2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Air traffic complexity and the interacting particle system method: An integrated approach for collision risk estimation

Maria Prandini, Henk A.P. Blom and G.J. (Bert) Bakker

Abstract— **In this paper, we explore the possibility of using air traffic complexity metrics to accelerate the Interacting Particle System (IPS) method for collision risk estimation.**

Collision risk estimation is an essential task to assess the performance and impact of, e.g., possible modifications of the current air traffic management system or new operational concepts. The standard Monte Carlo approach to probability estimation requires a number of simulations that scales as the inverse of the probability to be estimated. This makes it impracticable for estimating the probability of a rare event such as a collision, and calls for ad-hoc solutions. In the IPS method, the collision risk is estimated as the product of the conditional probabilities of an increasing sequence of conditionally not-so-rare events, where aircraft get at progressively smaller distances one from the other. Additional computational saving can be obtained by adopting importance sampling techniques where initial aircraft configurations that are more prone to lead to a collision are favored. The idea that we pursue in this paper is to select those configurations by using some complexity metric. In particular, we propose to combine IPS with a probabilistic complexity metric that explicitly accounts for the uncertainty affecting the aircraft motion. Preliminary results obtained by applying this integrated approach to a free flight scenario are presented in the paper.

I. INTRODUCTION

The availability of new technologies that enable the aircraft to broadcast and receive information about their own position and velocity and the position and velocity of surrounding aircraft has stimulated the rethinking of the current centralized and ground-based Air Traffic Management (ATM) system. In prospective ATM systems, aircrews will be allowed to choose their preferential trajectory while taking the responsibility of keeping at a safe distance from the other aircraft. This conceptual idea is known as free flight [1] and involves significant changes to the current ATM system: centralized control becomes distributed, responsibilities are transferred from ground to air, fixed air traffic routes are removed, and appropriate new technologies are brought in to assist aircrews in performing their navigation and self separation tasks.

The free flight concept idea has motivated the study of different operational concepts and implementation choices. One of the key issues of free flight design is safety verification, in particular for high traffic densities. For en-route traffic, the International Civil Aviation Organization (ICAO) has established thresholds on the acceptable probability of a mid-air collision. Hence, the en-route free flight safety verification problem consists of estimating the collision probability of free flight operations, and comparing this estimate with the ICAO established thresholds [2].

The problem of estimating the collision probability can be reformulated as a reachability analysis problem for the system modeling the air traffic: a collision occurs when the state of the overall system reaches a region of the state space where the distance of at least two aircraft is smaller than the aircraft size. The numerical approach to reachability analysis in [18] could be applied, in principle, to collision probability estimation in free flight operations. The problem is that the need of a fine gridding of the space and the prospective high density scenarios make the grid points blow up to a practically unmanageable large number. A seemingly simple approach toward the estimation of mid-air collision probability is to run many Monte Carlo (MC) simulations with a free flight model and count the fraction of runs for which a collision occurs. The advantage of a MC simulation approach is that it is not sensitive to the dimension of the state space and does not require specific assumptions on the system under consideration. A key problem is that in order to obtain accurate estimates of rare event probabilities, say about 10^{-9}, the outlined MC simulation approach requires to run 10^{11} simulation or more. Taking into account that an appropriate model for free flight operations is quite complex, this would require an impractically huge simulation time. More sophisticated approaches for probabilistic risk analysis adopted in safety critical industries [11] appear not suitable for free flight, since they cannot be applied to the class of Generalized Stochastic Hybrid Systems (GSHS) that are used for safety modeling of air traffic operations [17]. GSHS are continuous-time stochastic hybrid systems, whose state has two components: a continuous state component and a discrete state component (mode). The continuous state evolves according to a stochastic differential equation whose vector field and drift factor depend on both the hybrid state components. Switching from one mode to another is either governed by a probability law (spontaneous transitions) or occurs when the continuous state hits a pre-specified boundary (forced transitions). Whenever a switching occurs, the hybrid state is reset to a new state according to a probability measure which depends itself on the past hybrid state.

In [5], [6], [15] a sequential MC simulation approach for estimating small reachability probabilities has been developed and its convergence properties has been characterized.

This work was supported by the European Commission under the iFly project.

M. Prandini is with Dipartimento di Elettronica e Informazione, Politecnico di Milano, Italy - prandini@elet.polimi.it

H.A.P. Blom is with National Aerospace Laboratory NLR and with Delft Technical University, The Netherlands - blom@nlr.nl

G.J. (Bert) Bakker is with National Aerospace Laboratory NLR, The Netherlands - bakker@nlr.nl

The idea behind this approach is to express the small probability of interest as the product of a certain number of larger probabilities, which can be efficiently estimated by the MC approach. This can be achieved by introducing nested sets of intermediate states that are visited one set after the other before reaching the final set of states of interest. The reachability probability to be estimated is then given by the product of the conditional probabilities of reaching a set of intermediate states given that the previous one has been reached. Each conditional probability is estimated by simulating in parallel several copies of the system, i.e., each copy is considered as a *particle* following the trajectory generated through the system dynamics. To ensure unbiased estimation, the simulated process must have the strong Markov property.

