National Aerospace Laboratory NLR
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Stochastic Hybrid System Modeling, Bisimulation and Safety Verification of Airborne Self Separation

by

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Key Research Question

- Free Flight (or Airborne Self Separation) has been "invented" as a potential solution for high traffic demand airspace

- ATM community research trend has been to direct Airborne Self Separation research to situations of less demanding airspace (where mid-air safety risk is coming from pairwise encounters only)

- Key research question: Up to which traffic demand can Free Flight be designed sufficiently safe ?

iFly project

- Addresses the key research question for en-route airspace
- Approach:
  - Designing an advanced Free Flight concept of operation
  - Aiming for 3-6 times 2005 traffic demand over Europe
  - Analysing this advanced concept on mid-air safety risk
- This presentation:
  - First generation FF concept
  - Advanced FF concept
  - Safety analysis methods used
  - Results for a first generation FF concept
  - Initial results for an advanced FF concept

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- Bisimulation
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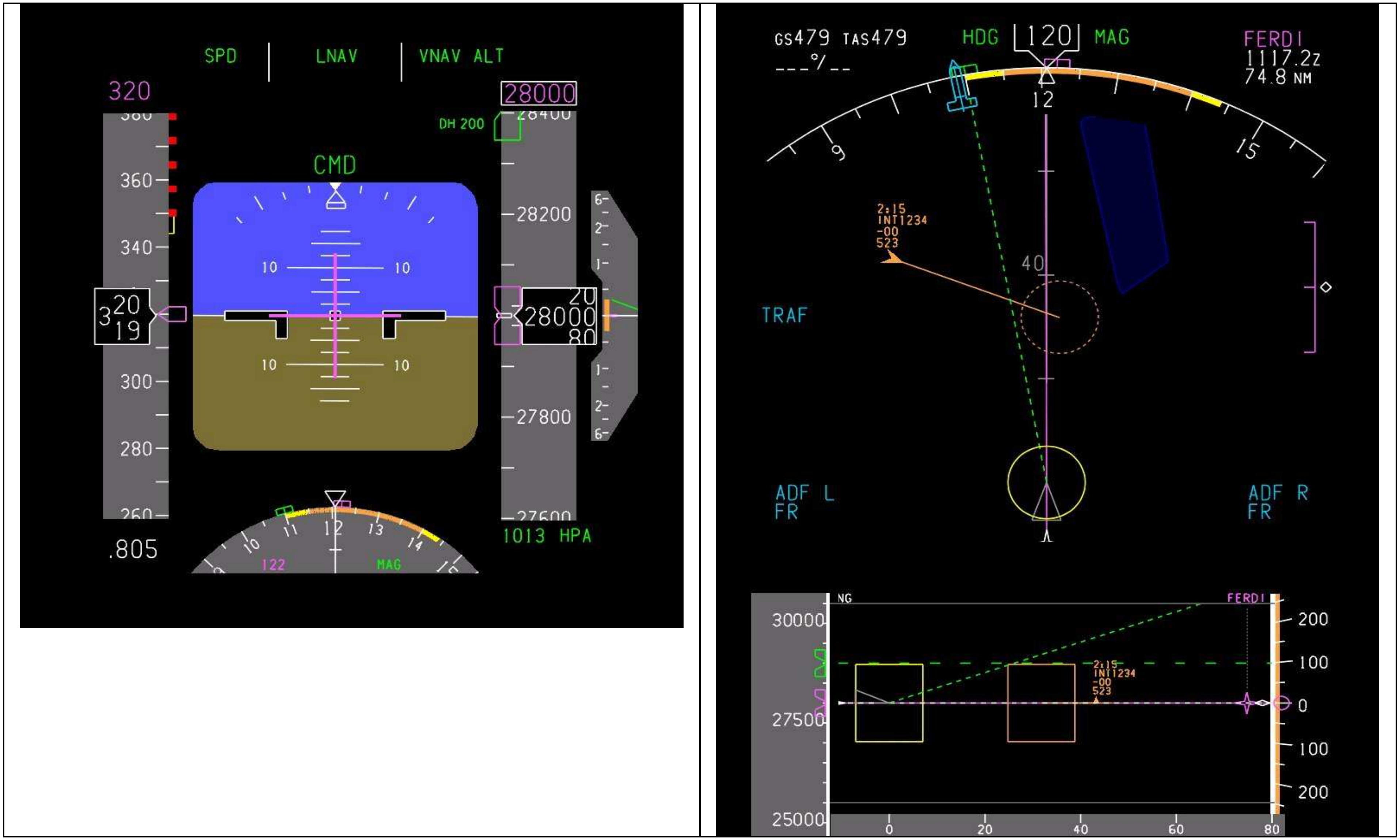
Autonomous Mediterranean Free Flight (AMFF)

- Future concept developed for traffic over Mediterranean area
- Aircrew gets freedom to select path and speed
- In return aircrew is responsible for self-separation
- Aircraft broadcast their states without delay to other aicraft
- Each a/c equipped with an Airborne Separation Assistance System
- In AMFF, conflicts are resolved one by one (pilot preference)
  - Medium term: priority a/c does nothing
  - Short term: both aircraft resolve conflict



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Evaluations performed for AMFF concept

- Real-time pilot-in-the-loop evaluations
- Eurocae/RTCA ED78a safety assessment

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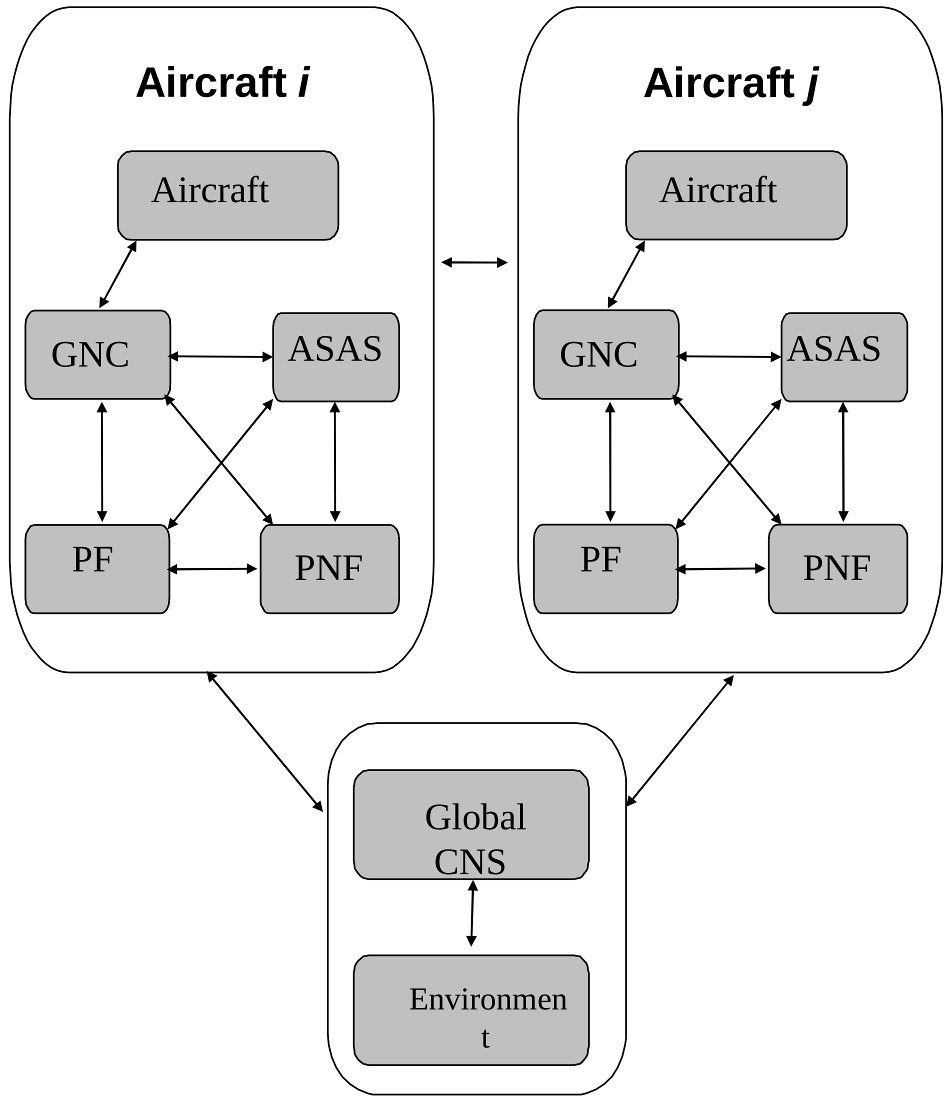
Multiple Agents in Airborne Self Separation

