) All agents apply $u_{j,k|k}$
3) Wait one timestep
4) increment k and communicate disturbances $\{w_{j,k}\}$
5) *while* $\sum_j t_{j,k} > 0$
   - Agent $\sigma(k)$ minimizes (19) subject to (20)-(23) and (12),
   - Agent $\sigma(k)$ applies $u_{\sigma(k),k|k}$. Other agents apply (12)
   - Increment k, remove completed agents so that $m = m - \sum_j (1 - t_{j,k})$, update $\sigma(k)$

   end

To establish robust feasibility of Algorithm 2.1, we require the following result, taken from [13] with a minor modification for our specific parameterisation of our disturbance rejection policy $M_{i,\sigma(k)}$:

Lemma 2.1: Given that

$$x_{k+1} = x_{k+1|k} + w_k \tag{25}$$

and

$$u_{j,k+i|k} = u_{j,k+i|k-1} + K_j L_{i-1,\sigma(k)} w_{k-1} \ \forall \ j \tag{26}$$

Then

$$x_{k+i+1|k+i} = x_{k+i+1|k} + L_{i,\sigma(k+1)} w_k.$$

Proof: This is proved by induction on i. Assume true for some i. Then,

$$\begin{aligned} x_{k+i+1|k+1} = \\ &\tilde{A}x_{k+i|k+1} + B_{\sigma(k+i)}u_{\sigma(k+i),k+i|k+1} \\ &+ \sum_{n \in m \setminus \sigma(k+i)} B_n(u_{n,k+i|k} + K_n L_{i-1,\sigma(k+1)} w_k) \end{aligned}$$

The summation term corresponds to the non-optimising agents applying the proposed disturbance feedback policy

$$x_{k+i+1|k+1} = \tilde{A}(x_{k+i|k} + L_{i-1,\sigma(k+1)}w_k) + B_{\sigma(k+i)}u_{\sigma(k+i),k+i|k+1} + \sum_{n\in m\setminus\sigma(k+i)} B_n(u_{n,k+i|k} + K_n L_{i-1,\sigma(k)}w_k).$$

By assumption,

$$u_{\sigma(k),k+i|k+1} = u_{\sigma(k),k+i|k} + K_{\sigma(k+i)}L_{i-1,\sigma(k+1)}w_k$$

so that

$$\begin{aligned} x_{k+i+1|k+1} &= \tilde{A}x_{k+i|k} + B_{\sigma(k+i)}u_{k+i|k} + \sum B_n u_{n,k+i|k} \\ &+ (\tilde{A} + B_{\sigma(k+i)}K_{\sigma(k+i)} + \sum_{n\in m\setminus\sigma(k+i)} B_n K_n)L_{i-1,\sigma(k+1)}w_k \\ &= x_{k+i+1|k} + (\tilde{A} + \tilde{B}\tilde{K})L_{i-1,\sigma(k+1)}w_k \\ &= x_{k+i+1|k} + L_{i,\sigma(k)}w_k. \end{aligned}$$

The last line follows from (18). The result is then true for $i+1$. From (25), the result is true for $i = 0$ and hence true $\forall\ i$. ■

Theorem 2.1: If the system of agents is controlled by Algorithm 2.1, and the initial optimisation can be solved, the optimisation remains feasible, namely the constraints $x_k \in \mathbb{X}$ are satisfied for all disturbances satisfying (2). If the control weighting is chosen to satisfy the condition

$$1 - m\gamma \max_w \sum_{i=0}^{\infty} \|(A + BK)^i w\| > 0, \qquad (27)$$

modified from [12] to accommodate multiple agents, the agents are driven to their targets in finite time.

