National Aerospace Laboratory NLR
The Netherlands

Rare-event probability estimation with application to air traffic

by

Henk A.P. Blom

e-mail : blom@nlr.nl

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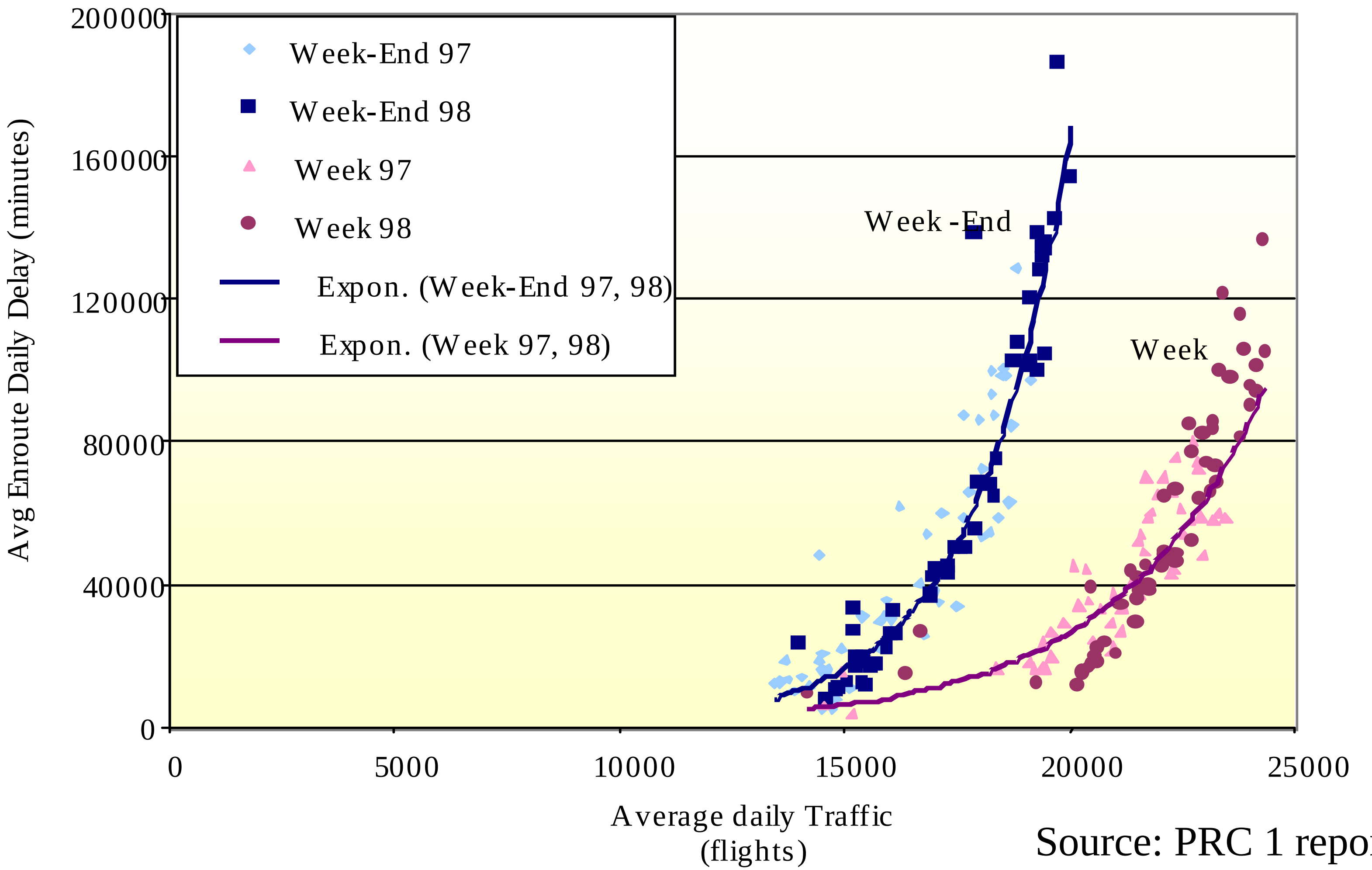
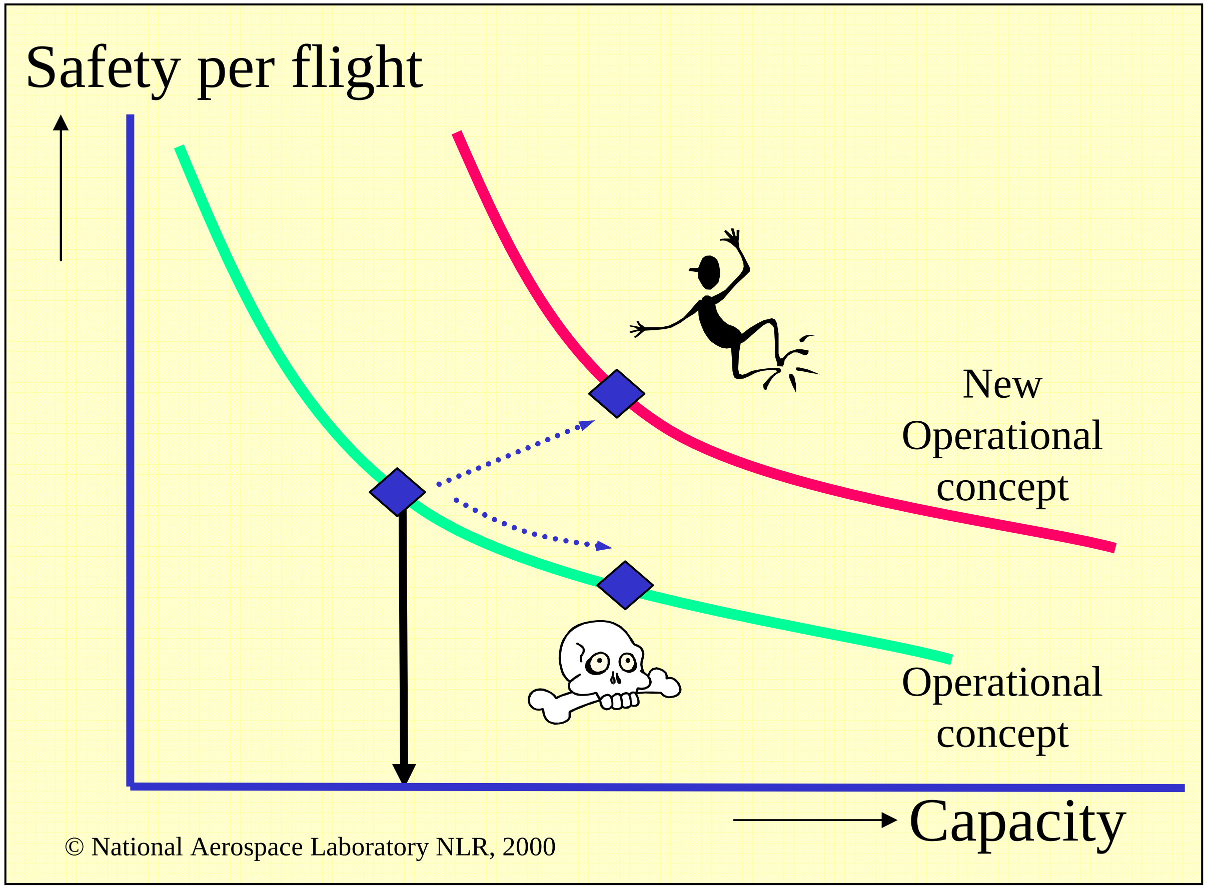
- Motivation
- Advanced air traffic example
- Interacting Particle System (IPS)
- Hierarchical Hybrid IPS
- Results for air traffic example
- Conclusions

The capacity 'wall'