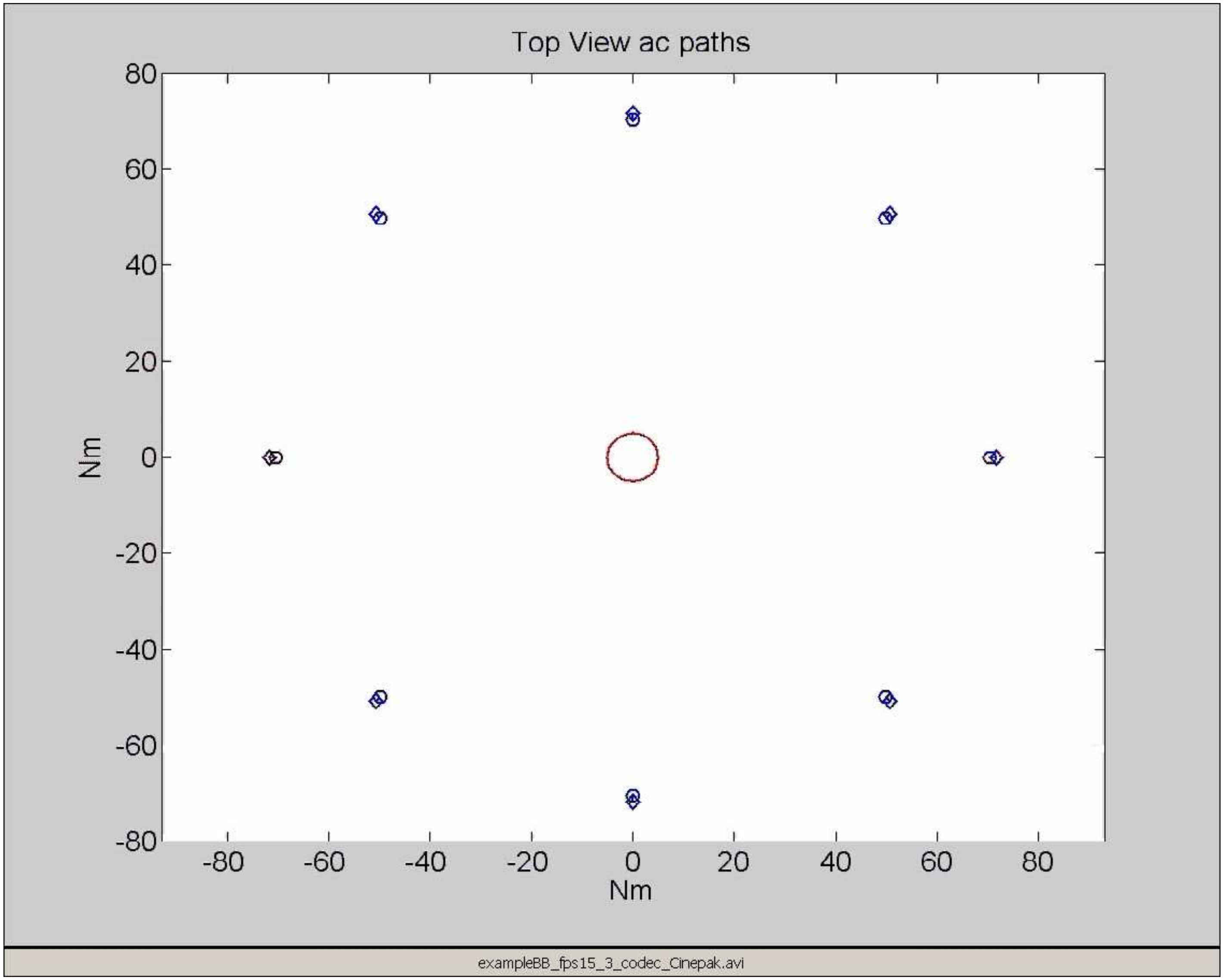


Development of SDCPN model

- Defining the relevant Agents
- Hazard identification
- Developing Petri net for each Agent
- Connecting Agent Petri nets
- Generate Monte Carlo simulation model
- Parametrization, Verification & Calibration