Proof: The result follows from combining results from [13] and [12]. Feasibility follows from the result of Lemma 2.1 and the construction of the tightened constraint sets, the details of which are in [13]. For robust completion, the basic idea is that while at least one agent has yet to reach its target region, the optimal cost must reduce by at least the amount given by the left hand side of (27). Non negativity of the cost dictates that all agents must reach their targets in finite time. ■

Remark 2.1: In the absence of the proposed modification of including disturbance feedback between updates, limits on the horizon length exist to ensure that the output constraint sets are nonempty. To see this, consider the following. After n steps of constraint tightening, given the evolution of the state transition matrices in (16) and the parameterisation of the disturbance rejection matrices in (17), the disturbance set $\bigoplus_{i=0}^{n} L_{i,\sigma(k)}\mathcal{W}$ is given by $\bigoplus_{i=0}^{n}\big(\prod_{j=0}^{i}(\tilde{A} + B_{\sigma(k+j)}K_{\sigma(k+j)})\big)\mathcal{W}$. For unstable subsystems, it is not possible to find a stabilising K for which $L_{i,\sigma(k)}$ is convergent.

In view of theorem 2.1, algorithm 2.1 has the interpretation that the disturbance feedback policy executed by non updating agents at a given time instant is a candidate feasible solution [11] formed from the tail of the solution obtained at the previous time step. The scheme requires a conventional 'synchronous' solution to initialise the algorithm, as shown in Figure 1. Agents perform a policy update at intervals of mT, and apply their candidate feasible solutions at intervals of T, in between updates. We note that this formulation is equivalent to a single update formulation of that found in [11].

Further benefit from multiplexing the MPC controls can be obtained by employing a multiplexed initialisation. We detail next an MMPC scheme in which the agents' feedforward controls are updated sequentially and held constant over the period T, with disturbance feedback applied between successive updates at intervals of T/m. The timing of the updates is depicted in Figure 1. The complexities of both schemes are equal, but employing a multiplexed initialisation permits a higher frequency of policy update, and potentially improved disturbance rejection.

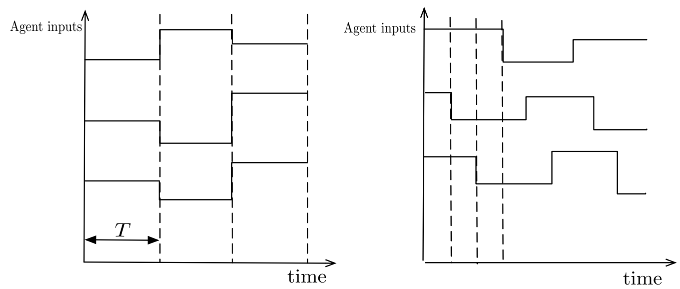

Fig. 1. Patterns of agents' input moves for a conventional 'synchronous' MPC initialisation (left) and a multiplexed initialisation (right).

Robust MMPC with multiplexed initialisation

This variant of robust multiplexed MPC involves optimisation over Δu, so we introduce the augmented state vector $\xi_k \in \mathbb{R}^{8m}$:

$$\xi_k = \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix}, \hat{B}_{\sigma(k)} = \begin{bmatrix} B_{\sigma(k)} \\ I_{\sigma(k)} \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ 0 & I \end{bmatrix} \hat{K}_{\sigma(k)} = \begin{bmatrix} K^T_{\sigma(k)} \\ I_{\sigma(k)} \end{bmatrix}.$$

Given that the disturbances only act on the state, and not the applied inputs, we can express the disturbance as $\hat{w}$, where $\hat{w} = [w\ 0]^T$. The change in control action $\Delta u_{j,k}$ executed by a non-optimising agent j at time k is

$$\Delta u_{j,k} = \sum_{i=0}^{l-1} P_i w_{j,k-l+i} \qquad (28)$$

where, as previously $P_i = K(A+BK)^i$ and l is the number of steps since agent j last performed an optimisation, with $0 \le l \le m$. The dynamics can be described as

$$\xi_{k+1} = \hat{A}\xi_k + \hat{B}_{\sigma(k)}\Delta u_k + \sum_{n\in N_m\setminus\sigma(k)} \hat{B}_n\hat{K}_n\hat{w}_{k-1} + \hat{w}_k. \qquad (29)$$

