Project no. TREN/07/FP6AE/S07.71574/037180 IFLY

iFly

Safety, Complexity and Responsibility based design and validation of highly automated Air Traffic Management

Specific Targeted Research Projects (STREP)

Thematic Priority 1.3.1.4.g Aeronautics and Space

iFly Deliverable D7.2g

Final Report on Monte Carlo speed-up studies

Version: 0.6

H.A.P. Blom and G.J. Bakker
NLR

Due date of deliverable: 22 January 2011
Actual submission date: 22 August 2011

Start date of project: 22 May 2007 Duration: 51 months

Project co-funded by the European Commission within the Sixth Framework Programme (2002-2006)		
Dissemination Level		
PU	Public	✓
PP	Restricted to other programme participants (including the Commission Services)	
RE	Restricted to a group specified by the consortium (including the Commission Services)	
CO	Confidential, only for members of the consortium (including the Commission Services)	

DOCUMENT CONTROL SHEET

Title of document:	*Final Report on Monte Carlo speed-up studies*
Authors of document:	*H.A.P. Blom and G.J. Bakker*
Deliverable number:	*D7.2g*
Project acronym:	*iFly*
Project title:	*Safety, Complexity and Responsibility based design and validation of highly automated Air Traffic Management*
Project no.:	*TREN/07/FP6AE/S07.71574/037180 IFLY*
Instrument:	*Specific Targeted Research Projects (STREP)*
Thematic Priority:	*1.3.1.4.g Aeronautics and Space*

DOCUMENT CHANGE LOG

Version #	Issue Date	Sections affected	Relevant information
0.1	12 August 2011	All	First draft
0.2	16 August 2011	All	Second draft
0.3	18 August 2011	All	Third draft
0.4	19 August 2011	Section 4-8	Fourth draft
0.5	20 August 2011	Sections 3-8	Fifth draft
0.6	31 Dec 2011	All	Internal review incorporated

Authors	H.A.P. Blom	NLR	
	G.J. Bakker	NLR	
Internal reviewers	M. Prandini	PoliMi	
	F. Le Gland	INRIA	
	P. Lezaud	ENAC	
	J. Krystul	UTwente	
External reviewers			

Abstract

Within WP7 of the iFLY project, several studies have been performed on the development of various complementary methods that aim to improve the speed-up performance of rare event Monte Carlo simulation of advanced ATM concept of operations. The aim of the current report is to provide an overview of these complementary speed-up results, and to show how this has been exploited, within the iFly project, in the rare event Monte Carlo simulation of the A3 ConOps.

Table of Contents

Acronyms

Acronym	Definition
A^3	Autonomous Aircraft Advanced
a/c	Aircraft
ACAS	Airborne Collision Avoidance System
ADS-B	Automatic Dependant Surveillance - Broadcast
AFR	Autonomous Flight Rules
AMFF	Autonomous Mediterranean Free Flight
ANP	Actual navigation performance
ANSP	Air Navigation Services Provider
AOC	Airline Operations Centre
ASAS	Airborne Separation Assistance System
ATM	Air Traffic Management
CD	Conflict Detection
CD&R	Conflict Detection and Resolution
CDTI	Cockpit Display of Traffic Information
CNS	Communication, Navigation & Surveillance
ConOps	Concept of Operations
CR	Conflict Resolution
CTA	Controlled Time of Arrival
DCPN	Dynamically Coloured Petri Net
FMS	Flight Management System
GNC	Guidance, Navigation and Control
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
GSHS	General Stochastic Hybrid System
ICAO	International Civil Aircraft Association
IPN	Interaction Petri Net
IPS	Interacting Particle System
IRS	Inertial Reference System
LOS	Loss of Separation
LPN	Local Petri Net
MAC	Mid-Air Collision
MC	Monte Carlo
MCMC	Markov Chain Monte Carlo
MSI	Minimum Separation Infringement
MTC	Medium Term Conflict
MTCR	Medium Term Conflict Resolution
n.a.	not applicable
Nm	Nautical mile
NMAC	Near Mid-Air Collision
OSED	Operational Services and Environmental Description
P-ASAS	Predictive Airborne Separation Assurance System

Acronym	Definition
PBC	Periodic Boundary Condition
PF	Pilot Flying
PNF	Pilot Non-Flying
RBT	Reference Business Trajectory
RNP1	Required Navigation Performance of 1 NM
RTD	Research, Technology and Development
SA	Situation Awareness
SDCPN	Stochastically and Dynamically Coloured Petri Net
SESAR	Single European Sky ATM Research
SSA	Self Separation Airspace
STC	Short Term Conflict
STCR	Short Term Conflict Resolution
SWIM	System Wide Information Management
TCAS	Tactical Collision Avoidance System
TCP	Trajectory Change Point
TMA	Terminal Area
TOPAZ	Traffic Organization and. Perturbation AnalyZer
WP	Work Package

1 Introduction

1.1 iFly project

Air transport throughout the world, and particularly in Europe, is characterised by major capacity, efficiency and environmental challenges. With the predicted growth in air traffic, these challenges must be overcome to improve the performance of the Air Traffic Management (ATM) system. The iFly project addresses these critical issues by developing a paradigm step change in advanced ATM concept development through a systematic exploitation of state-of-the-art mathematical techniques including stochastic modelling, analysis, optimisation and Monte Carlo simulation.

The iFly project will develop a highly automated ATM design for en-route traffic, which takes advantage of autonomous aircraft operation capabilities and which is aimed to manage a three to six times increase in current en-route traffic levels.

iFly will perform two operational concept design cycles and an assessment cycle comprising human factors, safety, efficiency, capacity and economic analyses. The general work structure is illustrated in Figure 1. During the first design cycle, state of the art Research, Technology and Development (RTD) aeronautics results will be used to define a "baseline" operational concept. For the assessment cycle and second design cycle, innovative methods for the design of safety critical systems will be used to refine the operational concept with the goal of managing a three to six times increase in traffic demand of 2005. These innovative methods find their roots in robotics, financial mathematics and telecommunications.