Size of an airborne self separation model

Agent	# of product places	Maximum colour Product state space
Aircraft	24^N	R^{13N}
Pilot-Flying (PF)	490^N	R^{28N}
Pilot-not-Flying (PNF)	7^N	R^{3N}
AGNC	$(15\times 2^{16})^N$	R^{45N}
ASAS	48^N	$R^{37N+21N^2}$
Global CNS	16	R^0
PRODUCT	$\approx 16\times(3.88\times 10^{12})^N$	$R^{126N+21N^2}$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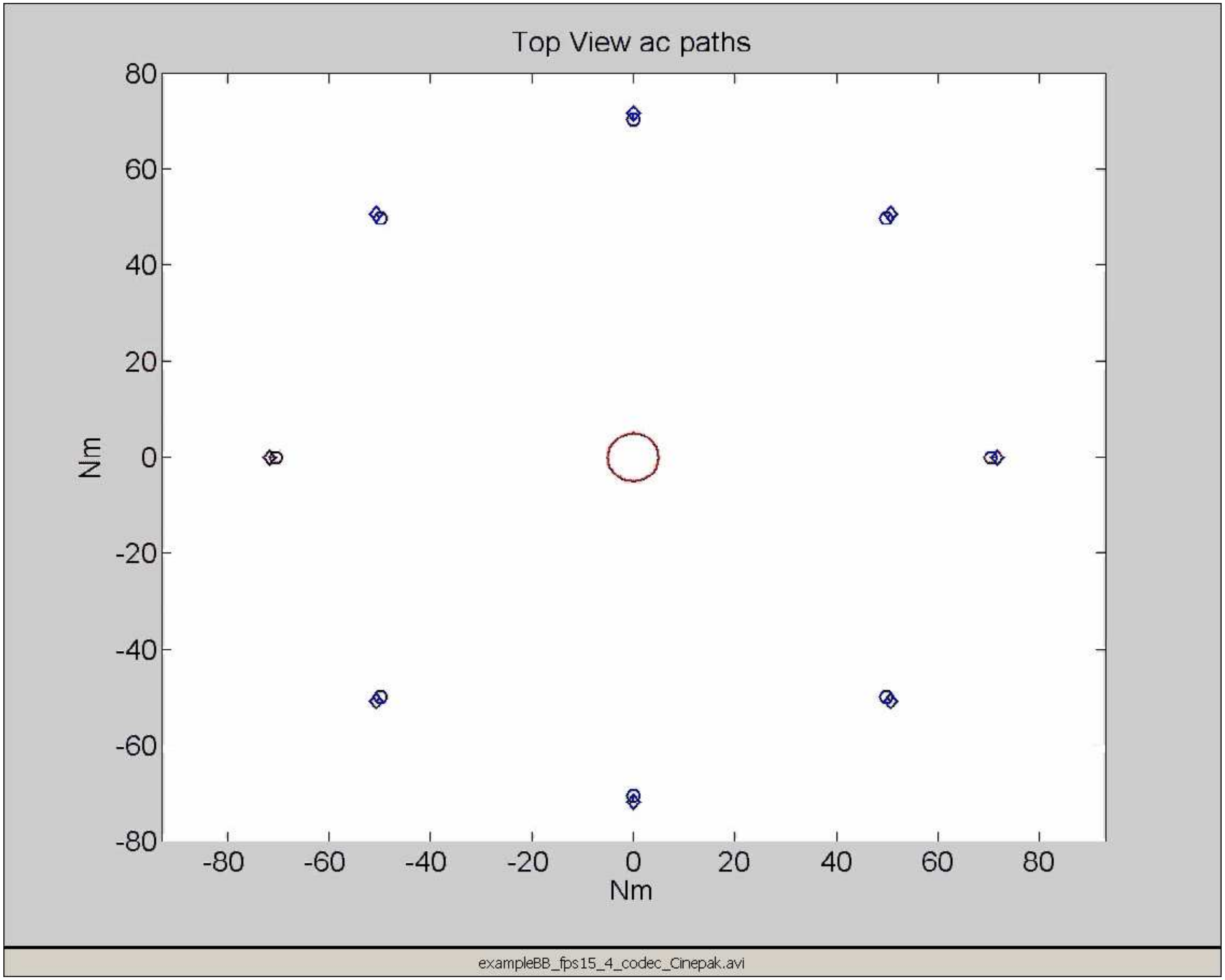
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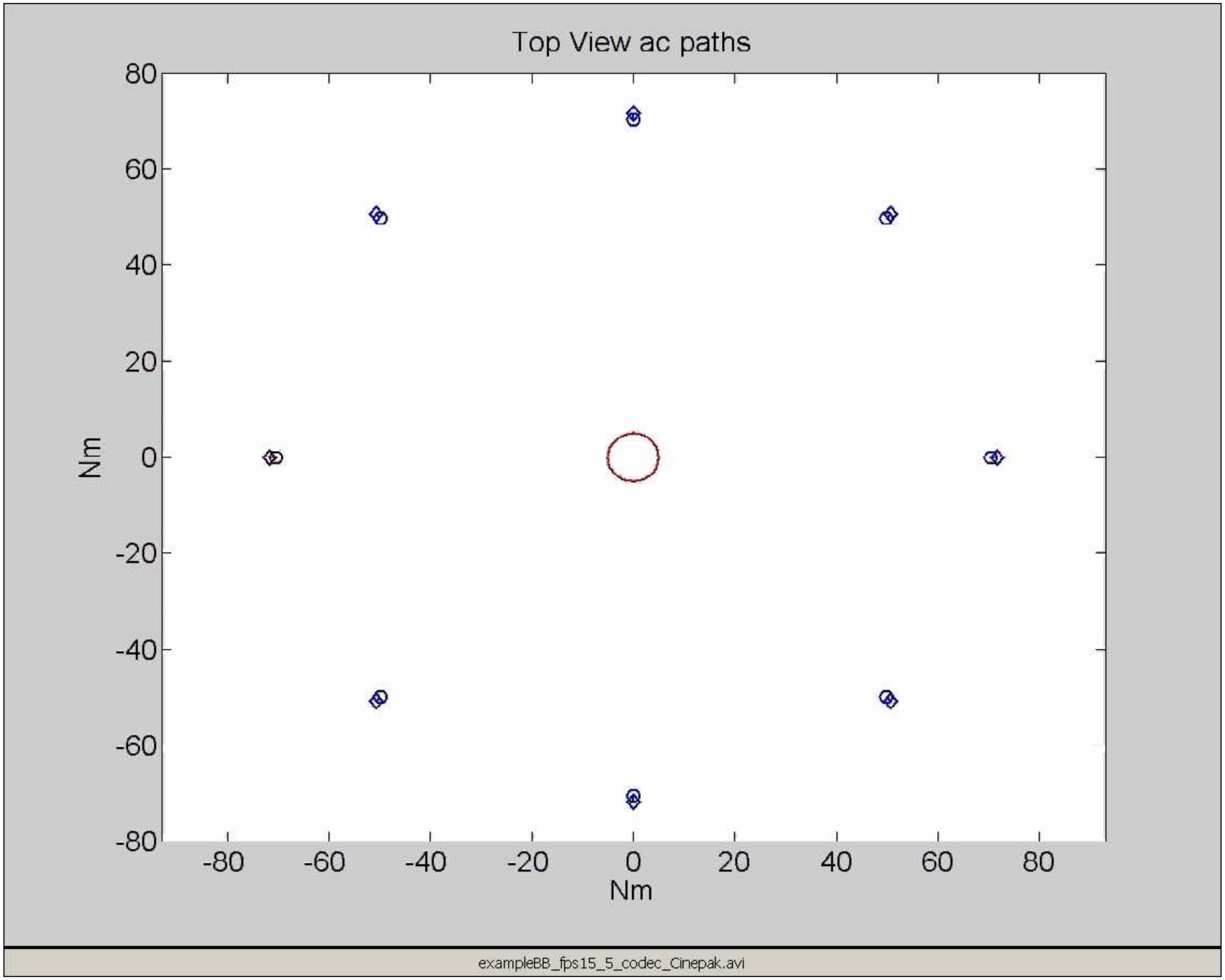

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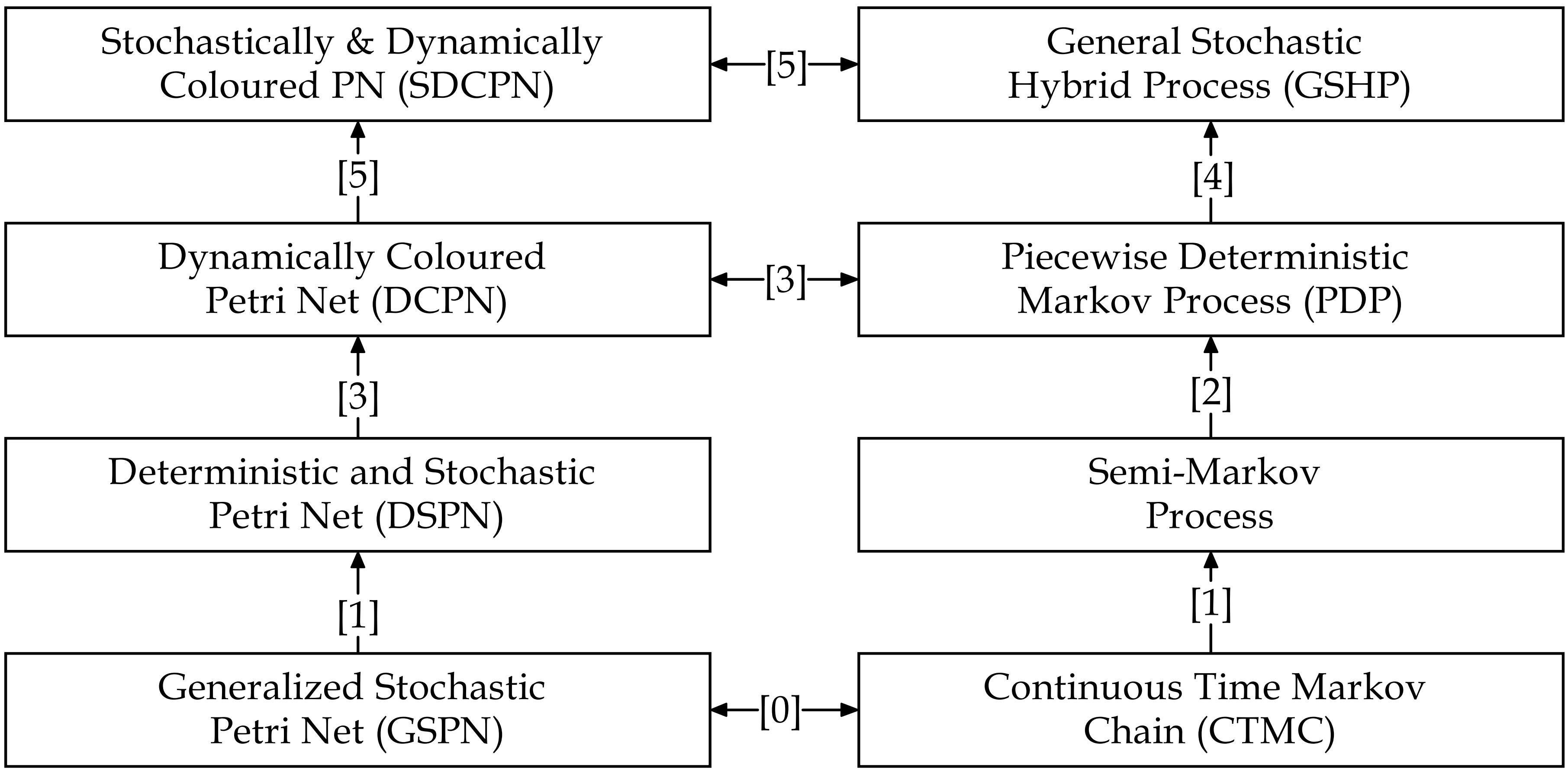
Modelling Power Hierarchy