In [3], [4], the sequential MC simulation approach was extended to mid-air collision risk estimation in free flight by developing an Interacting Particle System (IPS) algorithm where the sequential MC approach is applied to a Stochastically and Dynamically Coloured Petri Net (SDCPN) model of a free flight operation. The adopted Petri net formalism is quite powerful for developing a compositional model of a complex multi-agent system such as that involved in free flight operations. More importantly, SDCPN has been shown in [9] to be equivalent to GSHS, which satisfies the strong Markov property needed for the sequential MC simulation approach, [10]. The results of the IPS algorithm can provide some valuable feedback to the operational concept designers as discussed in [4]. The aspect of interest here is that, although the sequential MC simulation approach outperforms the standard MC approach for free flight safety verification, it still poses very high requirements on the availability of dynamic computer memory and simulation time.

In this paper, we explore the possibility of improving the performance of the IPS algorithm for collision risk estimation by selecting among the initial aircraft configurations those that are more prone to lead to a collision. This idea is inspired by the variance reduction technique known in the MC simulation literature as *importance sampling*. In importance sampling, samples are extracted according to a "biased" distribution that is higher in those regions making the most important contribution to the quantity to be estimated. The outcome of the simulations is then re-scaled to get an unbiased estimate.

The identification of the configurations that are more prone to lead to a collision is based on the evaluation of the air traffic complexity: the larger is the complexity, the more difficult is to guarantee safety. The probabilistic complexity measure recently introduced in [20]is chosen to this purpose. The results of a preliminary study on the proposed integrated approach to collision risk estimation are presented. Directions of further investigation are also discussed.

II. IPS-BASED COLLISION RISK ESTIMATION

This section is based on [4, Section 10.2].

We assume that air traffic operations are represented by a stochastic hybrid process $\{x_t, q_t\}$ which satisfies the strong Markov property.

For an N-aircraft free flight traffic scenario the stochastic hybrid process $\{x_t, q_t\}$ consists of Euclidean valued components $x_t{:=}(x_t^0, x_t^1, \ldots, x_t^N)$ and discrete valued components $q_t := (q_t^0, q_t^1, \ldots, q_t^N)$, where x_t^i assumes values from $\mathbb{R}^{n_i}$, and q_t^i assumes values from a finite set M_i. Physically, $\{x_t^i, q_t^i\}$, $i = 1, \ldots, N$, is the hybrid state process related to the i-th aircraft, and $\{x_t^0, q_t^0\}$ is a hybrid state process of all non-aircraft components. The process $\{x_t, q_t\}$ is $\mathbb{R}^n \times M$-valued with $n = \sum_{i=0}^N n_i$ and $M = \bigotimes_{i=0}^N M_i$.

In order to model collisions between aircraft, we introduce mappings from the Euclidean valued process $\{x_t\}$ into the relative position and velocity between two aircraft (i, j). Relative horizontal position and velocity are obtained through the mappings $y^{ij}(x_t)$ and $v^{ij}(x_t)$, respectively. The relative vertical position is obtained through $z^{ij}(x_t)$, and the relative vertical rate of climb through $r^{ij}(x_t)$.

A collision between aircraft (i, j) means that the process $\{y^{ij}(x_t), z^{ij}(x_t)\}$ hits the boundary of an area where the distance between aircraft i and j is smaller than their physical size. For simplicity, we approximate the volume of an aircraft by a cylinder whose orientation does not change in time. Then, under the assumption that all aircraft have the same size and their length is equal to their width, aircraft (i, j) have zero separation if $x_t \in D^{ij} = \{x \in \mathbb{R}^n : |y^{ij}(x)| \leq l \text{ and } |z^{ij}(x)| \leq h\}$, $i \neq j$, where l and h are the length and height of the aircraft. If x_t hits D^{ij} at time τ^{ij}, then we say that a collision event between aircraft (i, j) occurs at τ^{ij}, i.e., $\tau^{ij} = \inf\{t > 0 \ : \ x_t \in D^{ij}\}$, $i \neq j$. The first moment τ^i of collision of aircraft i with any other aircraft can then be expressed as $\tau^i = \inf_{j \neq i}\{\tau^{ij}\} = \inf\{t > 0 : x_t \in D^i\}$, with $D^i \triangleq \cup_{j \neq i} D^{ij}$. From τ^i on, we can assume that $\{x_t^i, q_t^i\}$ stop evolving.

An unbiased estimation procedure of the risk would be to run many simulations over a period of length T and count all cases in which the realization of τ^i is smaller than T. An estimator for the collision risk of aircraft i per unit T of time then is the fraction of simulations for which $\tau^i < T$.