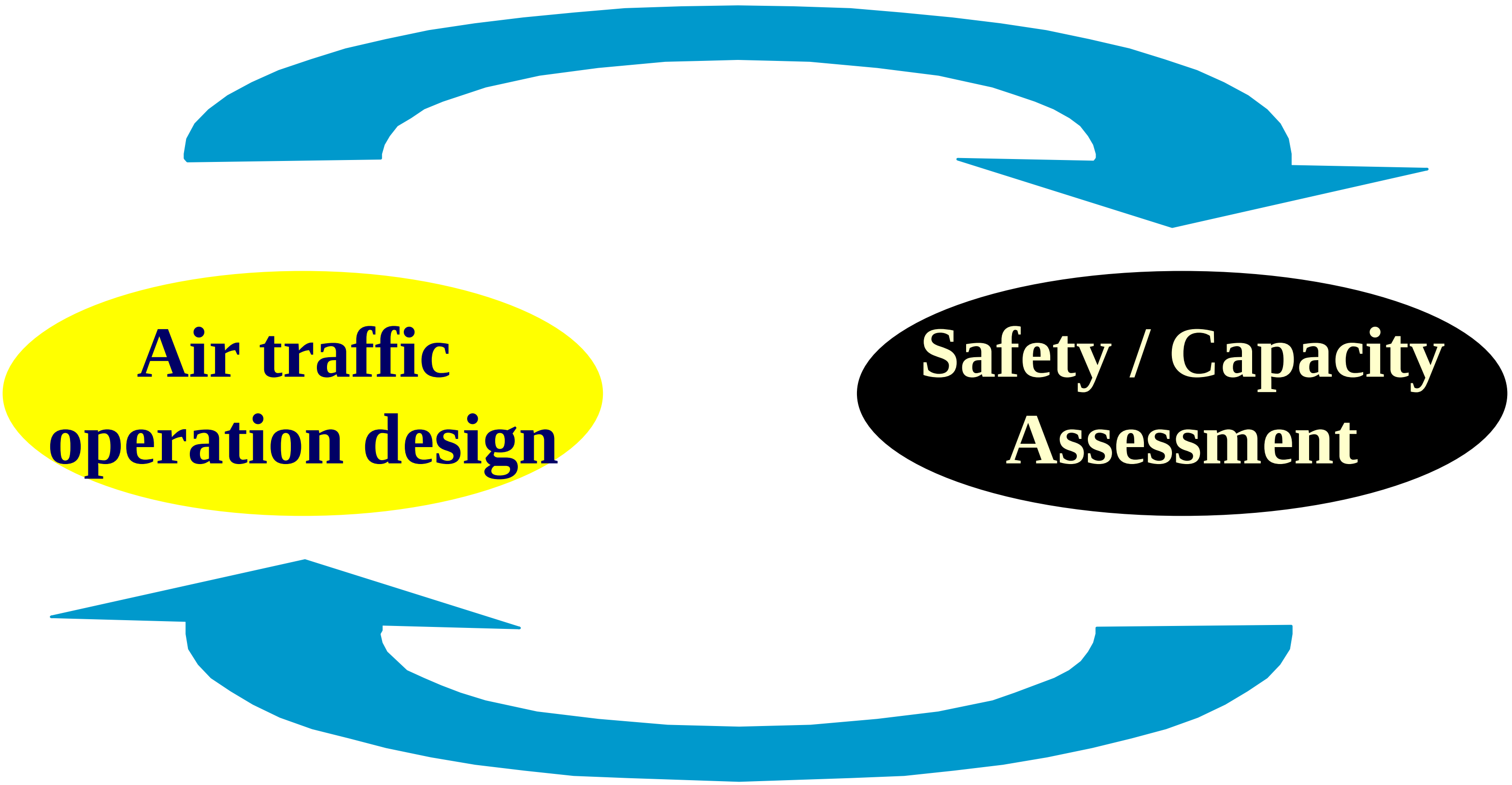
Source: PRC 1 report

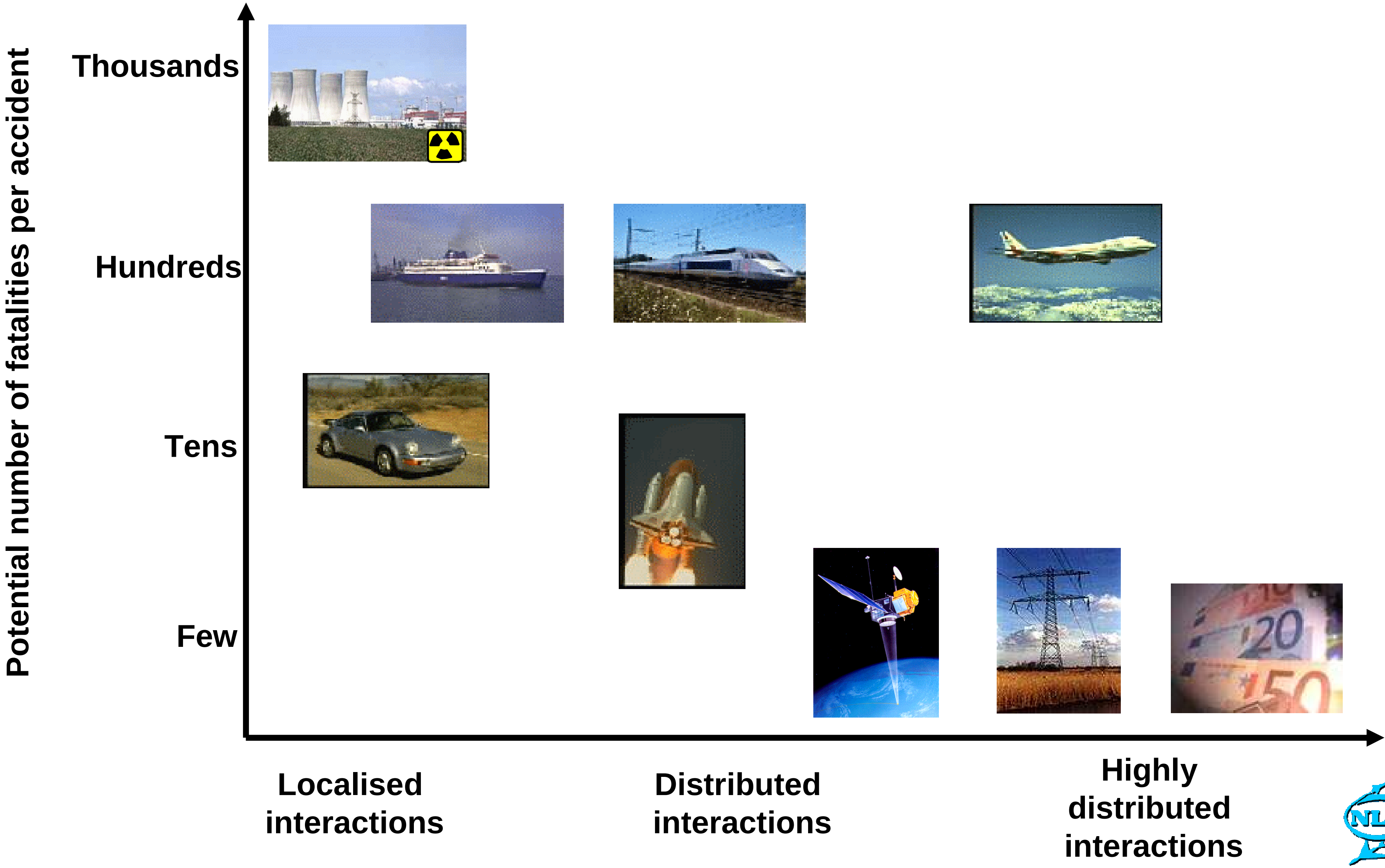
The capacity "wall" is a safety "wall"



- Capacity relates directly to safety
- The question usually is: how to increase capacity whilst at the same time manage the safety?
- The answer is:
by *improving* both capacity and safety per flight

Safety feedback based design





Potential number of fatalities per accident
Thousands
Hundreds
Tens
Few
Localised interactions
Distributed interactions
Highly distributed interactions

iFly

- Innovative project for European Commission
  - Follow-up of HYBRIDGE project
  - Partners: 11 universities + 7 from AirTraffic/Aviation
  - NLR is coordinator
  - iFly project duration: May 2007- August 2011
  - Web site http://iFly.nlr.nl

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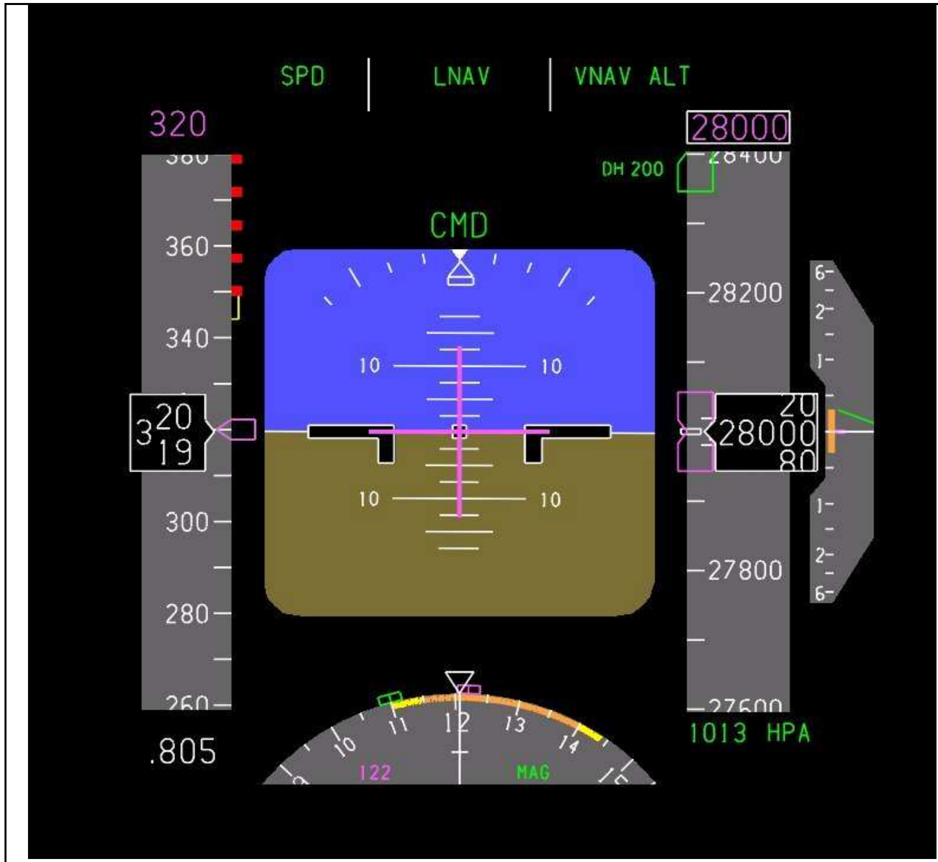
Autonomous Mediterranean Free Flight (AMFF)