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[1]: [Malhotra & Trivedi, 2004], [Muppala et al, 2000]

[2]: [Davis, 1984]

[3]: [Everdij & Blom, 2005]

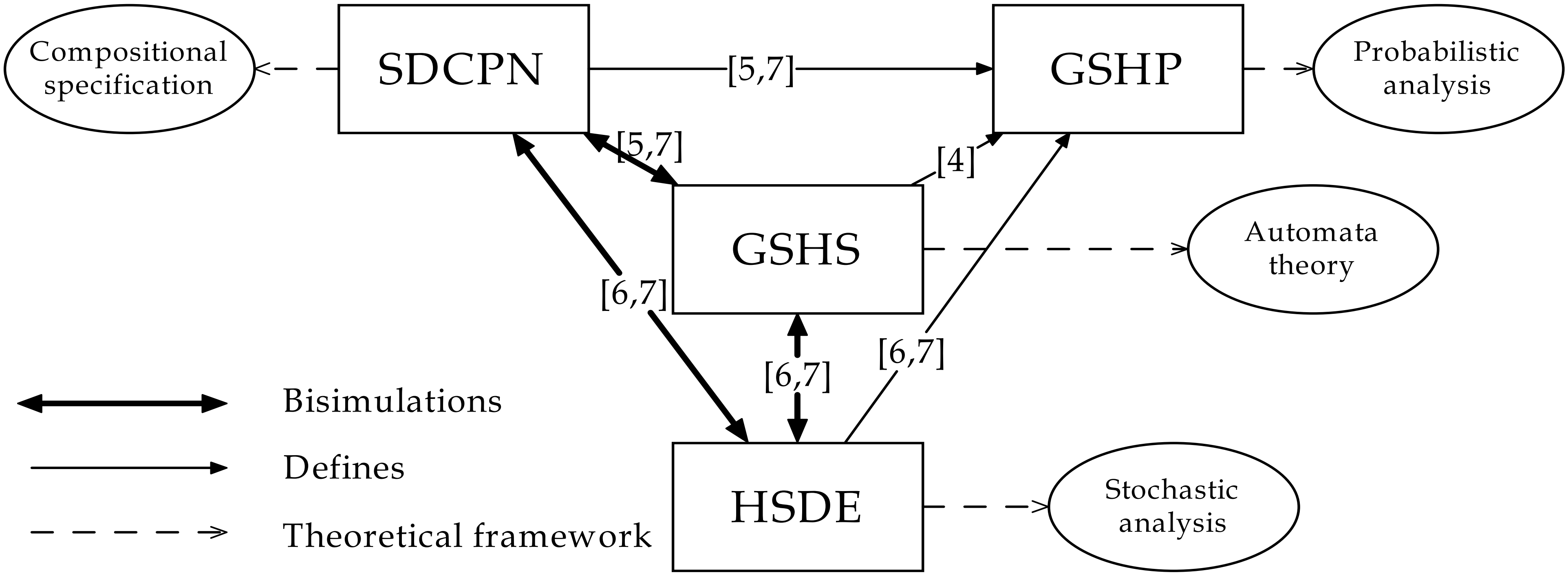
[4]: [Bujorianu & Lygeros, 2006]

[5]: [Everdij & Blom, 2006]

Bisimulation

- Two systems are bisimulations when their executions are equivalent in probabilistic sense
  - VanDerSchaft, 2004; Bujorianu et al., 2005
- Systems with GSHP executions:
  - SDCPN = Stochastically and Dynamically Coloured Petri Net
  - GSHS = General Stochastic Hybrid System
  - HSDE = Hybrid Stochastic Differential Equation

SDCPN inherits analysis power of SDE's and formal verification power of automata



[4]: [Bujorianu & Lygeros, 2006]

[5]: [Everdij & Blom, 2006]

[6]: [Everdij & Blom, 2010]

[7]: [Everdij, 2010]

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Approaches in Reach Probability Computation

- Markov Chain (MC) approximation (Prandini&Hu, 2006)
- Dynamic Programming (DP) approach (Abate, Amin, Prandini, Lygeros & Sastry, 2006)
- Interacting Particle System (IPS) approach (Cerou et al., 2005)

Interacting Particle System (IPS)

- Define a sequence of conflict levels decreasing in urgency $(D_k\text{'s})$
  - Most urgent level represents collision $(D_m = D)$
- Simulate N_p particles; initially all outside D_1 (less urgent level)
- Freeze each particle that reaches the next urgent level before T
- Make N_p copies of frozen particles
- Repeat this until the most urgent level has been reached
- Count the simulated fraction $\tilde{\gamma}_k$ that reaches level k
- Estimated collision risk = $\tilde{\gamma}_1 \times \tilde{\gamma}_2 \times \tilde{\gamma}_3 \times \ldots \times \tilde{\gamma}_m$

IPS convergence

Cerou, Del Moral, Legland and Lezaud (2002, 2005) have shown that the product of these fractions $\tilde{\gamma}_k$ forms an unbiased estimate of the probability of $\{s_t\}$ to hit the set D within the time period $[0,T)$, i.e.

$$\mathbb{E}[\prod_{k=1}^{m} \tilde{\gamma}_k] = \prod_{k=1}^{m} \gamma_k = P(\tau < T)$$

In addition there is a bound on the L^1 estimation error, i.e.:

$$\mathbb{E}(\prod_{k=1}^{m} \tilde{\gamma}_k - \prod_{k=1}^{m} \gamma_k) \leq \frac{c_p}{\sqrt{N_p}}$$

Hybrid IPS versions

1. Importance switching (Krystul&Blom, 2005)

2. Rao-Blackwellization, using exact equations for $\{\theta_t\}$ and particles for Euclidian state (Krystul&Blom, 2006)

- Both handle rare mode switching well
- Large scale SHS scalability problem remains
  - Huge number of discrete product places

Hierarchical Hybrid IPS (HHIPS)

(Blom, Bakker & Krystul, 2007, 2009)

- ✓ Define an aggregated mode process $\{\kappa_t\}$ with $\{\mathbb{M}_k,\ \kappa \in \mathbb{K}\}$ a partition of $\mathbb{M}$

$$\kappa_t = \kappa \ \text{ if } \theta_t \in \mathbb{M}_k$$

- ✓ Apply Importance switching to $\{\kappa_t\}$
- ✓ Rao-Blackwellization, i.e. use exact equations for $\{\kappa_t\}$ and particles for the other process elements $\{x_t, \theta_t\}$

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Scenarios

- Two aircraft encounter
- Eight aircraft encounter
- Random traffic

Sequence of conflict levels for air traffic

k	1	2	3	4	5	6	7	8
D_k (Nm)	4.5	4.5	4.5	4.5	2.5	1.25	0.5	0.054
h_k (ft)	900	900	900	900	900	500	250	131
Δ_k (min)	8	2.5	1.5	0	0	0	0	0

Medium Term Conflict (MTC)