A. Risk factorization using multiple conflict levels

Prior to a collision of aircraft i with aircraft j, a sequence of conflicts ranging from long term to short term always occurs. In order to incorporate this explicitly in the MC simulation, we formalize this sequence of conflict levels through a sequence of closed subsets of $\mathbb{R}^n$: $D^{ij} = D_m^{ij} \subset D_{m-1}^{ij} \subset \ldots \subset D_1^{ij}$ with for $k = 1, \ldots, m$:

$$D_k^{ij} = \{x \in \mathbb{R}^n : |y^{ij}(x) + \Delta v^{ij}(x)| \leq d_k \ \text{ and } \\ |z^{ij}(x) + \Delta r^{ij}(x)| \leq h_k, \ \text{ for some } \Delta \in [0, \Delta_k]\},$$

with d_k, h_k and Δ_k the parameters of the conflict definition at level k satisfying $d_{k+1} \leq d_k$, $h_{k+1} \leq h_k$ and $\Delta_{k+1} \leq \Delta_k$, and $d_m = l$, $h_m = s$ and $\Delta_m = 0$. If x_t hits D_k^{ij} at time τ_k^{ij}, then we say the first level k conflict event between aircraft (i, j) occurs at τ_k^{ij}, i.e., $\tau_k^{ij} = \inf\{t > 0 \ : \ x_t \in D_k^{ij}\}$.

Similarly as we did for the collision situation, we set $D_k^i \triangleq \cup_{j \neq i} D_k^{ij}$ and define the first moment τ_k^i that aircraft

i reaches conflict level k with any of the other aircraft as $\tau_k^i = \inf_{j \neq i}\{\tau_k^{ij}\} = \inf\{t > 0 \ : \ x_t \in D_k^i\}$.

Introducing the $\{0,1\}$-valued random variables $\{\chi_k^i, k = 0, 1, \ldots, m\}$

$$\chi_k^i = \begin{cases} 1, & \text{if } \tau_k^i < T \text{ or } k = 0 \\ 0, & \text{otherwise,} \end{cases}$$

then, the probability of collision of aircraft i with any of the other aircraft can be expressed as a product of conditional probabilities of reaching the next conflict level given that the current conflict level has been reached:

$$\mathbb{P}\left(\tau_m^i < T\right) = \mathbb{E}[\chi_m^i] = \mathbb{E}\left[\prod_{k=1}^{m} \chi_k^i\right] = \prod_{k=1}^{m} \gamma_k^i, \qquad (1)$$

where $\gamma_k^i := \mathbb{P}\left(\tau_k^i < T \,|\, \tau_{k-1}^i < T\right) = \mathbb{E}\left[\chi_k^i \,|\, \chi_{k-1}^i = 1\right]$.

With this, the problem can be seen as that of estimating the conditional probabilities γ_k^i in such a way that the product of these estimators is unbiased. Because of the multiplication of the various individual γ_k^i estimators, which depend on each other, in general such a product may be heavily biased. The key novelty in [5] was to show that such a product can be evaluated in an unbiased way when $\{x_t\}$ is a component of a larger stochastic process that satisfies the strong Markov property. This approach is explained next.

B. Characterization of the risk factors

Let $E' = \mathbb{R}^{n+1} \times M$ and $\mathcal{E}'$ be the Borel σ-algebra of E'. For any $B \in \mathcal{E}'$, $\pi_k^i(B)$ denotes the conditional probability of $\xi_k \triangleq (\tau_k, x_{\tau_k}, q_{\tau_k}) \in B$ given $\chi_l^i = 1$ for $1 \leq l \leq k$.

Define $Q_k^i = (0, T) \times D_k^i \times M$, $k = 1, \ldots, m$. Then the estimation of the probability for ξ_k to arrive at the k-th nested Borel set Q_k^i is characterized through the following recursive set of transformations

$$\begin{array}{ccccc} & \text{prediction} & & \text{conditioning} & \\ \pi_{k-1}^i(\cdot) & \longrightarrow & p_k^i(\cdot) & \longrightarrow & \pi_k^i(\cdot) \\ & & \downarrow & & \\ & & \gamma_k^i & & \end{array}$$

where $p_k^i(B)$ is the conditional probability of $\xi_k \in B$ given $\chi_l^i = 1$ for $0 \leq l \leq k-1$.

Because $\{x_t, q_t\}$ is a strong Markov process, $\{\xi_k\}$ is a Markov sequence. Hence the one step prediction of ξ_k satisfies a Chapman-Kolmogorov equation:

$$p_k^i(B) = \int_{E'} p_{\xi_k \,|\, \xi_{k-1}}(B|\xi) \; \pi_{k-1}^i(d\xi), \; B \in \mathcal{E}'. \qquad (2)$$

The conditional probability of reaching the next level satisfies

$$\gamma_k^i = \int_{E'} 1_{\{\xi \in Q_k^i\}} \; p_k^i(d\xi), \qquad (3)$$

and, as a consequence, the conditioning can be expressed as

$$\pi_k^i(B) = \frac{\int_B 1_{\{\xi \in Q_k^i\}} \; p_k^i(d\xi)}{\int_{E'} 1_{\{\xi' \in Q_k^i\}} \; p_k^i(d\xi')}, \; B \in \mathcal{E}'. \qquad (4)$$

With this, each of the m terms γ_k^i in (1) is characterized as a solution of a sequence of "filtering" kind of equations (2)–(4), which differ from the standard filtering equations because they do not present any stochastic term.