- Future concept developed for traffic over Mediterranean area
- Aircrew gets freedom to select path and speed
- In return aircrew is responsible for self-separation
- Each a/c equipped with an Airborne Separation Assistance System
- In AMFF, conflicts are solved one by one (pilot preference)
- Can AMFF safely accommodate high traffic demand ?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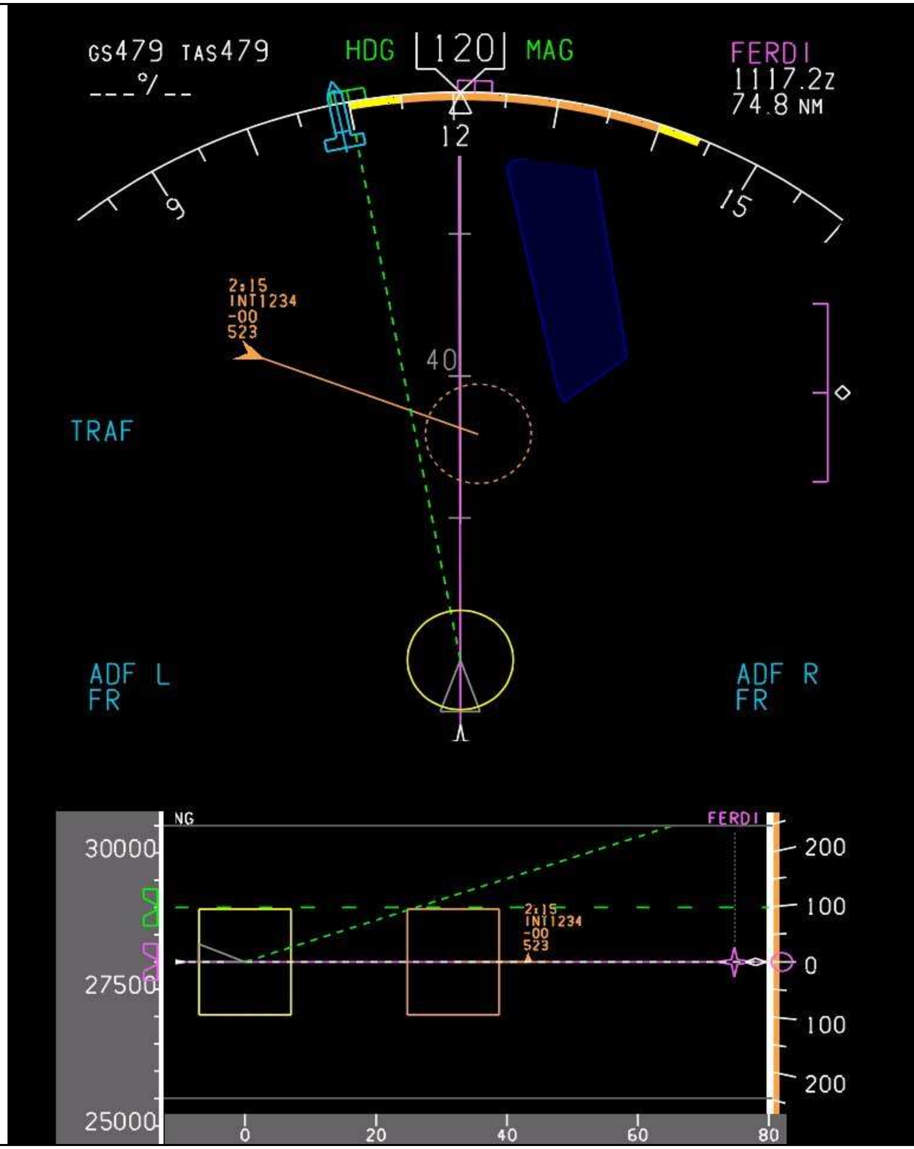
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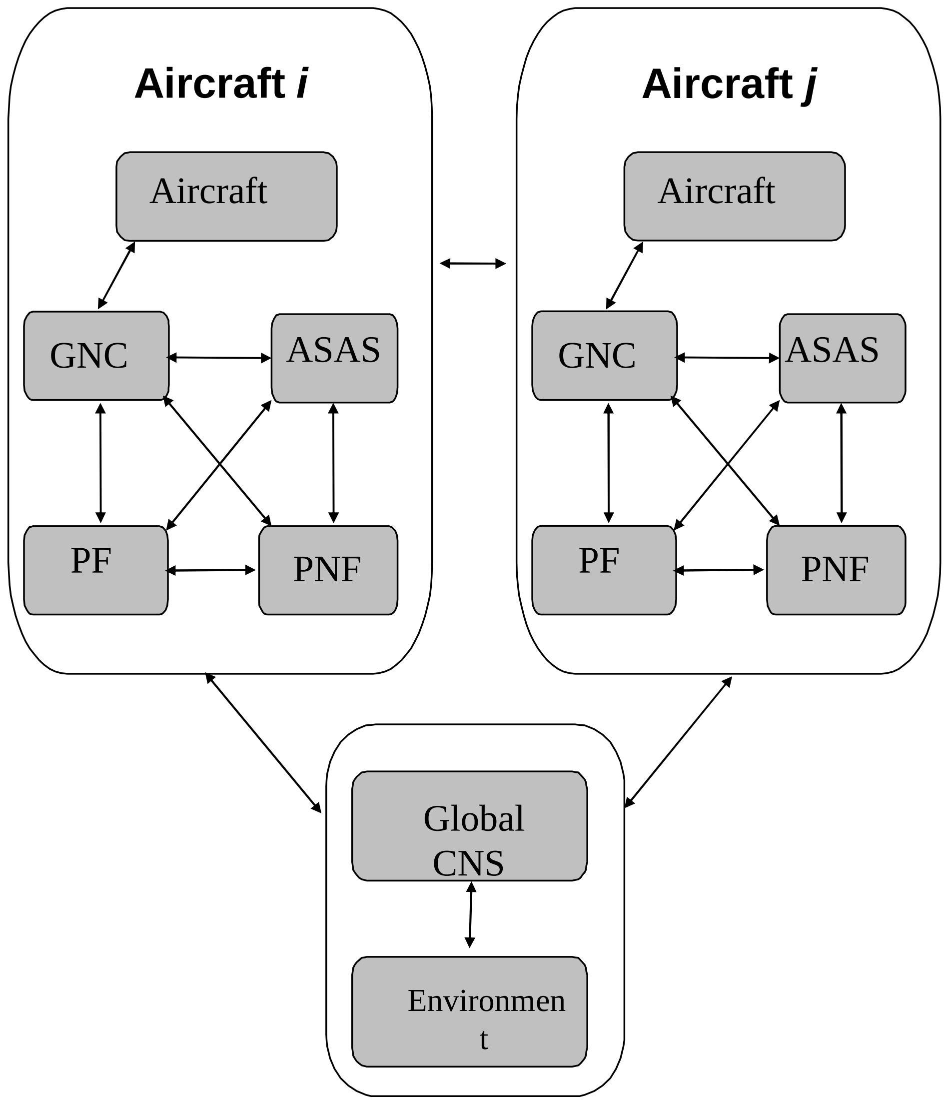
Development of MC simulation model

- Hazard identification
- Defining the relevant Agents
- Developing Petri net for each Agent
- Connecting Agent Petri nets
- Parametrization, Verification & Calibration
- Verification & Calibration