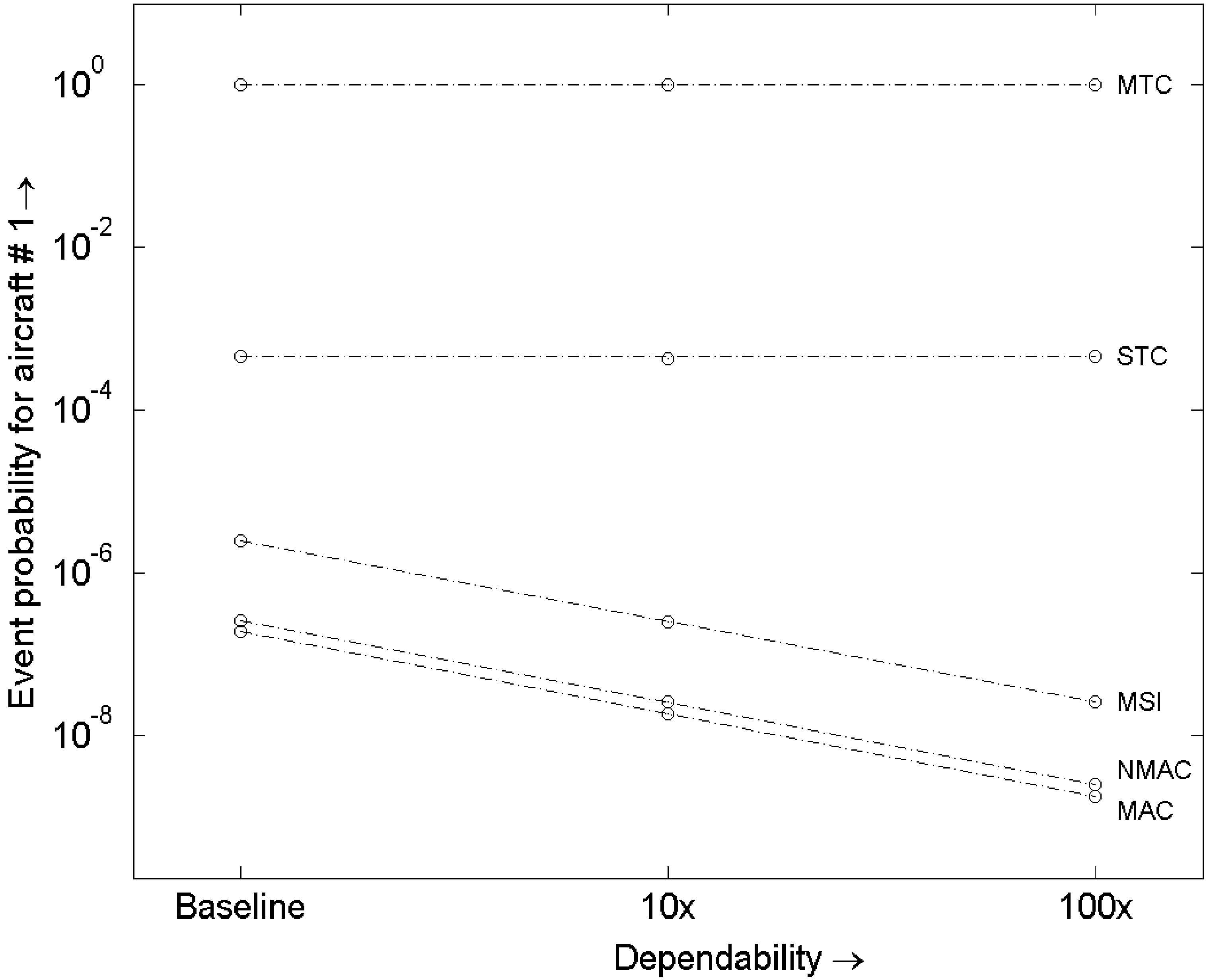
Short Term Conflict (STC)

Minimum Separation Infringement (MSI)

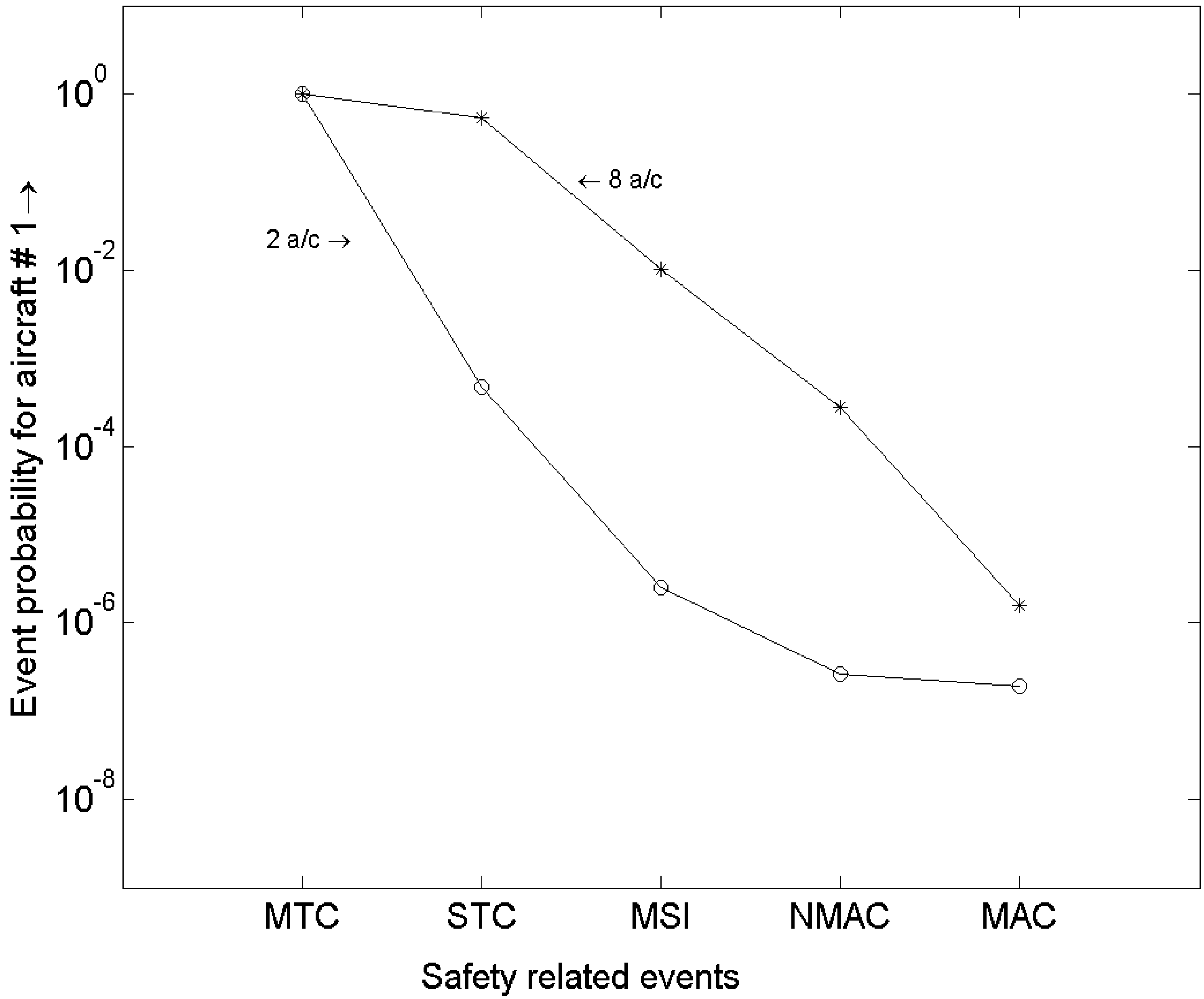
Near Mid-Air Collision (NMAC)

Mid-Air Collision (MAC)