C. IPS algorithm

Based on these theoretical results, we next describe an IPS simulation algorithm for an arbitrary hybrid state strong Markov process modeling air traffic. The IPS algorithm involves running m times the numerical version of the filtering equations (2)–(4). The numerical approximations of γ_k^i, p_k^i and π_k^i are denoted as $\bar{\gamma}_k^i$, $\bar{p}_k^i$ and $\bar{\pi}_k^i$, respectively. When simulating from D_{k-1}^i to D_k^i, a fraction $\bar{\gamma}_k^i$ only of the trajectories will reach D_k^i within the time period $(0, T)$.

IPS Step 0. Initial sampling for $k = 0$.

- For $l = 1, \ldots, N_p$ generate initial state value outside Q_1^i by independent drawings (x_0^l, q_0^l) from $p_{x_0, q_0}(\cdot)$ and set $\xi_0^l = (0, x_0^l, q_0^l)$.
- For $l = 1, \ldots, N_p$, set the initial weights: $\omega_0^l = 1/N_p$.
- Then $\bar{\pi}_0^i = \sum_{l=1}^{N_p} \omega_0^l \, \delta_{\{\xi_0^l\}}$.

IPS Iteration cycle: For $k = 1, \ldots, m$ perform step 1 (prediction), step 2 (assess fraction), step 3 (conditioning), and step 4 (resampling).

IPS Step 1. Prediction of $\pi_{k-1}^i \longrightarrow p_k^i$, based on (2);

- For $l = 1, \ldots, N_p$ simulate a new path of the hybrid state Markov process, starting at ξ_{k-1}^l until the k-th set Q_k^i is hit or $t = T$ (the first component of ξ_k^l counts time).
- This yields new particles $\{\hat{\xi}_k^l, \omega_{k-1}^l\}_{l=1}^{N_p}$.
- $\bar{p}_k^i$ is the empirical distribution associated with the new cloud of particles: $\bar{p}_k^i = \sum_{l=1}^{N_p} \omega_{k-1}^l \delta_{\xi_k^l}$.

IPS Step 2. Assess fraction γ_k^i, based on (3);

- The particles that do not reach the set Q_k^i are killed, i.e., we set $\hat{\omega}_k^l = 0$ if $\hat{\xi}_k^l \notin Q_k^i$ and $\hat{\omega}_k^l = 0$ if $\hat{\omega}_k^l = w_{k-1}^l$ if $\hat{\xi}_k^l \in Q_k^i$.
- Approximation: $\gamma_k^i \approx \bar{\gamma}_k^i = \sum_{l=1}^{N_p} \hat{\omega}_k^l$. If all particles are killed, i.e., $\bar{\gamma}_k^i = 0$, then the algorithm stops without $\mathbb{P}(\tau^i < T)$ estimate.

IPS Step 3. Conditioning of $p_k^i \longrightarrow \pi_k^i$, based on (4);

The non-killed particles form a set S_k^i, i.e., iff $\hat{\xi}_k^l \in Q_k^i$, then particle $\{\hat{\xi}_k^l, \hat{\omega}_k^l\}$ is stored in S_k^i.

Renumbering the particles in S_k^i yields a set of particles $\{\tilde{\xi}_k^l, \tilde{\omega}_k^l\}_{l=1}^{N_{S_k}}$ with N_{S_k} the number of particles in S_k^i. Hence, we also have $\bar{\gamma}_k^i = \sum_{l=1}^{N_{S_k}} \tilde{\omega}_k^l$.

IPS Step 4. Resampling of π_k^i;

Resample N_p particles from S_k^i as follows:

If $\frac{1}{2}N_p \leq N_{S_k} \leq N_p$, then copy the N_{S_k} particles, i.e., $\xi_k^l = \tilde{\xi}_k^l$ and set $\omega_k^l = \tilde{\omega}_k^l \, N_{S_k}/(\bar{\gamma}_k^i \, N_p)$ for $l = 1, \ldots, N_{S_k}$; the total weight of these particles is N_{S_k}/N_p. Subsequently, draw $N_p - N_{S_k}$ particles ξ_k^l independently from the empirical measure $\bar{\pi}_k^i = \sum_{l=1}^{N_{S_k}} \tilde{\omega}_k^l \, \delta_{\{\tilde{\xi}_k^l\}}$ and set $\omega_k^l = 1/N_p$; the total weight of this is $1 - N_{S_k}/N_p$.