At time k, agent $\sigma(k) = j$ minimises the cost function

$$V(k) = \sum_j \sum_{i=0}^{N-1} (\gamma|u_{j,k+i|k} + \Delta u_{j,k+i|k}| + it_{j,k+i|k})$$

with respect to inputs $\Delta u_{k+i|k}$ for all $\sigma(k+i) = j$ and binary inputs $t_{j,k+i|k}$, subject to the constraint

$$\xi_{k|k} = \xi_k,$$

the nominal prediction model

$$\xi_{k+1} = \hat{A}\xi_k + \hat{B}_{\sigma(k)}\Delta u_k$$

the constraint on the non-optimising agents, (28), and the tightened constraints (20)-(23). The binaries $t_{j,k+i|k}$ are fixed over the sampling interval. We also require the assumption that the dynamics in the terminal set are not multiplexed.

Proposition 2.1: Given that

$$\xi_{k+1} = \xi_{k+1|k} + \hat{w}_k$$

and the following potential candidate feasible solution for the change in control:

$$\Delta u_{j,k} = \sum_{i=0}^{l-1} P_i w_{j,k-l+i} \tag{30}$$

Then

$$x_{k+i+1|k+i} = x_{k+i+1|k} + L_{i,\sigma(k+1)} w_k.$$

Proof: The argument largely follows that associated with the proof of lemma 2.1. ■

The proof of robust feasibility and finite time completion is closely related to that of theorem 2.1.

Remark 2.2: We note that ensuring robustness of both the MMPC schemes outlined is not limited to constraint tightening. An alternative with similar complexity but increased conservatism would be employing a tube formulation for robustness [10], [17]. The first robust MMPC scheme would then reduce to the single update tube formulation employed by [17], with a different cost penalising deviation from a target set.

Remark 2.3: As noted in [12], no restrictions are placed on the choice of feedback controller K. Clearly it is desirable to reduce the conservatism of the formulation to avoid excessive constraint tightening. The advantages of choosing a nilpotent policy are outlined in [12].

III. Results

We present now results obtained from application of the MMPC algorithm with disturbance feedback in a realistic constant altitude collision avoidance problem in an Air Traffic Control (ATC) setting, using a simulator reported in [8]. The examples were generated using Matlab, with CPLEX solving the MILP optimisations, and YALMIP [7] as an interface between CPLEX and Matlab.

The ATC model is hybrid, with continuous dynamics arising from the aircraft dynamics, and discrete dynamics arising from the flight plan and Flight Management System (FMS). A point mass model based on realistic aircraft parameters is used. The effects of unpredicted wind disturbances are included in an internal wind correlation model. The FMS controller is modelled as a 3D -FMS, where along-track errors are neglected. Further details of the simulator can be obtained from [8]. Each aircraft is an agent which plans its own trajectory according to the MMPC algorithm detailed in 2.1. The sampling interval used is 1 minute, so that plans are exchanged every time a policy update is performed, every $m$minutes, and the disturbances experienced are exchanged every 1 minute.

The flight plan comprises a reference path of straight lines, formed from a sequence of waypoints and a sequence of airspeeds. At each timestep, the MMPC algorithm is executed to produce a waypoint obtained from the state predictions. This waypoint is used as a flight plan input to the simulator. The aircraft state evolves according to the dynamics incorporated in the simulator model, and the state measurement is used as input to the MMPC, which then produces a waypoint for the next time step. The procedure is repeated until all aircraft have reached their target destinations. The along-track speed errors are bounded at 1km per minute, and are accounted for with the wind velocity disturbances in our robustification.

500 simulations were performed with different wind fields for systems of a number of aircraft. The aircraft are initialised on the boundary of a circle with a collision predicted to occur in the absence of corrective action. The aircraft initialisation speeds are at 454 knots each, and their required minimum separation distance is 5 nautical miles (nmi). Disturbances enter in the form of wind velocities obeying a 2σ truncated Gaussian distribution with standard deviation $5.17 m/s$. A maximal prediction horizon T_f of 30 minutes is used. Representative plots of trajectories obtained with four aircraft and six aircraft are shown in Figures 2 and 4. The points displayed are aircraft positions plotted at equal 1 minute intervals. The initialisation yielding the closed loop response plotted in Figure 4 is suboptimal, as evident from the path taken by agent 6. Figure 3 shows the pairwise separation distances between the four aircraft whose trajectories are plotted in Figure 2. Statistics of the results obtained are summarised in Table I. It can be seen that safe separation is maintained in all cases. The minimum separation observed is in excess of the minimum threshold of $5nmi$, indicating the offset added to the minimum separation constraint to allow for discretisation errors was too high. The solution times for the multiplexed scheme scale favourably, in contrast to the centralised solution.