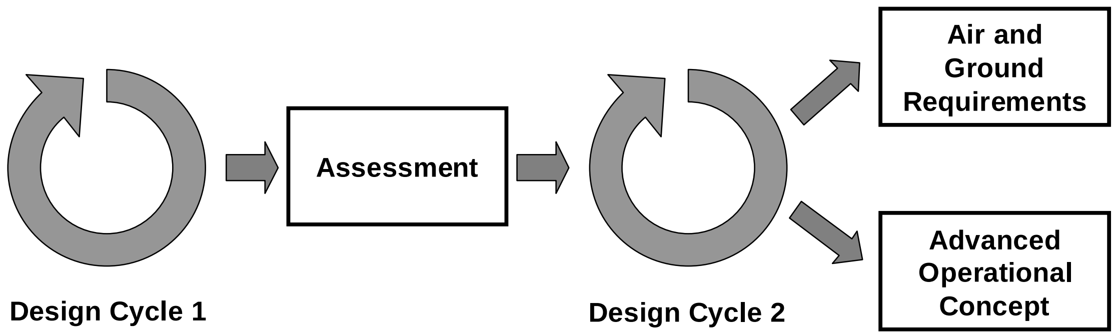

Figure 1. iFly Work Structure.

As depicted in Figure 2, iFly work is organised through nine technical Work Packages (WPs), each of which belongs to one of the four types of developments mentioned above:

Design cycle 1
The aim is to develop an Autonomous Aircraft Advanced (A^3) en-route operational concept which is initially based on the current "state-of-the-art" in aeronautics research. The A^3 ConOps is developed within WP1. An important starting and reference point for this A^3 ConOps development is formed by the human responsibility analysis in WP2.

Innovative methods
Develop innovative architecture free methods towards key issues that have to be addressed by an advanced operational concept:

- Develop a method to model and predict complexity of air traffic (WP3).
- Model and evaluate the problem of maintaining multi-agent Situation Awareness (SA) and avoiding cognitive dissonance (WP4).
- Develop conflict resolution algorithms for which it is formally possible to guarantee their performance (WP5).

Assessment cycle

Assess the state-of-the-art in Autonomous Aircraft Advanced (A^3) en-route operations concept design development with respect to human factors, safety and economy, and identify which limitations have to be mitigated in order to accommodate a three to six times increase in air traffic demand:

- Assess the A^3 operation on economy, with emphasis on the impact on organisational and institutional issues (WP6).
- Assess the A^3 operation on safety as a function of traffic density increase over current and mean density level (WP7).

Design cycle 2

The aim is to refine the A^3 ConOps of design cycle 1 and to develop a vision how A^3 equipped aircraft can be integrated within SESAR concept thinking (WP8). WP9 develops preliminary safety and performance requirements on the applicable functional elements of the A^3 ConOps, focused on identifying the required technology.

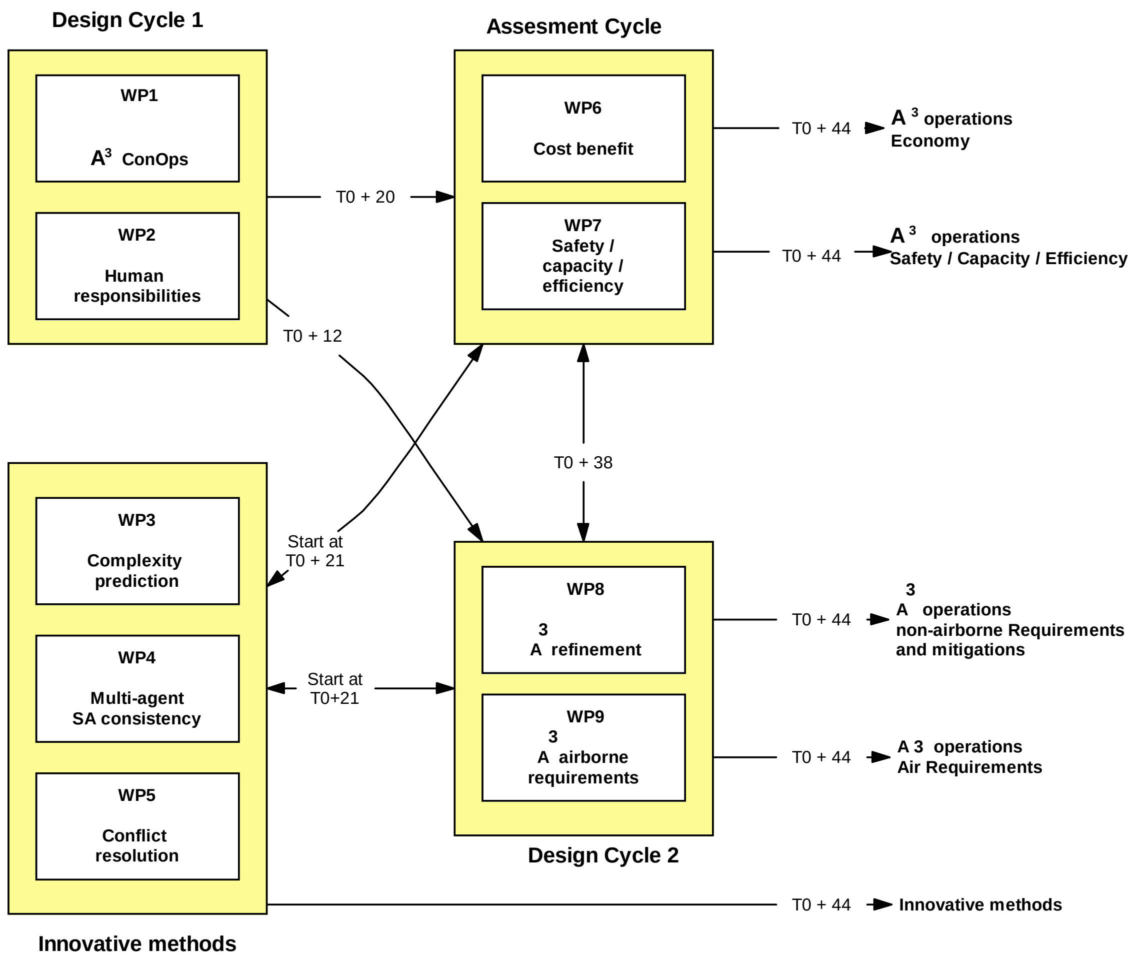

Figure 2. Organisation of iFly research.