If $N_{S_k} < \frac{1}{2}N_p$, then copy the N_{S_k} particles, i.e., $\xi_k^l = \tilde{\xi}_k^l$, and set $\omega_k^l = \frac{1}{2}\tilde{\omega}_k^l/\bar{\gamma}_k^i$ for $l = 1, \ldots, N_{S_k}$; the total weight of these particles is $\frac{1}{2}$. For the remaining weight of $\frac{1}{2}$, independently draw $N_p - N_{S_k}$ particles ξ_k^l from the empirical measure $\bar{\pi}_k^i = \sum_{l=1}^{N_{S_k}} \tilde{\omega}_k^l \, \delta_{\{\xi_k^l\}}$ and set $\omega_k^l = \frac{\frac{1}{2}}{N_p - N_{S_k}}$.

After step 4, the new set of particles is $\{\xi_k^l, \omega_k^l\}_{l=1}^{N_p}$. If $k < m$ then repeat steps 1, 2, 3, 4 for $k := k+1$. Otherwise, stop with $\mathbb{P}(\tau^i < T) \approx \prod_{k=1}^{m} \bar{\gamma}_k^i$.

III. AIR TRAFFIC COMPLEXITY METRIC

The concept of air traffic complexity has been originally introduced to evaluate the difficulty perceived by the air traffic controllers in handling safely a certain air traffic situation (ATC workload). The idea is that assessing the impact on the ATC workload of different air traffic configurations can help to evaluate how the current ground-based ATM system is operated, and can also provide guidelines on how to obtain more manageable sectors by reconfiguring the airspace and by modifying traffic patterns. In the next generation ATM systems where the trajectory management and separation functions will be distributed on board of the aircraft, the availability of complexity metrics can help to predict air traffic situations that could over burden the distributed ATM system, and also benefit the trajectory management operations (see [19] for a detailed discussion).

Most studies on air traffic complexity have been developed with reference to ground-based ATM. Those complexity metrics as the dynamic density [12], [21] where workload and air traffic measurements are incorporated within a single aggregate indicator depend on the adopted notion and measure of workload, and inherently incorporate various human factors aspects. Workload-oriented metrics are sector-based, and often show structural dependence on the sector characteristics, which further limits their applicability to a sector-free context such as free flight. The difficulty in obtaining reliable workload measures has been one of the strongest motivations for investigating complexity metrics independent of the ATC workload, such as the input-output approach in [13], the fractal dimension in [14], and the intrinsic complexity measures in [7], [8]. These metrics are actually those that appear more portable to a free flight context.

In this paper we adopt the complexity measure introduced in [20] for possible application to self-separation airspace. Such a measure is based on the notion of probabilistic occupancy of the airspace. Complexity is in fact evaluated in terms of proximity in time and space of the aircraft as determined by their intent and current state, while taking into account uncertainty in the aircraft future position. Specifically, air traffic complexity at a point s in an airspace region $\mathcal{S} \subset \mathbb{R}^3$ and at time t within the look-ahead time horizon $[0, T]$ is evaluated as the probability that a certain buffer zone $\mathcal{V}(s)$ in the airspace surrounding s will be "congested" within $[t, t+\delta]$, with $\delta > 0$. By defining congestion as the simultaneous occupancy of the buffer zone by a certain number of aircraft and evaluating this complexity measure at all possible points in $\mathcal{S}$, a complexity map can be built. Forming the complexity maps associated with different consecutive time intervals allows to predict when the aircraft will enter and leave a particular zone in the airspace, and to identify regions of the airspace $\mathcal{S}$ with a limited inter-aircraft maneuverability space.

From a single aircraft perspective, the complexity experienced by aircraft A along its nominal trajectory $\bar{s}_A : [0, T] \to \mathcal{S}$ within the time interval $[t_0, t_f]$ can be evaluated by making the buffer zone $\mathcal{V}(s)$ move along $\bar{s}_A(t)$ and computing the probability that some other aircraft i, $i = 1, 2, \ldots, N$, present in the same airspace area will enter such moving zone:

$$\begin{aligned} \mathrm{c}_A(t_0, t_f) :=& P(s_i(t) \in \mathcal{V}(\bar{s}_A(t)) \text{ for some } t \in [t_0, t_f] \\ & \text{and } i \in \{1, 2, \ldots, N\}), \end{aligned}$$

where $s_i(t)$ is the predicted position of aircraft i at time t. From an operational perspective, the so-obtained single-aircraft complexity measure can be used by aircraft A to evaluate the maneuverability space surrounding its nominal trajectory and to eventually redesign it. Interestingly, if the time window $[t_0, t_f]$ extends to the whole look-ahead time horizon $[0, T]$ and the buffer zone reproduces the protection zone surrounding the aircraft, then, $\mathrm{c}_A(t_0, t_f)$ can as well be interpreted as the probability of aircraft A getting in conflict with another aircraft i within time horizon $[0, T]$.