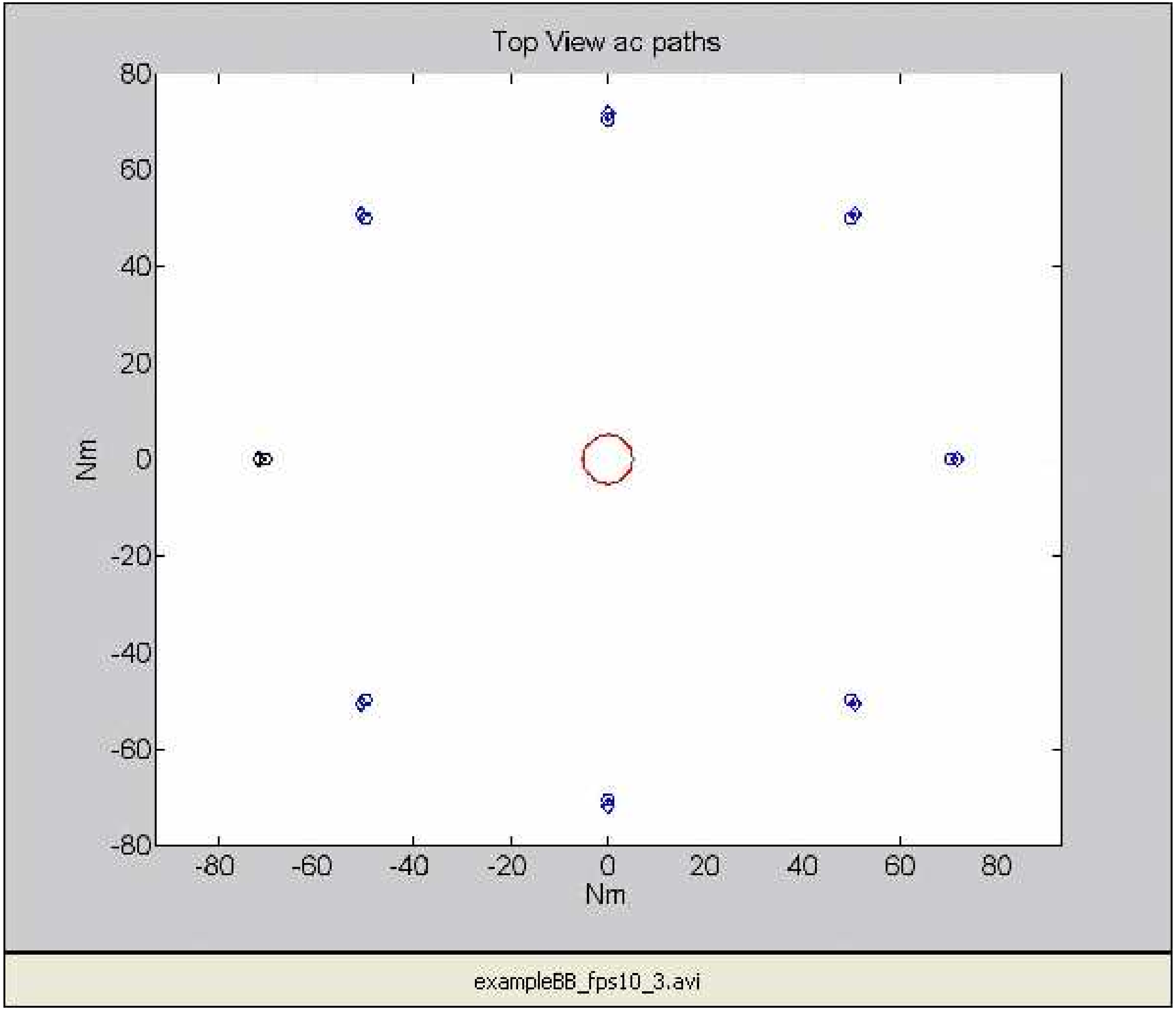
Agents in SHS model of AMFF

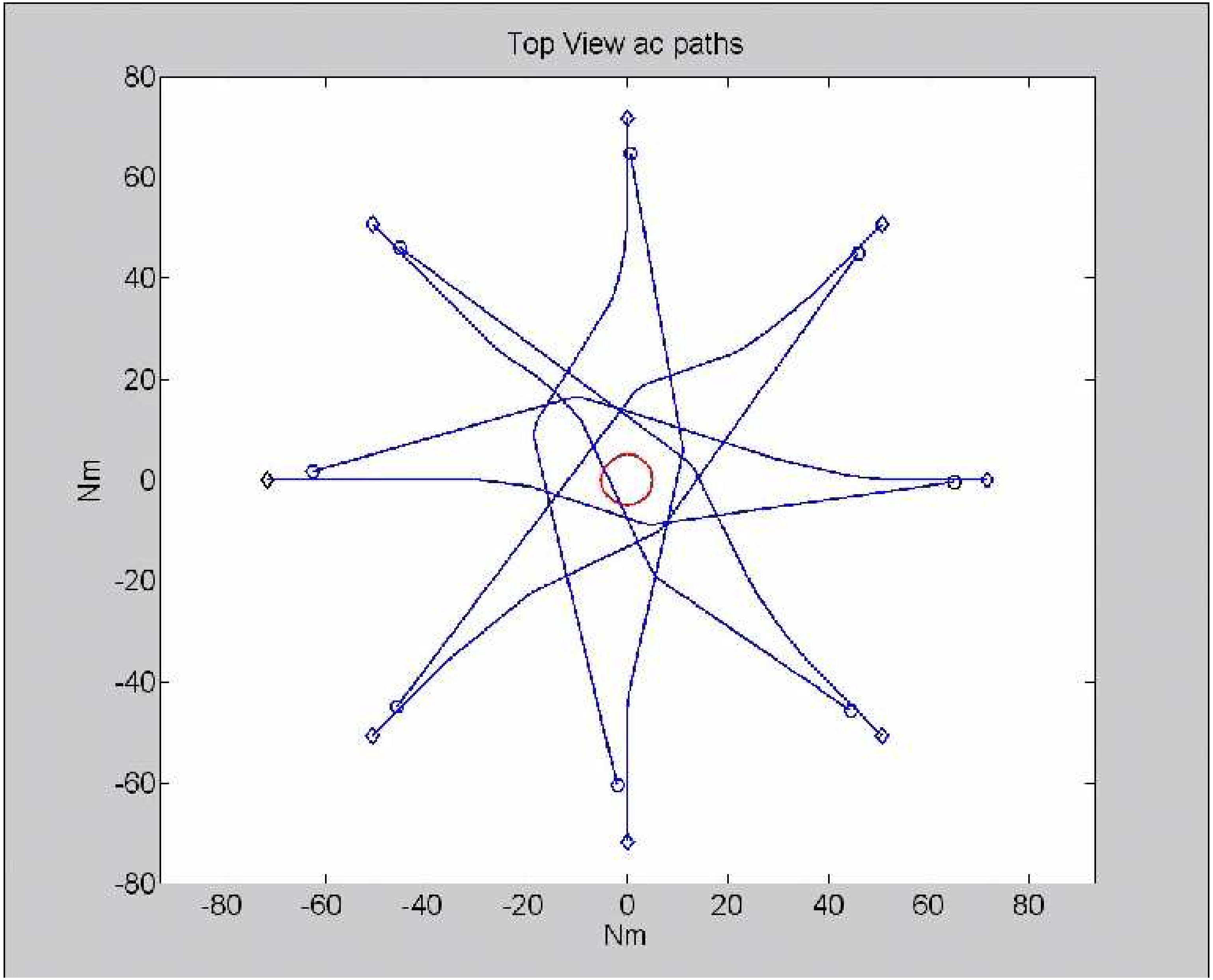


Dimensional analysis of SHS model of AMFF

Agent	**# of product places**	**Maximum colour product state space**
Aircraft	24^N	IR^{13N}
Pilot-Flying (PF)	490^N	IR^{28N}
Pilot-not-Flying (PNF)	7^N	IR^{3N}
AGNC	$(15\times 2^{16})^N$	IR^{45N}
ASAS	48^N	$IR^{37N+21N^2}$
Global CNS	16	$\{\}$
PRODUCT	$\approx 16\times(3.88\times 10^{12})^N$	$IR^{126N+21N^2}$