1.2 Objective of iFly work package 7

The objective of iFly WP7 is to assess the Autonomous Aircraft Advanced (A^3) operations developed by WP1 (A^3 Concept) and WP2 (Human responsibilities in autonomous aircraft operations), through hazard identification and Monte Carlo simulation on accident risk as a function of traffic demand, to assess what traffic demand can safely be accommodated by this advanced operational concept, and to assess the efficiency of the flights. The accident risk levels assessed should be in the form of an expected value, a 95% uncertainty area, and a decomposition of the risk level over the main risk contributing sources. The latter verifies which of these sources should be mitigated during the 2nd design cycle. In order to accomplish this assessment through Monte Carlo simulation, the complementary aim of this WP is to further develop the innovative HYBRIDGE speed up approaches in rare event Monte Carlo simulation. The work is organised in four sub-WPs:

- WP7.1: Monte Carlo simulation model of A^3 operation
- WP7.2: Monte Carlo speed up methods
- WP7.3: Perform Monte Carlo simulations
- WP7.4: Final report

1.3 WP7.1 Monte Carlo simulation model of A^3 operation

The development of a Monte Carlo simulation model of A^3 operation is accomplished through a sequence of steps. First, a scoping has to be performed regarding the desired risk and capacity simulation study. An important aspect of this scoping is to identify the appropriate safety requirements to be derived from safety regulation. This has been reported in iFly deliverable D7.1a on 'Scoping and safety target' [iFly D7.1a]. Then, a hazard identification and initial hazard analysis has been performed for the A^3 operation as has been developed by WP1 and WP2 [iFly D1.3],[iFly D2.2]. This has been reported in [iFly D7.1b]. In parallel to the initial hazard analysis, the development of a Monte Carlo simulation model has been started that aims to capture the accident risk and the flight efficiency of the A^3 operation. Such a simulation model covers the human and technical agents, their interactions and both the nominal and non-nominal aspects of the operation. This has been reported in [iFly D7.1c].

1.4 WP7.2 Monte Carlo speed up methods

Within HYBRIDGE novel Monte Carlo simulation speed up techniques have successfully been developed and applied. In [iFly F7.2a] a review has been provided of the Monte Carlo simulation based accident risk assessment situation. Subsequently, the following directions have been investigated for the development of complementary speed-up and bias and uncertainty assessment techniques:

- To combine Interacting Particle System based rare event simulation with Markov Chain Monte Carlo (MCMC) speed up technique. This has been reported in [iFly D7.2b].
- To study the relation between complexity of multiple aircraft encounter geometries and collision risk, and develop importance sampling approaches which take advantage of this relation. This has been reported in [Prandini, 2011], [iFly D7.2c].

- To study ways how Interacting Particle System speed up techniques that apply to a pair of aircraft can effectively be extended to situations of multiple aircraft. This has been reported in [iFly D7.2d].
- To extend Interacting Particle System based rare event simulation for application to hybrid systems. This has been reported in [iFly D7.2e].
- To study Monte Carlo simulation based bias and uncertainty assessment with operation design parameter optimization. This has been reported in [iFly D7.2f].

1.5 WP7.3 Perform Monte Carlo simulations

Monte Carlo simulations are performed to assess collision risk of the A^3 operation. At this stage of the work, the results were of point estimation type.

1.6 WP7.4 Final Reporting

The Monte Carlo simulations have been directed to a further elaboration of the results, including sensitivity analysis. This sensitivity analysis has revealed unexpectedly positive behaviour of the A3 ConOps. This is reported in [iFly D7.4].

1.7 Current report D7.2g

The current report is the final report of WP7.2. The aim of the current report is to provide an overview of the Monte Carlo speed-up results obtained within WP7.2, and to show how these results have been exploited, within the iFly project, in the rare event Monte Carlo simulation of the A3 ConOps, reported in [iFly D7.4].

This report is organised as follows. Section 2 provides an overview of the Monte Carlo speed-up results obtained within WP7.2. Section 3 explains how each of these speed-up results have been used for the evaluation of the A3 ConOps within the iFly project. Section 4 illustrates how the various speed-up methods in rare event estimation have helped in obtaining the risk assessment results reported in [iFly D7.4]. Finally, Section 5 draws conclusions.

2 Monte Carlo speed-up approaches studied within iFly

This section aims to provide a short overview of the iFly studied speed-up approaches for rare event Monte Carlo simulation of an advanced ATM design. All these methods have been aimed at the improvement of the Interacting Particle System (IPS) method that has originally been developed by [Cerou et al., 2002, 2005], and subsequently been elaborated for use to advanced ATM designs [Blom, CDC2006, CRC2007]. This Section is organized as follows. First a high level description of the IPS approach for ATM is given in Subsection 2.1. Subsequently, Subsection 2.2 through 2.7 shortly explain the results of the IPS enhancement studies performed within iFly WP7.2.

2.1 Interacting Particle System (IPS)

The background of rare event simulation and IPS has been documented in [iFly D7.2a]. The basic idea of assessing collision risk is to perform many Monte Carlo (MC) simulations with a stochastic model of the operation considered, and to estimate the collision risk by counting the number of collisions and dividing this by the number of simulated flight hours. Though this idea is simple, in order to make it work for rare events, we need an effective way of speeding up the MC simulation. This subsection describes the basic idea of how this works with the IPS method. The IPS method is a sequential MC simulation approach, i.e. one which consists of a series of MC simulation cycles, where each cycle uses the output of the previous cycle as input to its next cycle. This way it is possible per cycle to zoom further into the behavior of the simulated trajectories. During the first simulation round we are interested in counting events that happen quite regularly, i.e. say once in about 10 to 100 MC simulation runs. Each next cycle we are interested in events that happen an order of magnitude less frequent. To make this cyclic approach work, the MC simulation results that have been obtained by one cycle are going to be used to partly generate the seeds for the next MC simulation cycle. In [Cerou et al., 2002, 2005, 2006] a precise mathematical framework and algorithm has been developed for conducting such a sequential MC simulation well. It also has been proven that the estimated event probabilities converge to the true probabilities under some technical conditions. The main conditions are that the process to be assessed needs to satisfy semi-martingale and strong Markov properties. The specific Petri net formalism that has been used for the A3 model development and specification [Everdij&Blom, 2005, 2006],[Everdij, 2010], assures that the technical conditions are satisfied [Krystul, 2006], [Krystul et al., 2007], [Blom&Everdij, 2010]. Within the Hybridge project, this IPS method has been adapted to the safety analysis of the Autonomous Mediterranean Free Flight (AMFF) ConOps [Blom, ATC-Q2009]. Because it was anticipated that the A3 ConOps would pose additional challenges to the IPS method, several complementary approaches have been studied within WP7.2. An overview of the results of these studies is provided in this Section. Next, Section 3 describes the rationale for using these results (or not) for the safety analysis of the A3 ConOps within the iFly project, as has been reported in [iFly D7.4].