From a computational viewpoint, analytic – though approximate – expressions for $\mathrm{c}_A(t_0, t_f)$ are determined in [20] with reference to the case of an ellipsoidal buffer zone

$$\mathcal{V}(s) = \left\{ \hat{s} \in \mathbb{R}^3 : \ (\hat{s} - s)^T V (\hat{s} - s) \le 1 \right\},$$

with $V = \operatorname{diag}\left(\frac{1}{{r_h}^2}, \frac{1}{{r_h}^2}, \frac{1}{{r_v}^2}\right)$, and piecewise linear nominal trajectories. The model for predicting the future aircraft position when determining these formulas is characterized by a Gaussian prediction error whose variance grows not only linearly with time t but also faster in the along-track direction than in the cross-track directions.

IV. RESULTS OF A PRELIMINARY STUDY

In this section we present and discuss the results obtained by using the complexity metric in Section III to accelerate the IPS algorithm for collision risk estimation in free flight.

The free flight model considered is implemented in an SDCPN simulator of the Autonomous Mediterranean Free Flight (AMFF) [16], which was developed to study the introduction of autonomous free flight operation in Mediterranean airspace. The SDCPN model is composed of interconnected Local Petri Nets modelling each agent involved in the process (e.g., aircraft, pilot, navigation and surveillance equipment) and is described in details in [4].

The IPS algorithm was applied to an hypothetical AMFF air traffic scenario where one aircraft is flying through a virtual infinite airspace of randomly distributed aircraft. The traffic density was set equal to 2.5 times the density of 0.0032

TABLE I

IPS CONFLICT LEVEL PARAMETER VALUES.

k	1	2	3	4	5	6	7	8
d_k (nmi)	4.5	4.5	4.5	4.5	2.5	1.25	0.5	0.054
h_k (ft)	900	900	900	900	900	500	250	131
Δ_k (min)	8	2.5	1.5	0	0	0	0	0

aircraft per nmi^3 experienced on 23rd July 1999 in an en-route busy area near Frankfurt. To reproduce such a density, the airspace was divided into packed containers, each one having a length of 40 nmi, a width of 40 nmi, and a height of 3900 feet and containing 8 aircraft. The virtually infinite airspace is built according to the following procedure. A set of 8 aircraft ($i = 1, 2, \ldots, 8$) flying at arbitrary position and in arbitrary direction at a ground speed of about 466 nmi/h is generated first in a container. Duplicates of this container are then piled on top and next to each other.

The goal is to estimate the probability of collision of aircraft $i = 1$ in the central container with any of the other aircraft per unit time of flying.

The IPS conflict levels k are defined by the values for the lateral conflict distance d_k, conflict height h_k, and time to conflict Δ_k in Table I. These values have been determined through two steps. The first was to let an operational expert make a best guess of proper parameter values. Next, during initial simulations with the IPS some fine tuning of the number of levels and of parameter values per level has been done.

By running the IPS algorithm ten times over 15 minutes the collision probability per unit time of flying can be estimated. The number of particles per IPS simulation run is 10000. The results obtained by applying the standard IPS method are reported in Table II.

TABLE II

RESULTS OBTAINED THROUGH THE STANDARD IPS.

run	**risk estimate**
1	$7.28 \cdot 10^{-5}$
2	$8.83 \cdot 10^{-5}$
3	$3.54 \cdot 10^{-5}$
4	$1.03 \cdot 10^{-4}$
5	$1.22 \cdot 10^{-5}$
6	$7.21 \cdot 10^{-5}$
7	$2.66 \cdot 10^{-6}$
8	$3.94 \cdot 10^{-5}$
9	$1.41 \cdot 10^{-4}$
10	$8.03 \cdot 10^{-6}$
mean	$5.75 \cdot 10^{-5}$

In the integrated approach to collision risk estimation, for each particle the complexity $c_A(t_0, t_f)$ experienced by aircraft $i = 1$ in the central container during $[t_0, t_f] = [0, 15]$ min is computed. The value taken by $c_A(t_0, t_f)$ is used to decide whether some given airspace configuration has to be propagated through the system dynamics in the IPS algorithm or not. In the former case we call the configuration a *selected particle*. To the purpose of computing the complexity measure $c_A(t_0, t_f)$, the growth rates of the variance of the uncertainty affecting the future aircraft position were set equal to 0.0625 nmi^2/min in the along track direction, and 0.04 nmi^2/min in the cross track directions. The parameters r_h and r_v defining the ellipsoidal buffer region were set equal to $r_h = r_v = 0.05$ nmi to reproduce a condition of collision. The approximated formula in [20] obtained by making a zero-th order expansion of the integral involved in the exact computation of $c_A(t_0, t_f)$ was used to evaluate complexity from the perspective of aircraft $i = 1$ in the central container.