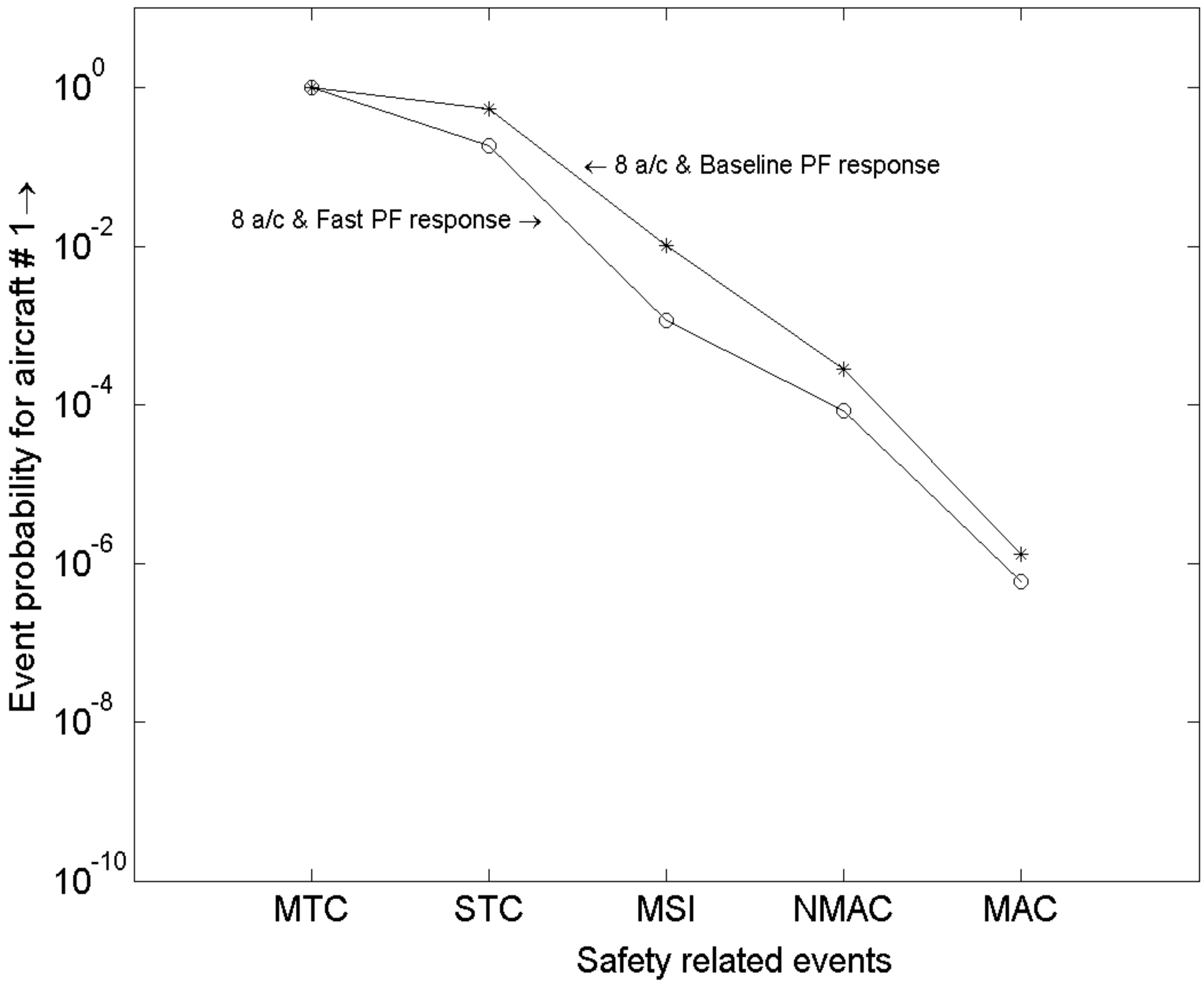
Two-aircraft encounter and dependable technical systems



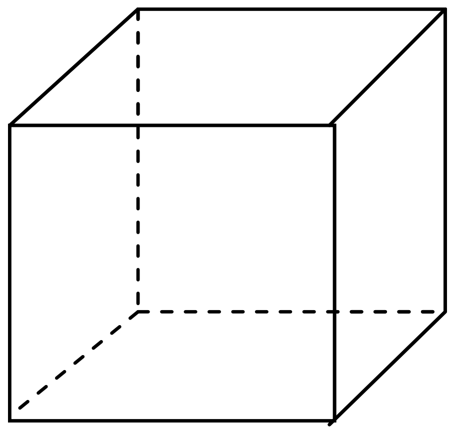
Two-aircraft vs. eight-aircraft encounter



Eight-aircraft encounter: Baseline PF response vs. Fast PF response

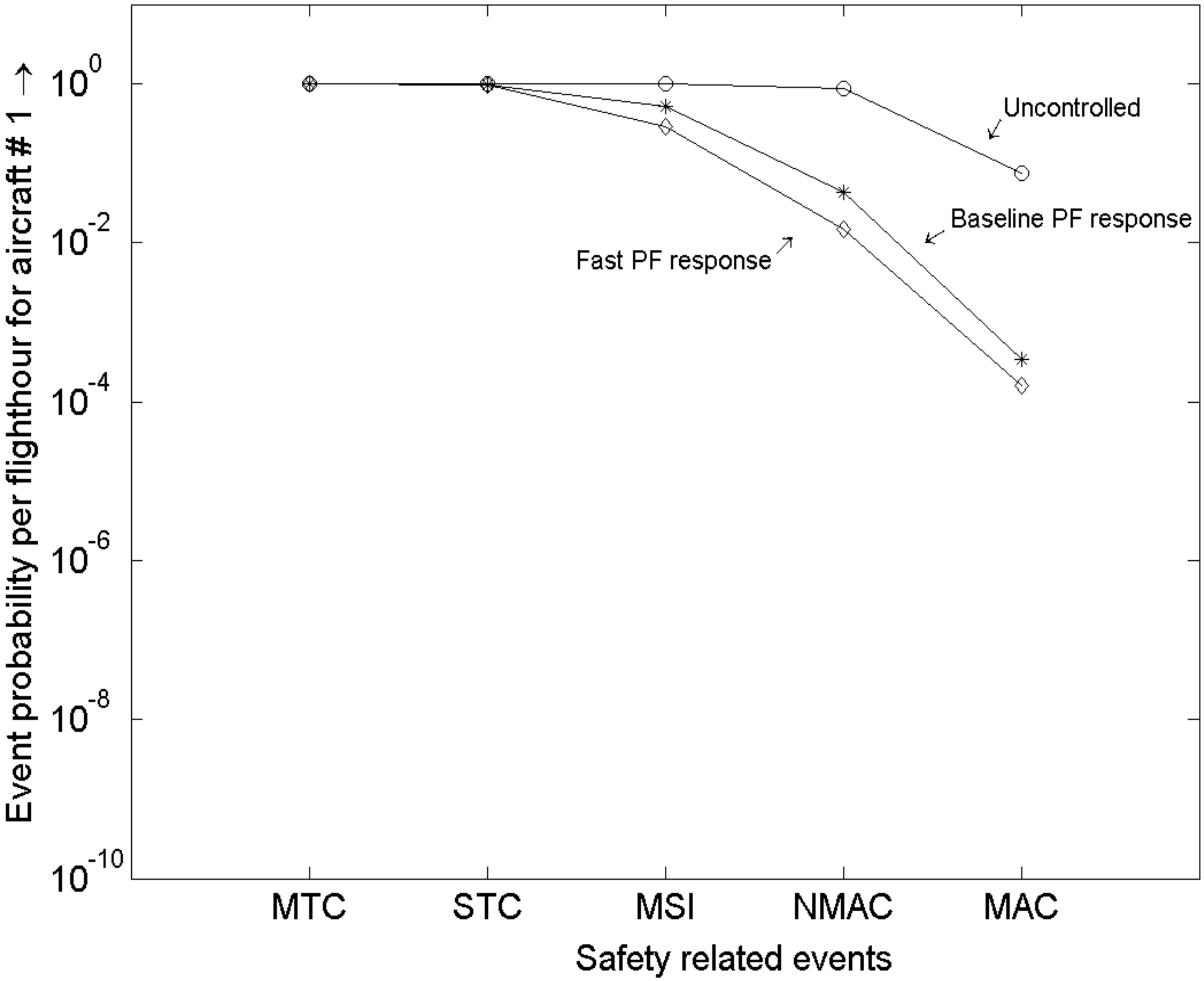


Random traffic, high density



- **Eight aircraft per packed container**
  - 3 times as dense above Frankfurt on 23rd July ’99

Random high traffic: Uncontrolled vs. AMFF controlled



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iFly developed advanced FF concept

- Future concept developed for high demand En Route areas

- Inputs used:
  - Advanced FF concept developed by NASA
  - Learning from AMFF safety analysis

- Communication means:
  - ADS-B between aircraft within line of sight
  - SWIM between aircraft over the horizon