Eight aircraft encounter

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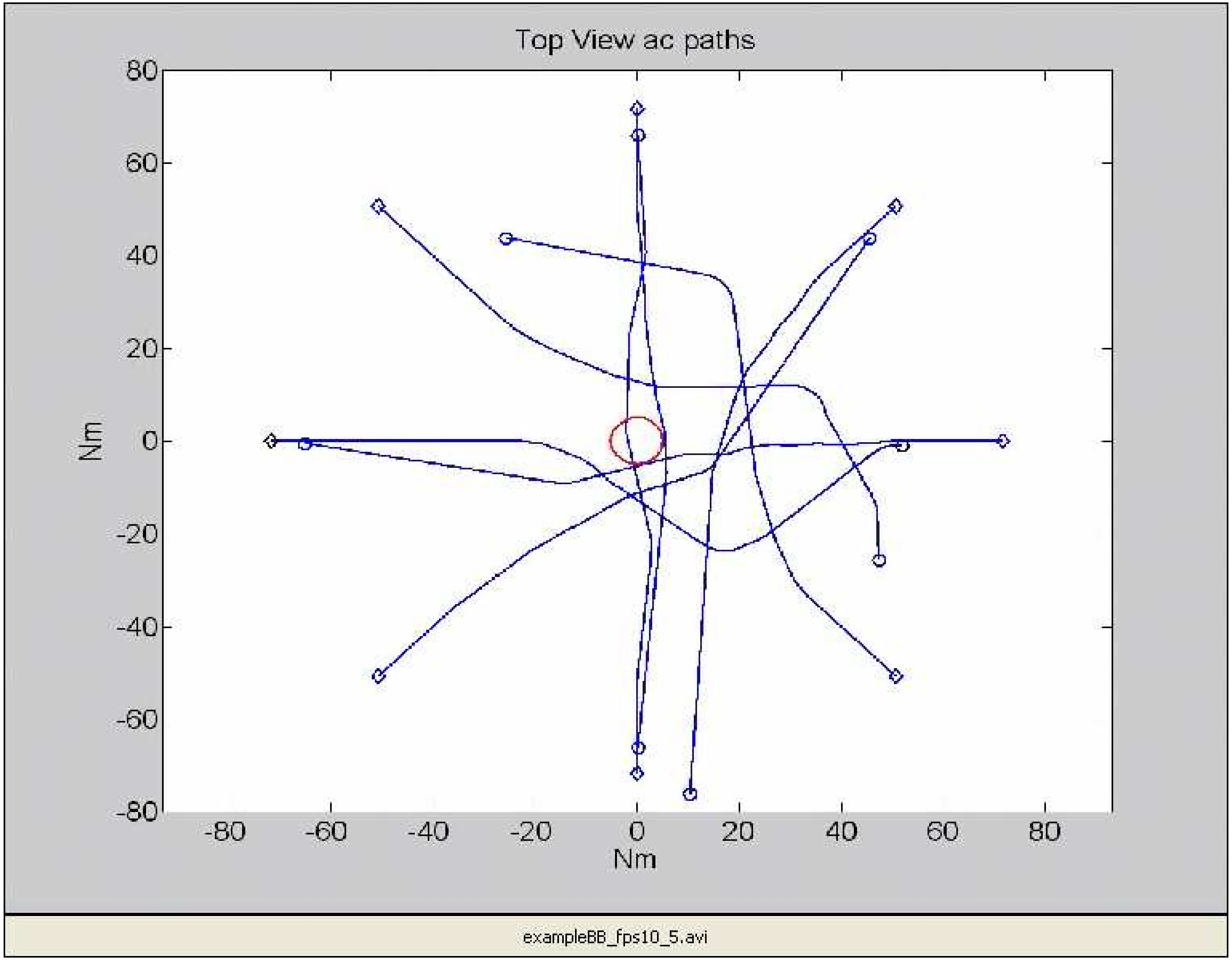
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Approaches in Reach Probability Computation

- Markov Chain (MC) approximation (Prandini&Hu, 2006)
- Dynamic Programming (DP) approach (Abate, Amin, Prandini, Lygeros & Sastry, 2006)
- Interacting Particle System (IPS) approach (Cerou et al., 2005)
- Hybrid IPS (Krystul & Blom, 2005, 2006)

Large scale SHS may cause scalability problems

- State space is too large to handle
- Relevant mode switching is rare

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SHS Reach probability

We consider a time-homogeneous strong Markov process which is a generalised stochastic hybrid process $\{x_t, \theta_t\}$, with $\{x_t\}$ assuming values in $\mathbb{R}^n$ and $\{\theta_t\}$ assuming values in discrete set $\mathbb{M}$. The first component of $\{x_t\}$ equals t and the other components of $\{x_t\}$ form an $\mathbb{R}^{n-1}$ valued cadlag process $\{s_t\}$. The problem considered is to estimate the probability that $\{s_t\}$ hits a given "small" closed subset $D \subset \mathbb{R}^{n-1}$ within a given time period $[0,T)$, i.e. $P(\tau < T)$ with $\tau = \inf\{t > 0;\ s_t \in D\}$.

Reach Probability Factorization

Assume nested sequence of closed subsets

$$D = D_m \subset D_{m-1} \subset \ldots \subset D_1 ,$$

with the constraints that $P(s_0 \in D_1) = 0$ and each component of $\{s_t\}$ that may hit $D_k, k = 1, 2, \ldots m$ has continuous paths P-a.s.

We set $\tau_0 = 0$ and define τ_k, $k = 1, .., m,$ as the first moment that $\{s_t\}$ hits subset k, i.e. $\tau_k = \inf\{t > 0;\ s_t \in D_k\}$

Then (e.g. L'Ecuyer et al., 2006):

$$P(\tau < T) = \prod_{k=1}^{m} \gamma_k \quad \text{with } \gamma_k \triangleq P(\tau_k < T \mid \tau_{k-1} < T)$$

Interacting Particle System (IPS)

- Define a sequence of conflict levels decreasing in urgency $(D_k\text{'}s)$
  - Most urgent level represents collision $(D_m = D)$
- Simulate N_p particles; initially all outside D_1 (less urgent level)
- Freeze each particle that reaches the next urgent level before T
- Make N_p copies of frozen particles
- Repeat this until the most urgent level has been reached
- Count the simulated fraction $\tilde{\gamma}_k$ that reaches level k
- Estimated collision risk = $\tilde{\gamma}_1 \times \tilde{\gamma}_2 \times \tilde{\gamma}_3 \times \ldots \times \tilde{\gamma}_m$

IPS convergence

Cerou, Del Moral, Legland and Lezaud (2002, 2005) have shown that the product of these fractions $\tilde{\gamma}_k$ forms an unbiased estimate of the probability of $\{s_t\}$ to hit the set D within the time period $[0,T)$, i.e.

$$\mathbb{E}[\prod_{k=1}^{m} \tilde{\gamma}_k] = \prod_{k=1}^{m} \gamma_k = P(\tau < T)$$

In addition there is a bound on the L^1 estimation error, i.e.:

$$\mathbb{E}(\prod_{k=1}^{m} \tilde{\gamma}_k - \prod_{k=1}^{m} \gamma_k) \leq \frac{c_p}{\sqrt{N_p}}$$

Hybrid IPS versions

1. Importance switching (Krystul&Blom, 2005)

2. Rao-Blackwellization, using exact equations for $\{ \theta_t \}$ and particles for Euclidian state (Krystul&Blom, 2006)

- Both handle rare mode switching well
- New large scale SHS scalability problem
  - Combinatorially many discrete modes

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Hierarchical Hybrid IPS (HHIPS)

- ✓ Define an aggregated mode process $\{ \kappa_t \}$

$$\kappa_t = \kappa \text{ if } \theta_t \in \mathbb{M}_k$$

$$\text{with } \{\mathbb{M}_k,\ \kappa \in \mathbb{K}\} \text{ a partition of } \mathbb{M}$$

- ✓ Apply Importance switching to $\{ \kappa_t \}$

- ✓ Rao-Blackwellization, i.e. use exact equations for $\{ \kappa_t \}$ and particles for the other process elements $\{x_t, \theta_t\}$

Hierarchical Hybrid IPS (HHIPS)

Step 0 generates per aggregation mode value $\kappa \in \mathbb{K}$, N_p initial particles for $k=0$, and then starts the cycling through steps 1 through 3:

Step 1 extrapolates each $(x_t, \theta_t, \kappa_t)$ -particle from $\tau_{k-1} \wedge T$ to $\tau_k \wedge T$

Step 2 evaluates the $(x_{\tau_k}, \theta_{\tau_k}, \kappa_{\tau_k})$ particles that have arrived at Q_k

Step 3 resamples from the $(x_{\tau_k}, \theta_{\tau_k}, \kappa_{\tau_k})$ particles that have arrived at Q_k

set k :=k+1 and go to step1

<u>Step 1</u> extrapolates each particle from $\tau_{k-1} \wedge T$ to $\tau_k \wedge T$ in time step of length h, using importance switching for the new κ - value and κ - conditional sampling of a new θ – value. For the latter use is made of the following theorem:

<u>Theorem 1</u> (κ - conditional θ –prediction)

Let τ be an arbitrary stopping time, then

$$p_{\theta_{\tau+h}|x_\tau,\theta_\tau,\kappa_{\tau+h}}(\eta \mid x,\theta,\kappa) = \frac{1_{\mathbb{M}_\kappa}(\eta)\, p_{\theta_{\tau+h}|x_\tau,\theta_\tau}(\eta \mid x,\theta)}{\sum_{\eta' \in \mathbb{M}} 1_{\mathbb{M}_\kappa}(\eta')\, p_{\theta_{\tau+h}|x_\tau,\theta_\tau}(\eta' \mid x,\theta)}$$