TABLE III

RESULTS OBTAINED WHEN THE THRESHOLD IS SET EQUAL TO 0.02.

run	**selected particles**	**estimate**	**gain**
1	398	$7.46 \cdot 10^{-8}$	0.0
2	415	$3.40 \cdot 10^{-5}$	9.3
3	385	$1.20 \cdot 10^{-5}$	8.8
4	368	$9.20 \cdot 10^{-5}$	24.3
5	347	$5.90 \cdot 10^{-7}$	1.4
6	392	0.0	0.0
7	358	0.0	0.0
8	377	0.0	0.0
9	386	$7.73 \cdot 10^{-5}$	14.2
10	382	0.0	0.0
mean	381	$2.16 \cdot 10^{-5}$	5.8

overall gain: (10000/381)/($5.75 \cdot 10^{-5}/2.16 \cdot 10^{-5}$) = 9.9

TABLE IV

RESULTS OBTAINED WHEN THE THRESHOLD IS SET EQUAL TO 0.025.

run	**selected particles**	**estimate**	**gain**
1	146	$7.46 \cdot 10^{-8}$	0.1
2	147	$3.39 \cdot 10^{-5}$	26.1
3	142	$2.45 \cdot 10^{-6}$	4.9
4	143	$9.20 \cdot 10^{-5}$	62.6
5	133	$5.90 \cdot 10^{-7}$	3.6
6	133	0.0	0.0
7	142	0.0	0.0
8	134	0.0	0.0
9	146	$7.73 \cdot 10^{-5}$	37.6
10	143	0.0	0.0
mean	141	$2.06 \cdot 10^{-5}$	13.5

overall gain: (10000/141)/($5.75 \cdot 10^{-5}/2.06 \cdot 10^{-5}$) = 25.5

TABLE V

RESULTS OBTAINED WHEN THE THRESHOLD IS SET EQUAL TO 0.03.

run	**selected particles**	**estimate**	**gain**
1	65	$7.46 \cdot 10^{-8}$	0.2
2	56	$3.39 \cdot 10^{-5}$	68.5
3	54	$2.45 \cdot 10^{-6}$	12.8
4	50	$9.20 \cdot 10^{-5}$	179.1
5	44	0.0	0.0
6	57	0.0	0.0
7	46	0.0	0.0
8	55	0.0	0.0
9	72	0.0	0.0
10	48	0.0	0.0
mean	55	$1.28 \cdot 10^{-5}$	26.1

overall gain: (10000/55)/($5.75 \cdot 10^{-5}/1.28 \cdot 10^{-5}$) = 40.9

In the integrated approach to collision risk estimation, the initial particles for which $c_A(t_0, t_f)$ is lower that some threshold are discarded and the IPS algorithm is run on the selected particles only. The impact of the complexity-based selection procedure in terms of reduction of the number of particles to simulate and degradation of the IPS collision risk estimate can be evaluated in each run through the following quantity

$$\text{gain} := \frac{\frac{\text{original number of particles}}{\text{number of selected particles}}}{\frac{\text{risk estimate with all particles}}{\text{risk estimate with the selected particles}}}.$$

The overall gain over the 10 runs of the IPS algorithm can be computed through the same formula applied to average quantities.

Increasing values of the threshold (0.01, 0.015, 0.02, 0.025, 0.03, and 0.035) have been considered in our experiments. Due to space limitations, we report here only the more representative results obtained when the threshold is equal to 0.02, 0.025 and 0.03 (see Tables III, IV and V). As expected, as the threshold increases, the number of selected particles decreases. This has the beneficial effect of reducing the computational effort in the IPS algorithm, but may lead to an excessive impoverishment of the set of initial particles and cause the collision risk estimate to be zero (i.e., no particle reaches the final level 8). In view of this consideration, the threshold value 0.03 (and, hence, also 0.035) can be considered too large since the collision risk estimate is zero in 6 runs over 10 (more than 50%). The speed-up factor for the threshold values smaller than 0.03 can be estimated by re-scaling the overall gain factor with the fraction of runs that correspond to a nonzero estimated risk. The larger effective overall gain is the speed-up factor, which turns out to be 15.3 ($= 25.5 \times 6/10$) and corresponds to the threshold value 0.025.

This result looks promising, especially since the parameters entering $c_A(t_0, t_f)$ have not been tuned to the model adopted in the IPS algorithm, which may further improve the speed-up factor. On the other hand, the obtained speed-up factor does not account for the time needed to compute the complexity measure $c_A(t_0, t_f)$. In this respect, it is worth noting that the time needed to compute $c_A(t_0, t_f)$ is independent of the length of the time horizon $[0, T]$ and of the complexity of the model for the IPS simulations. Also, it scales linearly with the number of aircraft present in the airspace. The computation of $c_A(t_0, t_f)$ can in fact be parallelized by estimating in isolation the contribution of each single aircraft. Aircraft with a zero contribution to $c_A(t_0, t_f)$ can be identified based on their relative position and velocity with respect to aircraft A. All these aspects play a role in the practical application of the approach.

V. CONCLUSIONS

In this paper, we have proposed an importance sampling approach to collision risk estimation by combining the IPS algorithm in [4] with the complexity assessment method in [20]. Though the performance of the approach has been tested only in a quite preliminary study, the results look promising. Further investigation is needed in the following directions: i) tuning of the prediction model used for complexity evaluation; and ii) assessment of the computational requirements of an efficient, possibly parallelized, implementation of the complexity evaluation procedure.