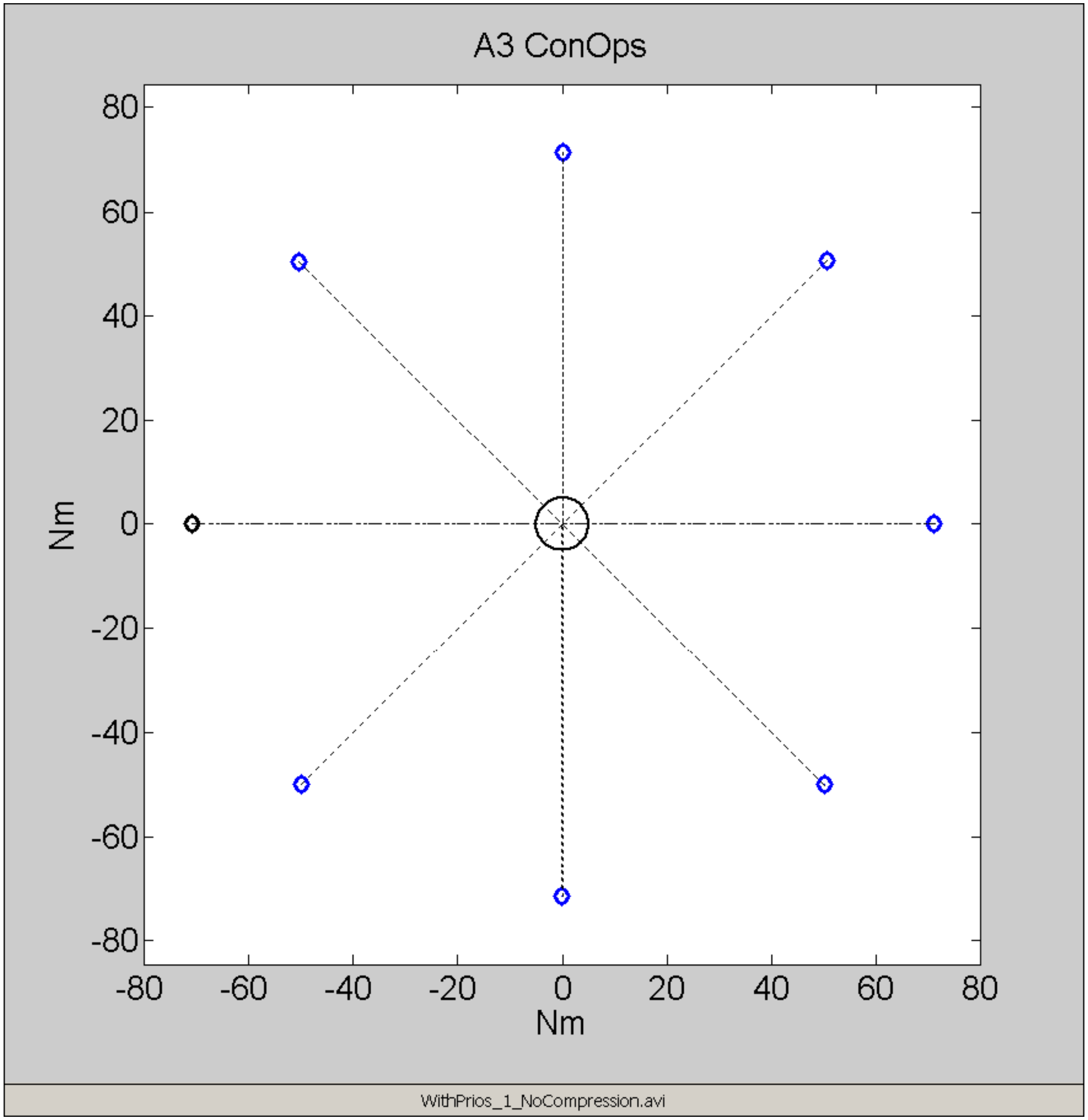
- Each aircraft receives
  - State updates by other aircraft without delay
  - Intent updates by other aircraft with some delay

- Each a/c equipped with an advanced ASAS, which resolves:
  - Medium term conflicts: a higher priority a/c does not need to resolve
  - Short term conflicts: all aircraft involved help resolving

Candidate resolution algorithms

- Short Term conflict resolution (3 minutes horizon):
  - Navigation Functions based escapes
  - Velocity Obstacles (VO) based escapes

- Medium Term conflict resolution (6 minutes horizon):
  - Genetic Algorithm based intents
  - Multiplexed Model Predictive Control based intents
  - Combinatorial Optimization based intents
  - Velocity Obstacles (VO) based intents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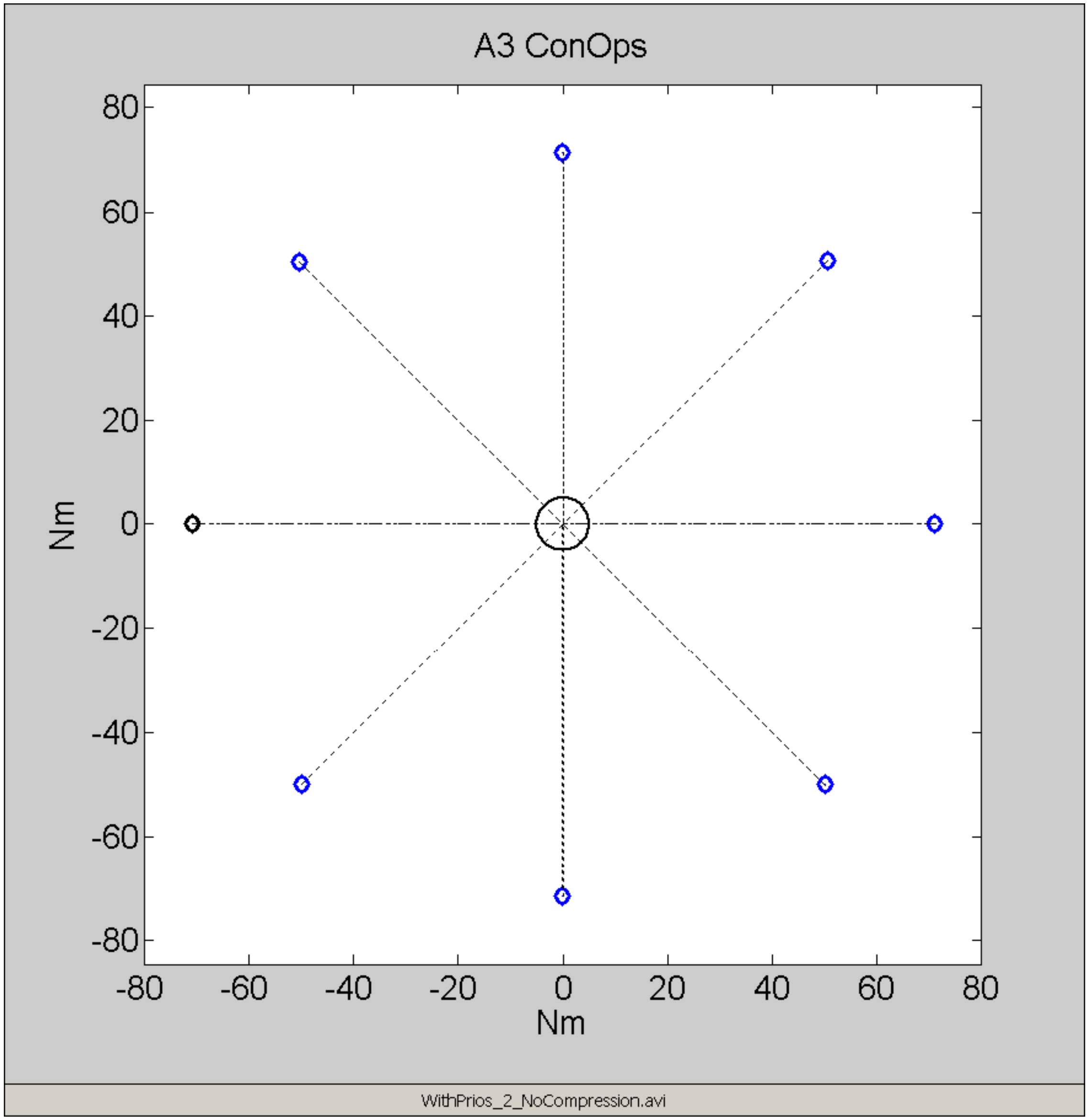