IV. Concluding Remarks

We have presented a novel formulation of robust multiplexed MPC for application to the control of systems of decoupled structure, with coupling present in the constraints. By introducing a disturbance feedback policy employed between updates, robust feasibility properties are retained. Though attention has been restricted to a specific cost in this paper, the approach is readily applicable to other cost formulations. The method has successfully been applied to a realistic ATM collision avoidance scenario, and the computation time of

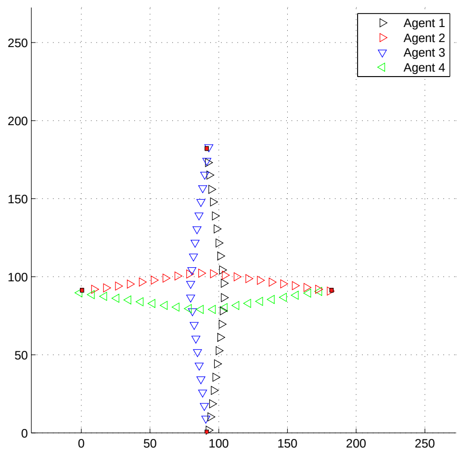

Fig. 2. Trajectories of 4 agents initialised optimally

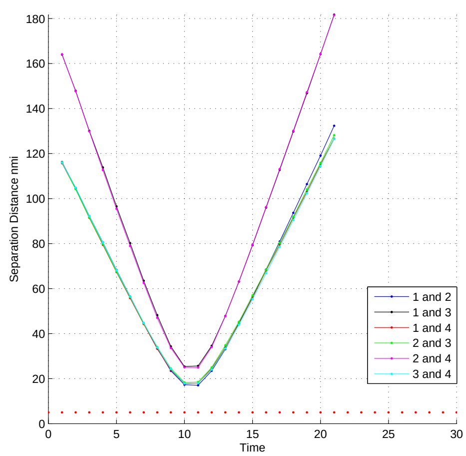

Fig. 3. 4 agents, pairwise separation distances. The minimum separation distance of $5nmi$ is displayed as a dotted red line.

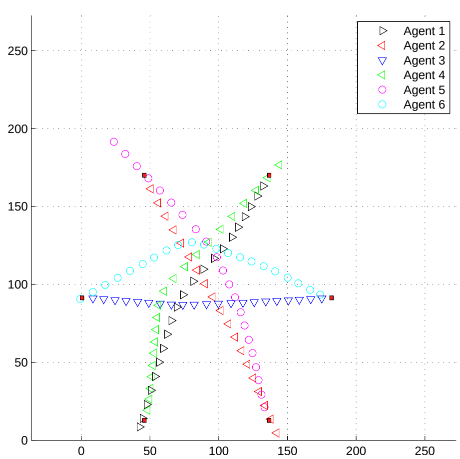

Fig. 4. Trajectories of 6 aircraft, initialised with a suboptimal but feasible solution.

TABLE I
MULTIPLEXED MPC WITH ATC SIMULATOR: STATISTICS SUMMARISING RESULTS OBTAINED WITH 500 SIMULATIONS PERFORMED WITH DIFFERENT WINDFIELDS

	Number of aircraft		
	2	**3**	**4**
Mean min separation /nmi	16.9	16.0	16.9
Variance min separation $/nmi^2$	0.145	0.291	0.128
Min separation over all sims /nmi	16.2	14.4	16.2
Mean MMPC computation time/stage/s	0.0306	0.0382	0.0514
Mean Centralised computation time/s	0.2143	2.80	20.9

the MMPC has been observed to scale favourably with the number of agents, in contrast to a centralised scheme.

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