2.2 IPS x Markov Chain Monte Carlo

This study has been documented in [iFly D7.2b]. The main outcome of this study is the proposal to include in each IPS cycle a Markov Chain Monte Carlo (MCMC) operation. Such

MCMC operation has as crucial property that it does not change the probabilistic characteristics of the statistical estimates. In [iFly D7.2b] it has been announced that the inclusion of this MCMC operator step requires the following changes to the IPS method as it has been used in [Blom, ATC-Q2009]. These changes are:

- The nested subsets have to be choosen much closer to each other; otherwise it is quite unrealistic that an appropriate MCMC operator step can be developed.
- In the original IPS, resampling is applied in each cycle. This should be avoided; resampling is only allowed if the diversity of the particles underscores some critical threshold.
- Because the MCMC operator step applies to the histories of the particles, there is a need to copy/restore the particle histories in/from computer memory.

Although [iFly D7.2b] provides guidelines for the development of such an MCMC operator, this remains to be done for the specific application considered, including a systematic evaluation of its working.

2.3 Importance Sampling of Initial Traffic Condition

This study has been documented in [Prandini, 2011]. The key idea is to perform an importance sampling of the initial condition of each particle, based on an evaluation of the complexity of initial traffic condition in a random sample. In [iFly D3.2] several proposals for such complexity measures have been evaluated, and the conflict probability prediction approach appeared to be the best choice. Subsequently, in [Prandini, 2011] it has been studied how well this probabilistic conflict prediction method can be combined with the IPS method. Based on simulations performed for the Autonomous Mediterranean Free Flight (AMFF) ConOps, it has been estimated that the IPS speed-up improvement may go up to a theoretical factor 15.6x [Prandini, 2011], although in practice this factor may be lower.

2.4 Periodic Boundary Condition

This study has been documented in [iFly D7.2d]. In order to simulate a realistic en-route situation, it is required to study a very large air space area containing hundreds of aircraft. Because for so many aircraft, the IPS method would run into dramatic dynamic computer memory limitations, a Periodic Boundary Condition (PBC) has been adopted. This has already been done in the study of the AMFF ConOps [Blom, ATC-Q2009]. The PBC used is to pack the airspace full with same size boxes, and to let fly a fixed number of aircraft in each box. Each moment one of the aircraft wants to fly out of the box, then a copy of that aircraft enters the very same box at the opposite side. This way of working is well known from performing simulations in theoretical physics [Rapaport, 2004]. Obviously when the sizes of the box become too small, then the practical use of this PBC approach will become useless. And when the sizes of the box become too large then too many aircraft have to be simulated. So the question is what sizes are right for the box. Unfortunately in literature there is no specific theory available regarding this aspect of PBC. Hence in [iFly D7.2d] minima to be posed on the sizes of the box have been studied, taking into account performance bounds of commercial aircraft in en-route airspace.

2.5 Managing IPS for large hybrid systems

This study has been documented in [iFly D7.2e]. The state space of an safety model of ATM operations is hybrid, i.e. it is a product of a Euclidean space and a discrete set. [Krystul &

Blom, 2005] have developed a Hybrid IPS (HIPS) method which is able to handle such a hybrid state space. The HIPS method uses a large number of particles per mode. And the safety model of the A3 ConOps comprises some 10 to the power 110 discrete modes [iFly D7.4]. This would mean that application of HIPS would require a particle system using more than 10 to the power 110 particles. Obviously this is practically impossible to manage. In order to resolve this problem, in [iFly D7.2e] combinatorially many modes are aggregated into a small number of high level modes. Subsequently an IPS has been developed which manages in the order of 10 thousand particles for each of these high level modes, and where each of the particles covers a realization of the full hybrid state space. This extension of IPS has been named Hierarchical Hybrid IPS (HHIPS) [Blom, CDC2007, Wiley2009], and its practical use has been illustrated for the AMFF ConOps [Blom, ATC-Q2009].

2.6 Parameter Sensitivity analysis

This study has been documented in [iFly D7.2f]. For risk analysis purposes it is important to assess the influence of parameter variation upon the variation in assessed safety risk levels. In general, a relatively simple approach in performing such a sensitivity analysis is to assess the safety risk once for the baseline value of each parameter, and subsequently change the value of each parameter, one at a time. The difference in parameter value also leads to a difference in assessed risk level. The problem in using this approach in combination with IPS is that the estimation errors in IPS are non-negligible, and this only gets worse for a difference between two IPS estimated risk values. In order to resolve this problem the idea is to exploit a multi-dimensional regression analysis for the assessment of model parameter sensitivities. In [iFly D7.2f] several multi-dimensional regression analysis methods have been evaluated, such as: Classical Least Squares (CLS), Least Squares with Moore Penrose (LS-MP), Partial Least Squares based on NIPALS (PLS-N) and Partial Least Squares based on SIMPLS (PLS-S). Complementary to this, two sampling approaches have been evaluated: Standard Random Sampling (SRS) and Latin Hypercube Sampling (LHS). These evaluations have shown that best performance is obtained with the LS-MP method, in combination with an SRS based sampling of parameter values for the input of the safety risk simulation model. The study has also shown that the number of random parameter samples for which an IPS should be performed is 2N or more, where N is the number of relevant (groups of) parameters. In [iFly D7.2f] this multi-dimensional regression analysis has not been tested out for the AMFF ConOps and neither for the A3 ConOps.

3 What has been used in rare event estimation of the A3 ConOps

This subsection explains for each of the methods described in Section 2 the rationale regarding their use (or not) in the evaluation of the A3 ConOps as reported in [iFly D7.4].