REFERENCES

[1] Final report of RTCA Task Force 3: Free flight implementation. Technical report, RTCA, Washington, D.C, 1995.

[2] Safety and ASAS applications, Co-operative R&D Action Plan 1 report, version 4.1, 2004.

[3] H.A.P. Blom, J. Krystul, and G.J. Bakker. A particle system for safety verification of free flight in air traffic. In *Proceedings of the 45th IEEE Conference on Decision and Control*, 2006.

[4] H.A.P. Blom, J. Krystul, G.J. (Bert) Bakker, M. B. Klompstra, and B. Klein Obbink. Free flight collision risk estimation by sequential MC simulation. In C.G. Cassandras and J. Lygeros, editors, *Stochastic hybrid systems*, Automation and Control Engineering Series 24, pages 249–282. Taylor & Francis Group/CRC Press, 2007.

[5] F. Cérou, P. Del Moral, F. Le Gland, and P. Lezaud. Genetic genealogical models in rare event analysis. Publications du Laboratoire de Statistiques et Probabilites, Toulouse III, 2002.

[6] F. Cérou, P. Del Moral, F. Le Gland, and P. Lezaud. Limit theorems for the multi-level splitting algorithms in the simulation of rare events. In *Proceedings of Winter Simulation Conference*, Orlando, FL, 2005.

[7] D. Delahaye and S. Puechmorel. Air traffic complexity: Towards intrinsic metrics. In *Proc. of the 3rd FAA/Eurocontrol ATM R&D Seminar*, Napoli, Italy, June 2000.

[8] D. Delahaye and S. Puechmorel. Air traffic complexity map based on nonlinear dynamical system. In *4th Eurocontrol Innovative Research Workshop & Exhibition*, 2005.

[9] M.H.C. Everdij and H.A.P. Blom. Hybrid petri nets with diffusion that have into-mappings with generalised stochastic hybrid processes. In H.A.P. Blom and J. Lygeros, editors, *Stochastic Hybrid Systems: Theory and Safety Critical Applications*, LNCIS, pages 31–64. Springer, Berlin, 2006.

[10] J. Krystul, H.A.P. Blom, and A. Bagchi. Stochastic differential equations on hybrid state spaces. In C.G. Cassandras and J. Lygeros, editors, *Stochastic hybrid systems*, Automation and Control Engineering Series 24, pages 15–46. Taylor & Francis Group/CRC Press, 2006.

[11] P.E. Labeau, C. Smidts, and S. Swaminathan. Dynamic reliability: towards an integrated platform for probabilistic risk assessment. *Reliability Engineering and System Safety*, 68:219–254, 2000.

[12] I.V Laudeman, S.G Shelden, R Branstrom, and C.L Brasil. Dynamic density: an air traffic management metric. Technical report, NASA TM-1998-112226, 1998.

[13] K. Lee, E. Feron, and A. Pritchett. Describing airspace complexity: Airspace response to disturbances. *Journal of Guidance, Control, and Dynamics*, 32(1):210–222, January-February 2009.

[14] S. Mondoloni and D. Liang. Airspace fractal dimension and applications. In Eurocontrol-FAA, editor, *Fourth USA/EUROPE Air Traffic Management R& D Seminar*, 2001.

[15] P. Del Moral. *Feynman-Kac Formulae. Genealogical and Interacting Particle Systems with Applications*. Springer Verlag, New York, 2004.

[16] B. Klein Obbink. MFF airborne self separation assurance OSED, Report MFF R733D. April 2005.

[17] G. Pola, M.L. Bujorianu, J. Lygeros, and M.D. Di Benedetto. Stochastic hybrid models: an overview with applications to air traffic management. In *IFAC Conf. Analysis and Design of Hybrid Systems (ADHS)*, Saint-Malo, Brittany, France, 2003.

[18] M. Prandini and J. Hu. A stochastic approximation method for reachability computations. In H.A.P. Blom and J. Lygeros, editors, *Stochastic hybrid systems: theory and safety applications*, Lecture Notes in Control and Information Sciences 337, pages 107–139. Springer Verlag, 2006.

[19] M. Prandini, L. Piroddi, S. Puechmorel, and S.L. Brázdilová. Towards air traffic complexity assessment in new generation air traffic management systems. *IEEE Trans. on Intelligent Transportation Systems*, 2011. To appear.

[20] M. Prandini, V. Putta, and J. Hu. A probabilistic measure of air traffic complexity in three-dimensional airspace. *International Journal of Adaptive Control and Signal Processing, special issue on Air Traffic Management: Challenges and opportunities for advanced control*, 24(10):813–829, 2010.

[21] B. Shridar, K.S. Seth, and S. Grabbe. Airspace complexity and its application in air traffic management. In *4th FAA/Eurocontrol Air Traffic Management R&D Seminar*, 2001.