Step 3 resamples from the particles that have arrived at Q_k
In order to draw N_p samples per κ – value, use is made of the following hierarchical interaction theorem :

Theorem 2 (Hierarchical interaction)

If $p_{\kappa_{\tau+h}}(\kappa) > 0$ for arbitrary stopping time τ, then

$$p_{x_\tau,\theta_\tau|\kappa_{\tau+h}}(dx,\theta\mid\kappa) = \sum_{\eta\in\mathbb{M}_\kappa} p_{\theta_{\tau+h}|x_\tau,\theta_\tau}(\eta\mid x,\theta).p_{x_\tau,\theta_\tau}(dx,\theta)\,/\,p_{\kappa_{\tau+h}}(\kappa)$$

$$p_{\kappa_{\tau+h}}(\kappa) = \sum_{\theta\in\mathbb{M}}\int_{\mathbb{R}^n}\sum_{\eta\in\mathbb{M}_\kappa} p_{\theta_{\tau+h}|x_\tau,\theta_\tau}(\eta\mid x,\theta).p_{x_\tau,\theta_\tau}(dx,\theta)$$

Key extensions of HHIPS over IPS for SHS

- Embedding of an aggregation mode process;
- Particles are maintained per aggregation mode;
- Importance switching of aggregation mode is used for the conditional prediction of SHS particles;
- Hierarchical interaction is used for the resampling of particles that reached $Q_k \triangleq (0,T) \times D_k \times \mathbb{M},\ k = 1,..,m-1.$

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Scenarios

- Two aircraft encounter using HHIPS
- Eight aircraft encounter using IPS
- Random traffic high density using IPS

Air traffic safety related events

Event	MTC	STC	MSI	NMAC	MAC
Prediction time (minutes)	8	2.5	0	0	0
Horizontal distance (Nm)	4.5	4.5	4.5	1.25	0.054
Vertical distance (ft)	900	900	900	500	131

MTC = Medium Term Conflict
STC = Short Term Conflict
MSI = Minimum Separation Infringement
NMAC = Near Mid-Air Collision
MAC = Mid-Air Collision

Sequence of conflict levels for air traffic

k	1	2	3	4	5	6	7	8
D_k (Nm)	4.5	4.5	4.5	4.5	2.5	1.25	0.5	0.054
h_k (ft)	900	900	900	900	900	500	250	131
Δ_k (min)	8	2.5	1.5	0	0	0	0	0

Medium Term Conflict

Short Term Conflict

Minimum Separation Conflict

Near Mid-Air Collision (NMAC)

Mid-Air Collision

$\tilde{\gamma}_k$ values estimated by HHIPS for Two-aircraft scenario

k	**Run 1**	**Run 2**	**Run 3**	**Run 4**	**Run 5**
1	1.000	1.000	1.000	0.991	1.000
2	5.77E-04	5.64E-06	6.24E-06	5.04E-06	6.13E-06
3	6.40E-03	7.25E-01	7.20E-01	6.84E-01	7.66E-01
4	0.566	0.569	0.596	0.540	0.608
5	0.344	0.256	0.223	0.401	0.198
6	0.420	0.452	0.402	0.459	0.429
7	0.801	0.845	0.929	0.710	0.949
8	0.814	0.827	0.841	0.828	0.802
Π	1.97E-07	1.89E-07	1.89E-07	2.00E-07	1.85E-07

IPS based estimation typically yields values 0.0 for $k \geq 4$

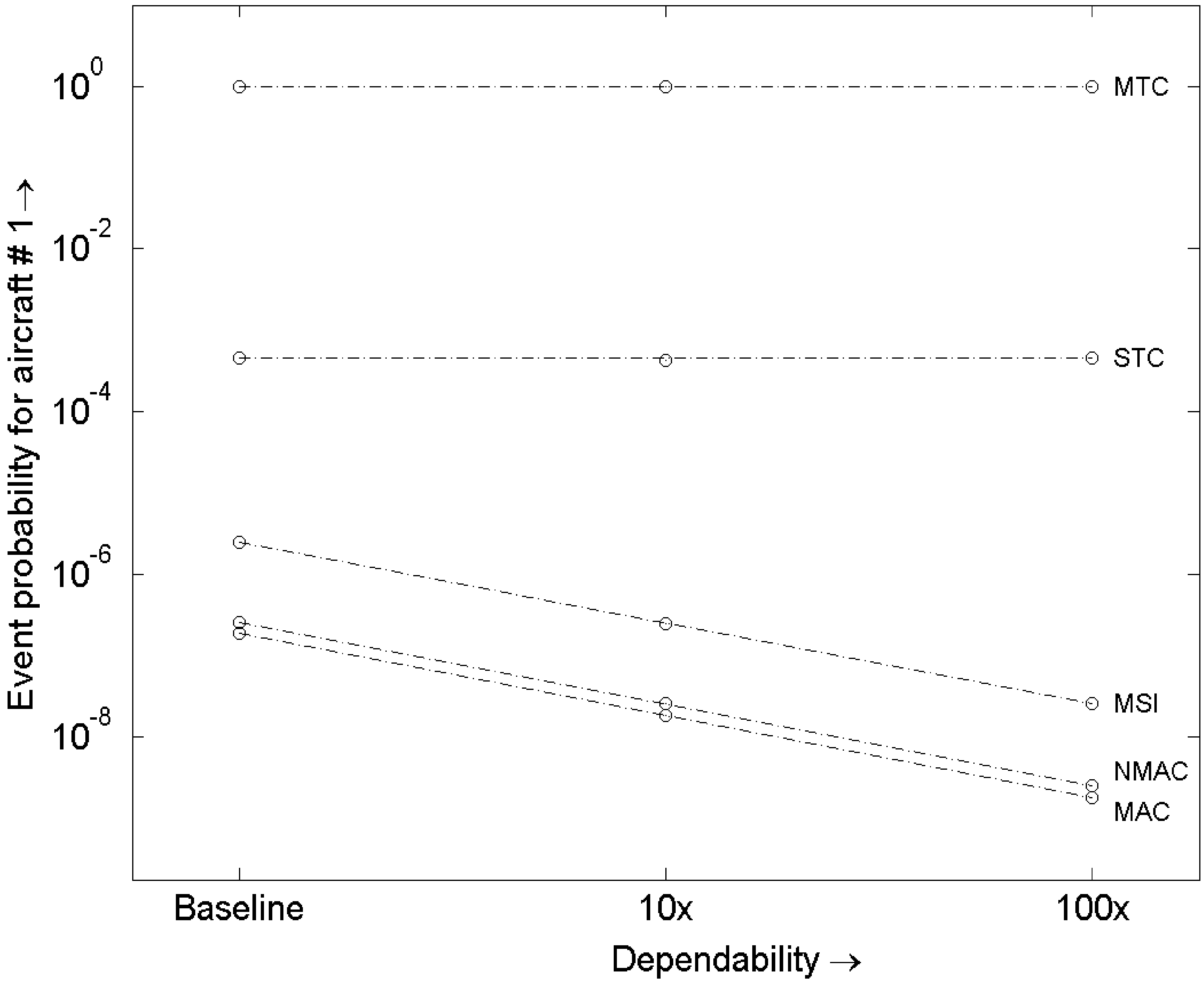
Reach probabilities estimated through 10 runs

Event	Mean	Std. dev.
MTC	1.0	4.8E-03
STC	2.35E-04	5.0E-04
MSI	2.57E-06	3.9E-07
NMAC	2.82E-07	4.5E-08
MAC	1.91E-07	1.6E-08