3.1 A3 ConOps dedicated IPS

In the earlier study of the AMFF ConOps, the IPS approach has played a crucial role in the analysis of the rare events. This also was the case for the rare event estimation of the A3 ConOps. However, it appeared necessary to define a new sequence of conflict severity levels. This old and new set of conflict severity levels are specified in Table I and Table II respectively. The key reason why there was a need to make this change is that due to the 4D trajectory planning and broadcasting, there hardly were any predicted conflicts anymore. At the same time there was a need to introduce additional prediction-free conflict severity levels.

TABLE I
IPS CONFLICT LEVEL PARAMETER VALUES USED FOR AMFF

k	1	2	3	4	5	6	7	8
d_k Nm	5.0	5.0	5.0	5.0	2.5	1.25	.50	.054
h_k ft	1000	1000	1000	1000	1000	500	250	131
Δ_k min	8	2.5	1.5	0	0	0	0	0

Table I. The conflict severity levels used for rare event estimation of the AMFF ConOps [Blom, CDC2007].

TABLE II
IPS CONFLICT LEVEL PARAMETER VALUES USEFD FOR A^3 CONOPS

k	1	2	3	4	5	6	7	8	9	10	11	12	13
d_k Nm	6.0	5.5	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	.054
h_k ft	900	900	900	900	900	900	900	750	600	500	400	300	131
Δ_k min	0	0	0	0	0	0	0	0	0	0	0	0	0

Table II. The conflict severity levels used for rare event estimation of the A3 ConOps [iFly, D7.4] for the two and eight aircraft encounters are defined by parameters d_k = horizontal distance, h_k = vertical distance, and Δ_k = prediction period.

An important improvement over rare event simulations performed for AMFF, and those to be performed for A3 ConOps, is that the computer system has been upgraded from two Dell Precision 390 to two Dell Precision T7500 (each with two Intel Xeon X5680 CPU's). This implies two key improvements: 1) availability of an order in magnitude more computer power; and 2) availability of an order in magnitude more dynamic memory.

Regarding the effectively available dynamic memory; with the novel this has gone up from about 2 GigaByte to about 40 GigaByte. This factor 20 improvement allowed to use many more particles in one sequential MC (IPS or HHIPS). At the other hand, because 4D intent of other aircraft is now incorporated in particle embedded information, the memory occupancy of one particle has also increased significantly. Nevertheless, the net improvement still is an order of magnitude.

Regarding the computer power, the largest improvement was that each new Dell has 12 CPU's each of which can work in parallel. When running a normal Monte Carlo simulation, each of these CPU's performs MC simulation runs, independently of the other CPU's. For IPS, however, theory tells that it is better to let these CPU's work in a coordinated way. In order to manage this, one IPS run has been divided over the 12 CPU's as follows. At the beginning of an IPS cycle, each CPU starts with the same number N_P of particles (e.g. 10 thousand). Based on this input, each of the 12 CPU's then performs independently all local steps of one IPS cycle for its own particles. Upon completion of the local IPS steps, each CPU will stop and has identified those particles that have reached the next level. Each CPU stores these particles in static memory. Next a global IPS step is performed by one of the 12 CPU's. This global step collects all arrived particles from static memory, adds extra copies of them (according to the IPS resampling step), and re-allocates all particles in this set randomly over 12 new particle subsets. Finally, each of these 12 subsets is allocated to one of the CPU's. Subsequent the next local steps of an IPS cycle are conducted by each CPU (as has been done during the previous IPS cycle). This approach in running one IPS by multiple CPU's has successfully been implemented and tested. A similar approach in running a HHIPS based SMC on multiple CPU's remains to be developed.

3.2 IPS x Markov Chain Monte Carlo

Although [iFly D7.2b] provides guidelines for the further development of the IPSxMCMC approach, it is not yet ready for application to the A3 ConOps. Prior to such a demanding application, the following aspects of the IPSxMCMC approach remain to be addressed:
- To develop an effective MCMC operator step;
- To develop an effective way to store particle histories in static memory;
- To develop a sequence of nested subsets which matches well to the MCMC operator step;
- To develop a particle resampling strategy which works well in combination with the above.
- To perform a systematic evaluation of this for the A3 ConOps.

These steps have been judged to be too much to be accomplished within the iFly project. Hence, within the iFly project, IPSxMCMC can not be used for the evaluation of the A3 ConOps.

3.3 Importance Sampling of Initial Traffic Conditions

In [Prandini et al., 2011] it has been estimated that theoretically this approach might deliver a significant factor extra speed-up for the AMFF ConOps. Unfortunately, we have not found a way to exploit this theoretical factor in a practically useful way. In contrast with AMFF ConOps, the medium term conflict-free 4D trajectory planning by the A3 ConOps changes the nature of medium term complexity prediction in such a dramatic way that the probabilistic complexity prediction method proposed by [Prandini et al., 2011] becomes less effective as an importance sampling method. Hence, within the iFly project, this complexity based importance sampling could not be used for the evaluation of the A3 ConOps. This finding does not mean anything regarding the potential use of this complexity prediction method to other problems, such as strategic traffic flow management.

3.4 Periodic Boundary Condition

The criteria identified by the PBC study [iFly D7.2d] have gratefully been used within the rare event Monte Carlo simulations of dense random traffic under the A3 ConOps. This has led to differences with the PBC used for the AMFF ConOps [Blom et al., ATC-Q2009]:

- For the A3 ConOps it is assumed that air traffic does not climb or descend;
- The highest traffic density has gone up by a factor 8/3.
- The size of a container has been changed from 40Nmx40Nmx4000ft to 62Nmx62Nmx1000ft.

3.5 Managing IPS for large hybrid systems

For two aircraft encounters under the AMFF ConOps, the HHIPS has been used to evaluate the impact of varying the dependability of ASAS supporting technical systems. For the A3 ConOps it also is desired to evaluate such dependability for more demanding aircraft encounters. However an effective approach remains to be developed in applying HHIPS to multi-aircraft encounter situations.

3.6 Parameter Sensitivity analysis

The A^3 model has a set of 164 scalar parameters. Based on expert judgement, the following six (groups of) parameters have been selected for sensitivity analysis:

- Flight crew response times (28 parameters)
- Dependability parameters of ASAS supporting systems (6 parameters)
- Actual Navigation Performance (ANP)
- Medium Term CD&R minimum horizontal separation
- Short Term CD&R minimum horizontal separation
- Aircraft groundspeed

For this selected (groups of) parameters, it has been judged to be more realistic to conduct a sensitivity analysis using a one-at-a-time approach, rather than the more involved multi-dimensional regression analysis of [iFly D7.2f]. The one-at-a-time approach changes the value of one parameter only by a factor two (or it changes all values of the parameters in one group by this factor), rather than randomly changing all parameter values.