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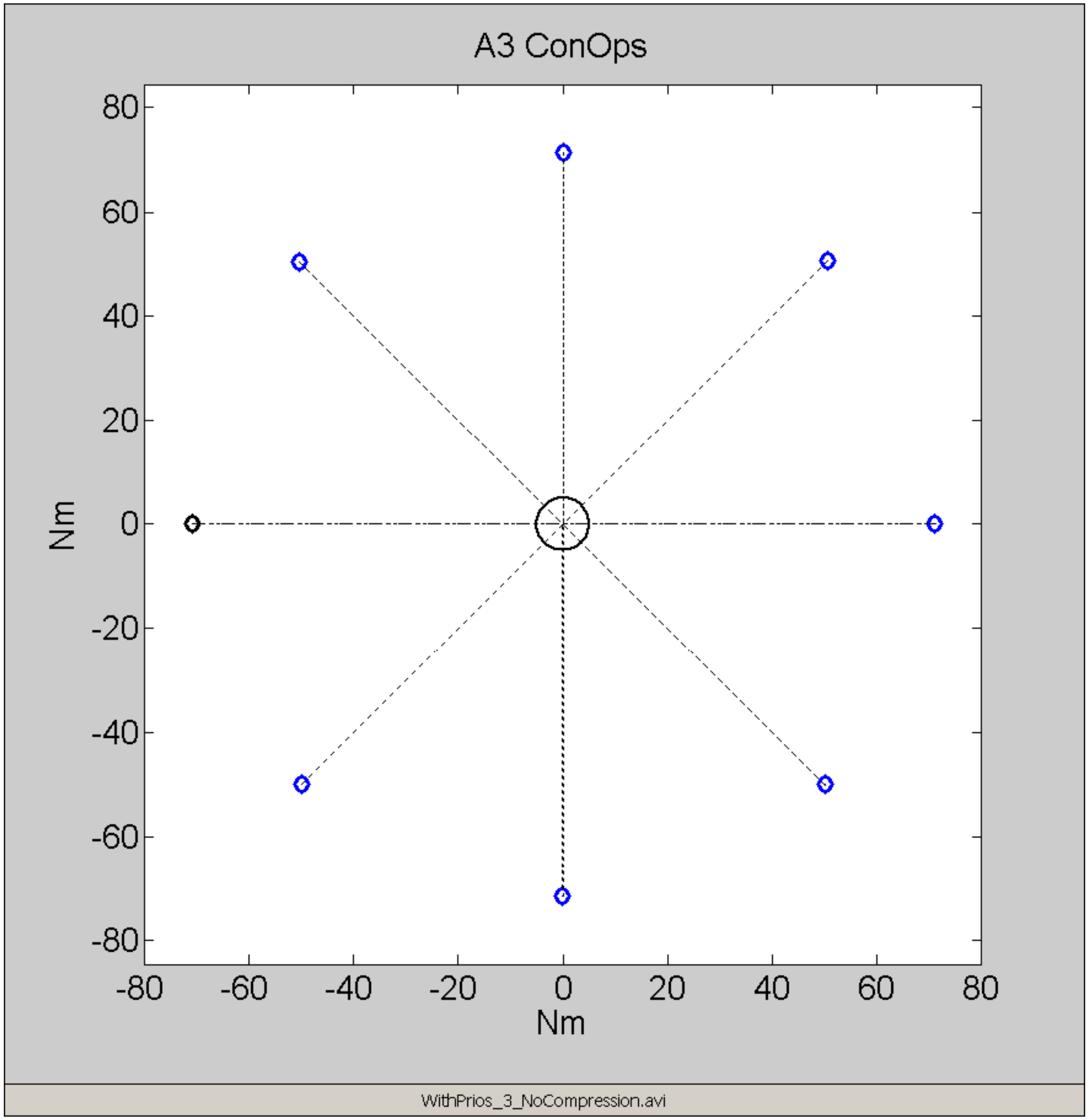
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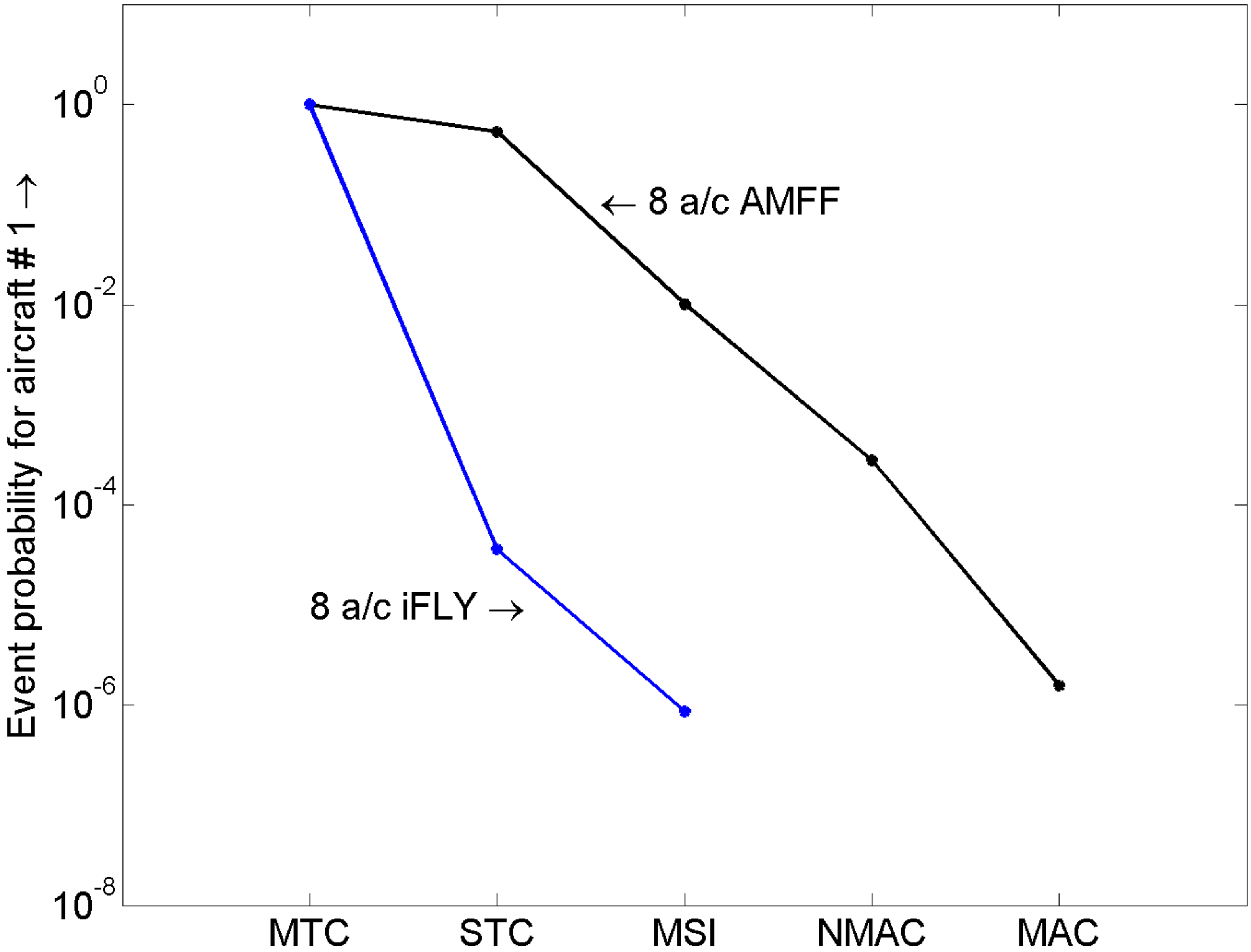
Eight-aircraft encounter; iFLY vs. AMFF



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Conclusions

- Outstanding Key research question: Up to which traffic demand can be safely accommodated by Free Flight (or Airborne Self Separation) ?

- This presentation has given an overview of safety verification approach:
  - SHS modelling framework
  - Bisimilarity relations
  - Monte Carlo speed-up

- Application to AMFF and advanced FF showed:
  - 1st Generation FF cannot accomodate high traffic demands
  - Advanced FF characteristics look very promising

Follow-up

- Follow-up work on advanced FF concept:
  - Continue resolution algorithm developments
  - Emphasis on efficiency of resolutions
  - Lygeros, Maciejowski, Kyriakopoulos and co-workers

- Follow-up work on risk assessment:
  - Continue evaluation of advanced FF concept using VO
  - Further enhancement of HHIPS
  - Include ACAS in simulation model
  - Validation of assessed risk level

- http://iFly.nlr.nl

Validation of assessed risk level

- **Simulation model ≠ Reality**
- Identify the differences
- Assess each difference individually (and conditionally)
  - use of statistical data and expert knowledge
- Assess model parameter sensitivities by Monte Carlo simulations
- Evaluate effect of each assumption at simulated risk level
  - use of statistical data and expert knowledge
- Evaluate combined effects of all model assumptions
  - Typical output: expected risk and 95% area
- Improve simulation model for large differences

Questions / Discussion

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