Contribution to reach probability

Global comm.	DM-loop	Share %
up	up	0.5
up	down	1.9
down	up	97.6
down	down	0.002

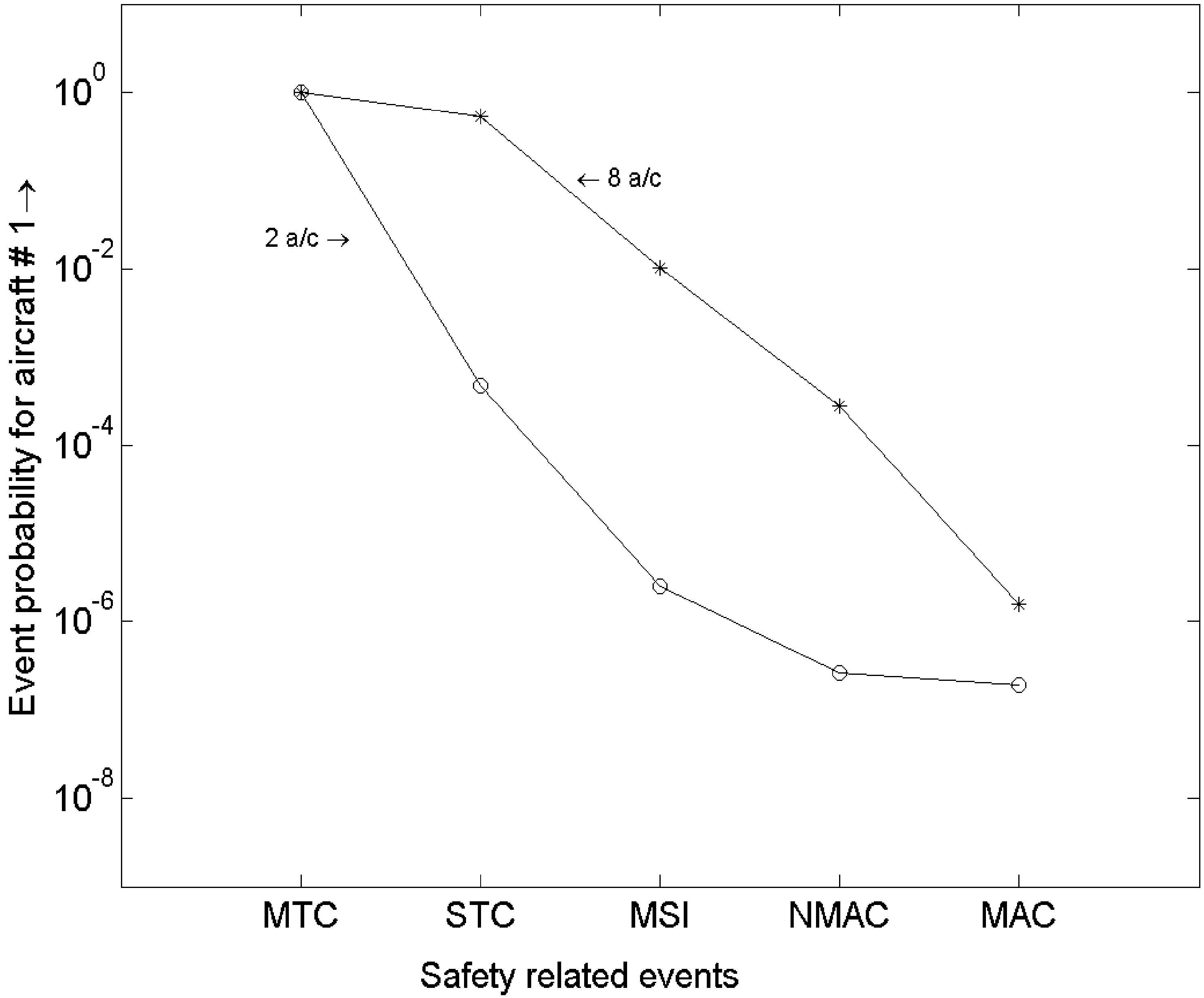
Two-aircraft encounter and dependable technical systems