4 Definition of scenarios to be evaluated

4.1 Air traffic scenarios

For the A^3 model, MC simulations will be conducted for the following encounter scenarios:

- Two-aircraft head-on encounter scenario
- Eight-aircraft encounter scenario
- Random traffic scenarios for various traffic densities

For each encounter scenario simulation results are obtained under A3 control as well as for the uncontrolled condition, i.e. without doing any conflict detection and resolution. Under the uncontrolled condition, the safety related event probabilities in the various encounter scenarios have also been calculated using the gas model [Alexander, 1970; Endoh & Odoni, 1983]; these results agreed with the estimated values obtained through MC simulation.

Because of the objective of the current report, the material in this report is focused on the working of the rare event simulation methods. This is in contrast to [iFly D7.4] where the focus is on the results obtained for the A3 ConOps. In order to stay well aware of the relation between the scenarios considered and the methods applied, Table III shows which SMC speed-up methods are used for the evaluation of each of the above mentioned traffic scenarios.

TABLE III. SMC speed-up methods used for the evaluation of which traffic scenario.

SMC method	**Two a/c**	**Eight a/c**	**Random**
IPS	-	Yes	Yes
IPS on multiple CPU’s	-	-	Yes
IPSxMCMC	-	-	-
Complexity importance sampling	-	-	-
PBC bounds	-	-	Yes
HHIPS	Yes	-	-
Sensitivity analysis	Yes	Yes	-
Regression analysis	-	-	-

4.2 Measurements per scenario

The aim is to estimate for each scenario, probabilities for the following safety related events:

- Minimum Separation Infringement (MSI)
- Loss Of Separation (LOS) = 2/3rd of MSI [NATS, 2011]
- Near Mid Air Collision (NMAC)
- Mid Air Collision (MAC)

These safety related events are defined through three parameters: a prediction time, a horizontal distance criterion, and a vertical distance criterion. The specific values adopted for MSI, LOS, NMAC and MAC are given in Table IV. The MSI value of 3 Nm has been proposed in [iFly D1.3].

TABLE IV. Definition of safety related events used in collecting statistics from the rare event MC simulations of the A^3 ConOps. For A^3 ConOps it appeared to be rather ineffective to use a non-zero prediction period.

Event	MSI	LOS	NMAC	MAC
Prediction period (minutes)	0	0	0	0
Horizontal distance (Nm)	3.0	2.0	1.0	0.054
Vertical distance (ft)	900	600	400	131

In addition to these specific safety related events, the frequency of occurrence is measured for various intermediate distance values also. An illustrative picture of a possible resulting curve is provided in Figure 3. In this Figure, the horizontal axis is linear and typically runs from 6.0 Nm to 0.0 Nm miss distance (from left to right the miss distance reduces, which means that time runs from left to right also). The MAC point is only some 100 m away from the 0.0 Nm point. The vertical axis is logarithmic and covers 10 orders of magnitude in frequency of events (either per encounter or per flight hour).

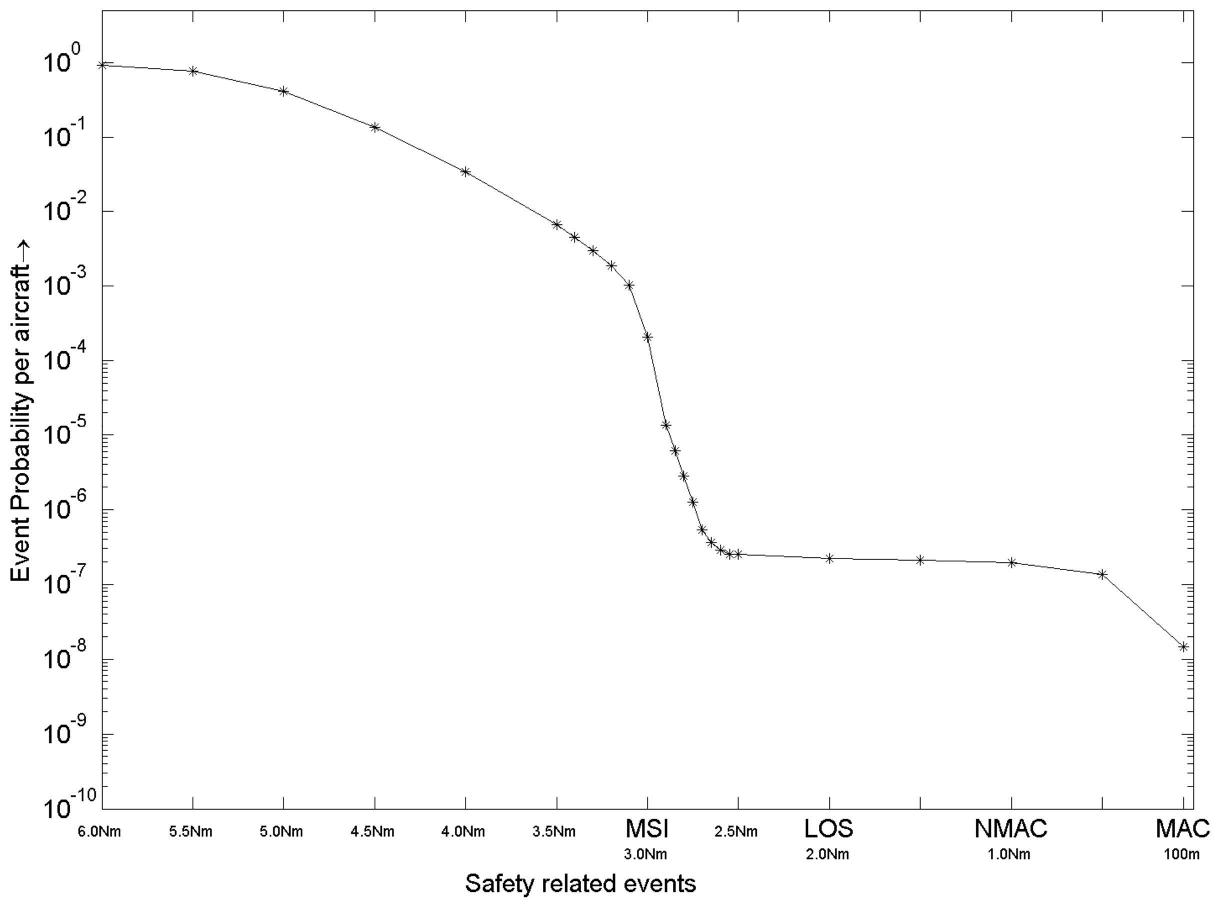

Figure 3. Illustration of the typical kind of results obtained through rare event simulation.

4.3 Model Parameter Values

Inherent to the early phase of design, for an A3 ConOps there are several parameters in the model for which it is not yet clear what the exact value should be. Therefore, one of the purposes of the early rare event simulation is to identify what the impact is of parameter values on the behaviour of the A3 ConOps. Because there are 164 model parameters, it is not realistic to start such analysis for all 164 parameter values. Instead we work as follows:

Step 1: With the help of literature sources and various experts, for each model parameter a baseline parameter value has been identified; these baseline values have been documented in Appendix B of [iFly D7.4]. For some key parameters in the A3 ConOps model, baseline parameter values are given in Table V and Table VI. Table V provides the baseline minimum separation values proposed in [iFly D1.3] for the MTCD&R and the STCD&R of the A3 ConOps. Table VI provides the baseline dependability values adopted for the main safety critical parameters of the A3 enabling technical systems (GNSS, ADS-B and ASAS). The baseline dependability values are based on [RTCA, 2002] and [Scholte & KleinObbink, 2005].

TABLE V. Baseline values of A^3 *ConOps model based MTCD&R and STCD&R parameters*

	Look ahead time	**Horizontal separation**	**Vertical separation**	**Info used**	**Max turn angle** $\varphi_{M,\max}$
STCD	3 minutes	3Nm	900ft	State & Intent	N.A.
STCR	3 minutes + 10 sec	3Nm	900ft	State & Intent	$\varphi_{S,\max} = 60°$
MTCD	10 minutes	5Nm	1000ft	Intent	N.A.
MTCR	15 minutes	5Nm	1000ft	Intent	$\varphi_{M,\max} = 60°$

TABLE VI. Baseline values of key dependability parameters of A^3 *enabling technical systems*

Math symbol	**Model parameters of** $\mathbf{A}^3$ **enabling technical systems**	**Baseline dependability**
p_{SAT}^{down}	Probability of GNSS down	1.0×10^{-5}
$p_{ADS,FRQ}^{occupied}$	Probability of Global ADS-B down[1]	1.0×10^{-6}
$p_{ADS,REC}^{down}$	Probability of Aircraft ADS-B Receiver down	5.0×10^{-5}
$p_{ADS,TRM}^{down}$	Probability of Aircraft ADS-B Transmitter down	5.0×10^{-5}
$p_{ASAS}^{corrupted}$	Probability of Aircraft ASAS performance corrupted	5.0×10^{-5}
p_{ASAS}^{fail}	Probability of Aircraft ASAS System down	5.0×10^{-5}

[1] Global ADS-B down refers to frequency congestion/overload of data transfer technology used by ADS-B.

TABLE VII. Parameter values identified for sensitivity analysis of A^3 *ConOps model*

Model parameter(s)	**Id**	**# settings**	**Specific setting(s)**
Baseline	0	1	Baseline parameter values [iFLY D7.4, Appendix B]
Crew response delay	1	1	All crew response times in the model are reduced by a factor 2, i.e. the crew is expected to respond twice as fast as has been assumed for the baseline.
Dependability	2	2	10x better, 100x better than the values in Table V
ANP	3	2	ANP0.5 and ANP2 in contrast to baseline ANP1
MTCD&R	4	1	Horizontal separation 6Nm instead of 5Nm
STCD&R	5	2	Horizontal separation 4Nm and 5Nm instead of 3Nm
Groundspeed	6	1	300m/s instead of baseline 250m/s

Step 2: The 164 model parameters have been walked through regarding their importance to assess the sensitivity of the assessed safety risk level to changes in their adopted value(s). This has led to the identification of the six (groups of) parameters specified in Table VII.

Step 3: Rare event simulations are repeated one-by-one for each of the six parameter changes in Table VII. Typically, both a standard MC simulation as well as a Sequential MC simulation is conducted. The reason for doing so is that a standard MC simulation typically is more detailed in the assessment of events that happen more frequent, whereas a Sequential MC simulation often is better in the estimation of the rare event frequencies.

Step 4: Evaluation of the simulation results obtained. In the current report the evaluation is directed to building a proper understanding of any differences in results obtained by running standard MC and Sequential MC (IPS or HHIPS) simulations. The evaluation of what the obtained results mean for the A3 ConOps is documented in [iFly D7.4].

5 SMC versus MC for Two-Aircraft encounters

5.1 Two-aircraft encounter scenario considered

In this encounter scenario, two aircraft start at the same flight level, some 320 km (173 Nm) away from each other, and fly on opposite direction flight plans head-on with a ground speed of approximately 250 m/s. The initial 3-dimensional position has standard deviations of 20m along the RBT centerline, 0.5Nm in the lateral direction (RNP1) and 20m in height.

The scenarios considered are specified in Table VIII. The identity number refers to the parameter settings adopted in Table VII. For each parameter setting both a standard MC and a HHIPS has been conducted. For the assessment of each scenario for one set of parameter values, we ran both a standard MC simulation and 10 times an HHIPS based Sequential MC (SMC) simulation [Blom CDC2007, Wiley2009]. Table VIII also shows the number of MC runs or the number of particles used, and the time-duration of the simulation.