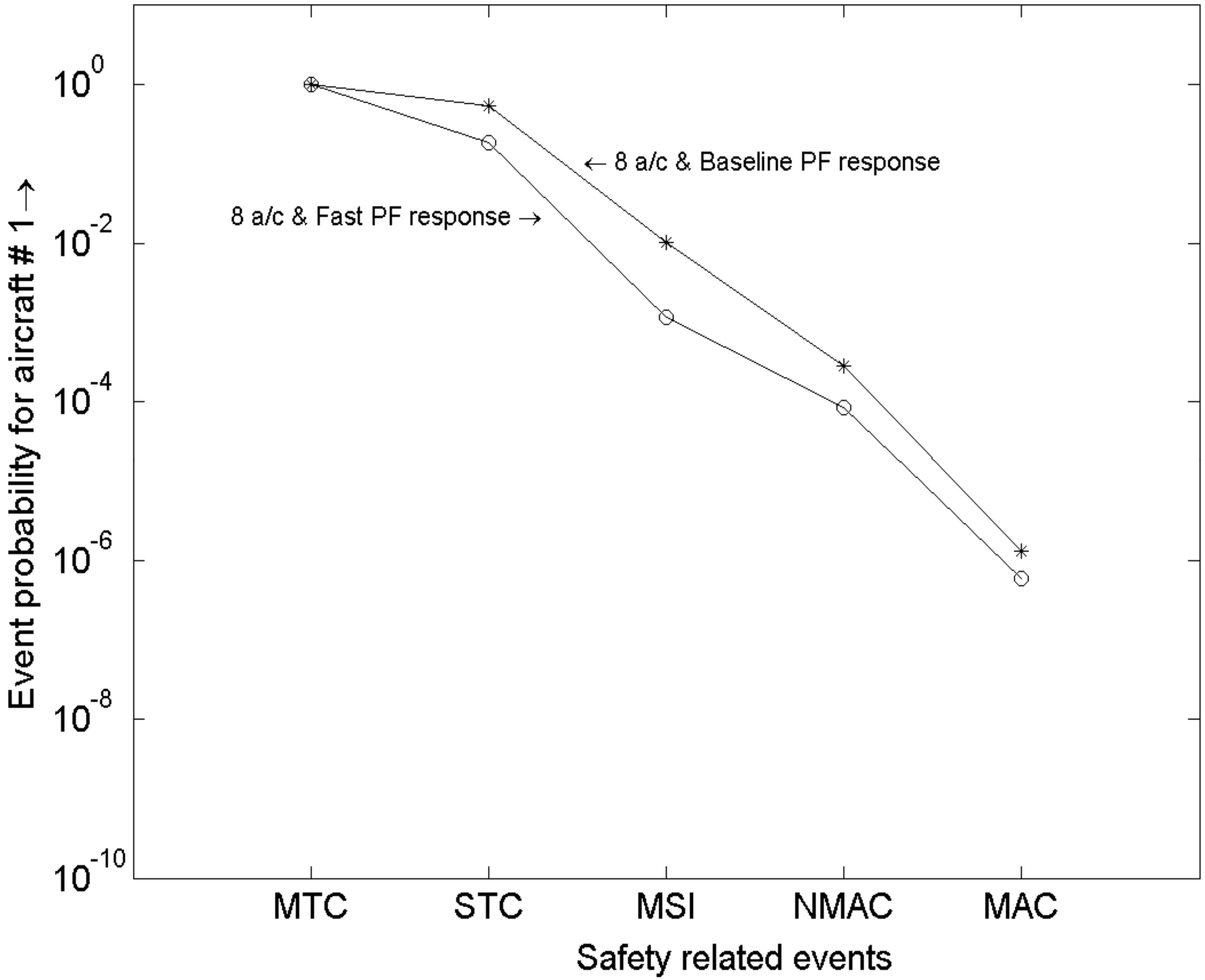
Eight aircraft encounter using IPS

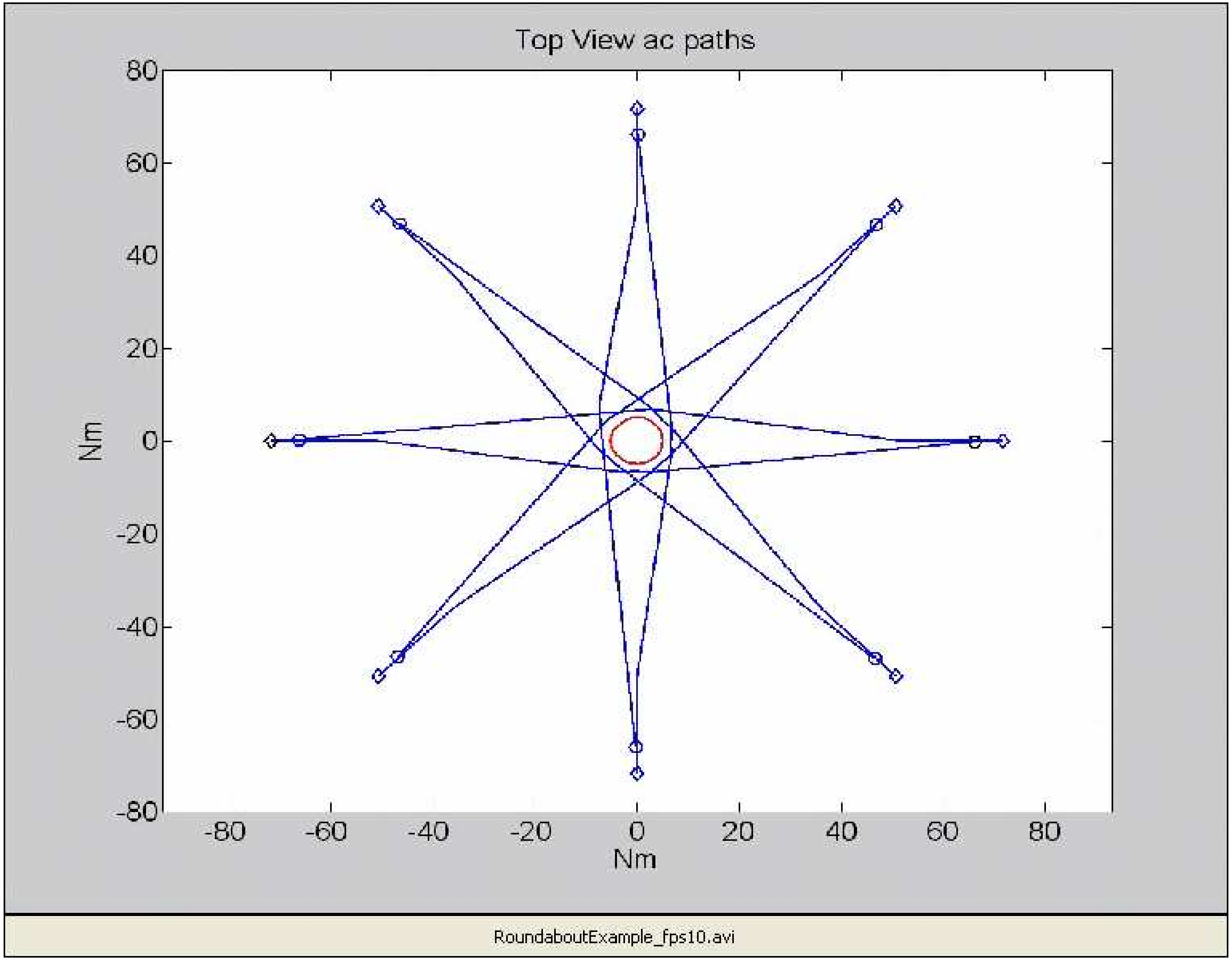
Level	1st IPS	2nd IPS	3rd IPS	4th IPS
1	1.000	1.000	1.000	1.000
2	0.528	0.529	0.539	0.533
3	0.426	0.429	0.424	0.431
4	0.033	0.036	0.035	0.037
5	0.175	0.180	0.183	0.181
6	0.267	0.158	0.177	0.144
7	0.150	0.268	0.281	0.427
8	0.000	0.009	0.233	0.043
Product of fractions	0.0	$5.58 \cdot 10^{-7}$	$1.67 \cdot 10^{-5}$	$4.01 \cdot 10^{-6}$

Two-aircraft vs. eight-aircraft encounter



Eight-aircraft encounter: Baseline PF response vs. Fast PF response

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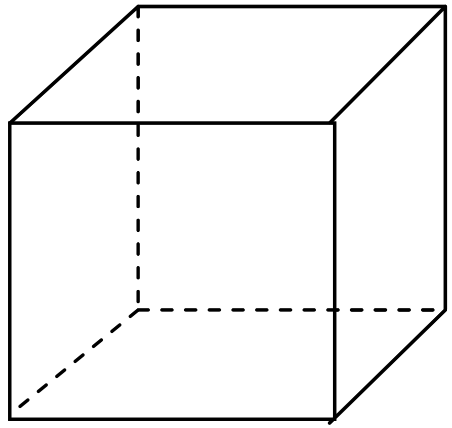
Random traffic scenario, high density

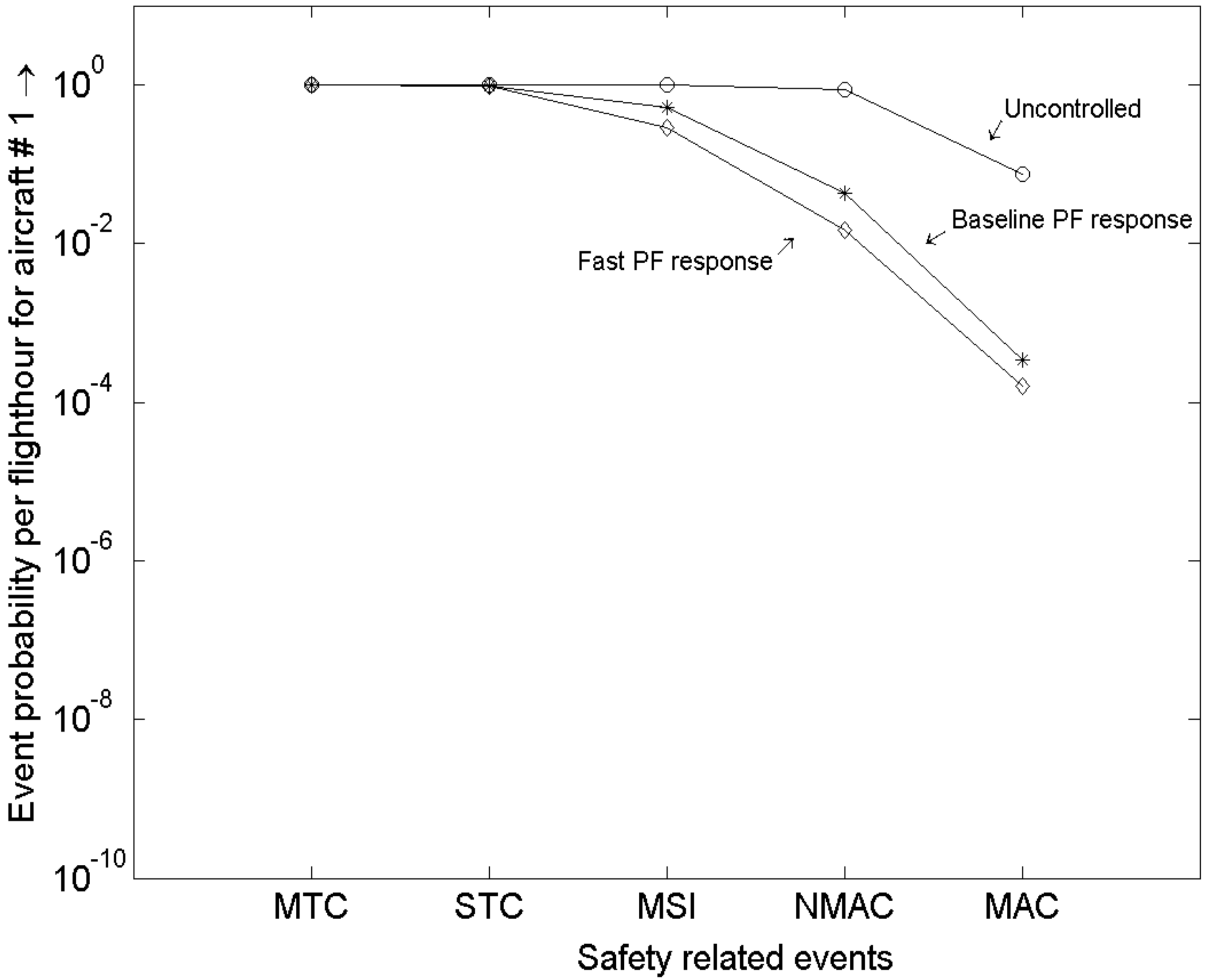


- **Eight aircraft per packed container**
  - 3 times as dense above Frankfurt on 23rd July '99
  - IPS, 10,000 particles, 30 hours per IPS run

IPS runs for random traffic scenario

Level	1st IPS	2nd IPS	3rd IPS	4th IPS
1	0.922	0.917	0.929	0.926
2	0.567	0.551	0.560	0.559
3	0.665	0.666	0.674	0.676
4	0.319	0.331	0.323	0.321
5	0.370	0.367	0.371	0.379
6	0.181	0.158	0.162	0.171
7	0.130	0.209	0.174	0.145
8	0.067	0.005	0.094	0.066
Product of fractions	$6.42 \cdot 10^{-5}$	$6.76 \cdot 10^{-6}$	$1.11 \cdot 10^{-4}$	$6.99 \cdot 10^{-5}$

Random high traffic: Uncontrolled vs. AMFF controlled



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Conclusions

- Thanks to IPS developments it has been shown that uncoordinated conflict resolution falls short in safely accommodating high en route traffic demand

- Follow-up work on risk assessment:
  - Continue developments of IPS and HHIPS
    - Extending Convergence Proof
    - Monte Carlo Markov Chain
    - Traffic complexity prediction
  - Evaluate advanced airborne self separation concept
  - Include ACAS in simulation model
  - Validation of assessed risk level

Validation of assessed risk level

- **Simulation model ≠ Reality**
- Identify the differences
- Assess each difference individually (and conditionally)
  - use of statistical data and expert knowledge
- Assess model parameter sensitivities by Monte Carlo simulations
- Evaluate effect of each assumption at simulated risk level
  - use of statistical data and expert knowledge
- Evaluate combined effects of all model assumptions
  - Typical output: expected risk and 95% area
- Improve simulation model for large differences

To be continued

http://iFly.nlr.nl