TABLE VIII. Parameter value scenarios simulated, MC types and time durations of the computations on two Dell precision T7500

Parameter value scenario	**Id**	**Figure**	**Standard MC**		**HHIPS**	
			# of runs	**Duration**	**# of particles**	**Duration**
Baseline	0	4	28 million	12 hrs	10×80 thousand	1 hr
Crew response	1	5	4 million	2 hrs	10×80 thousand	1 hr
Dependability	2	6	4 million	2 hrs	10×80 thousand	1 hr
ANP	3	7	0.7 million	<1 hr	10×80 thousand	1 hr
MTCD&R	4	8	1.5 million	<1 hr	10×80 thousand	1 hr
STCD&R	5	9	0.8 million	<1 hr	10×80 thousand	1 hr
Groundspeed	6	10	1.5 million	<1 hr	10×80 thousand	1 hr

The total duration of using both Dell machines for the running of simulations for two aircraft encounter scenarios amounts 24 hours, which comes down to running the two Dell computers 1 day full time. In practice, there also is an order in magnitude more days needed for the preparation of the simulations (including testing of the software adaptations), and for the evaluation and documentation of the results obtained.

5.2 Simulation results: MC versus SMC

The simulation results are shown in Figures 4 through 10. In this subsection we only address the differences observed between the results of MC and HHIPS based Sequential MC (SMC). In [iFly D7.4] the behavior of the A3 model itself will be discussed on the basis of the simulation results obtained.

Figure 4 presents the estimated probabilities for the baseline parameter values (Id. 0 in Table VIII). Because the standard MC simulation is so time demanding, only for the baseline parameter values a very large number of standard MC runs has been simulated. Hence the curve obtained using standard MC simulations is pretty accurate in Figure 4. Comparison of the standard MC simulation results with the HHIPS based Sequential MC results in Figure 4 shows that standard MC has an advantage in assessing regular event frequencies, and HHIPS has a significant advantage in assessing rare event frequencies. In figures 5 through 10, the total simulation time for both approaches is about the same: then the advantage of HHIPS for estimating rare event frequencies becomes really clear.

Figure 6 presents sequential MC estimated probabilities for the safety related events defined in Table VIII under A^3 control for three sets of dependability parameter values. The lowest probability that has been estimated this way is 1.4E-10. In order to estimate this value similarly well through straightforward MC simulation, this would require 7×10^{10} standard MC runs. For two Dell Precision T7500 this would take $\frac{7\times10^{10}}{28\times10^{6}}\times12\approx30.000$ hrs. (approximately 3.5 years). The HHIPS accomplished this in 1 hour, which comes down to a speed-up factor of 30 thousand times.

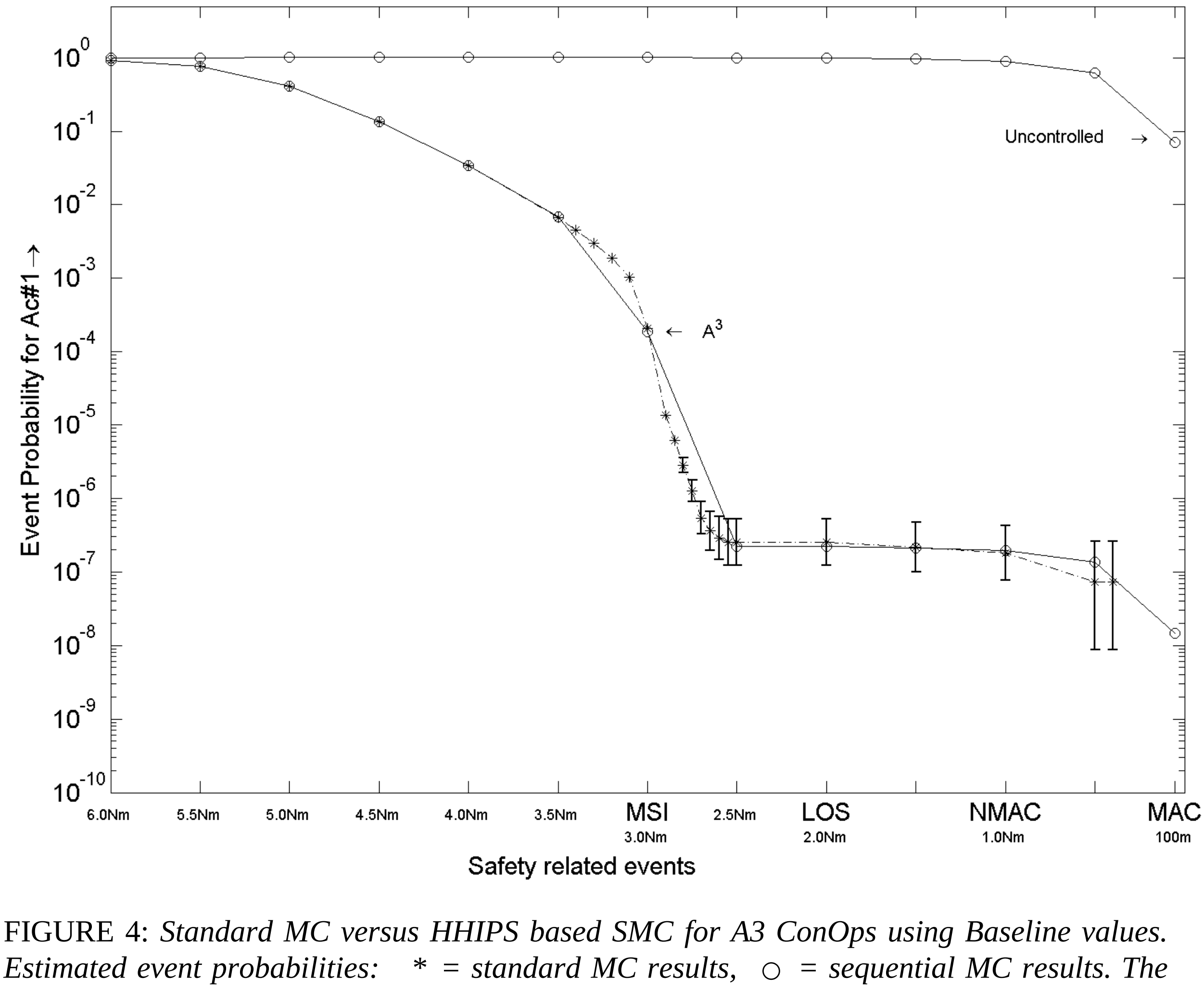

FIGURE 4: *Standard MC versus HHIPS based SMC for A3 ConOps using Baseline values. Estimated event probabilities: * = standard MC results, ○ = sequential MC results. The vertical segments show the 95% uncertainty intervals for the standard MC simulation based estimated frequencies.*

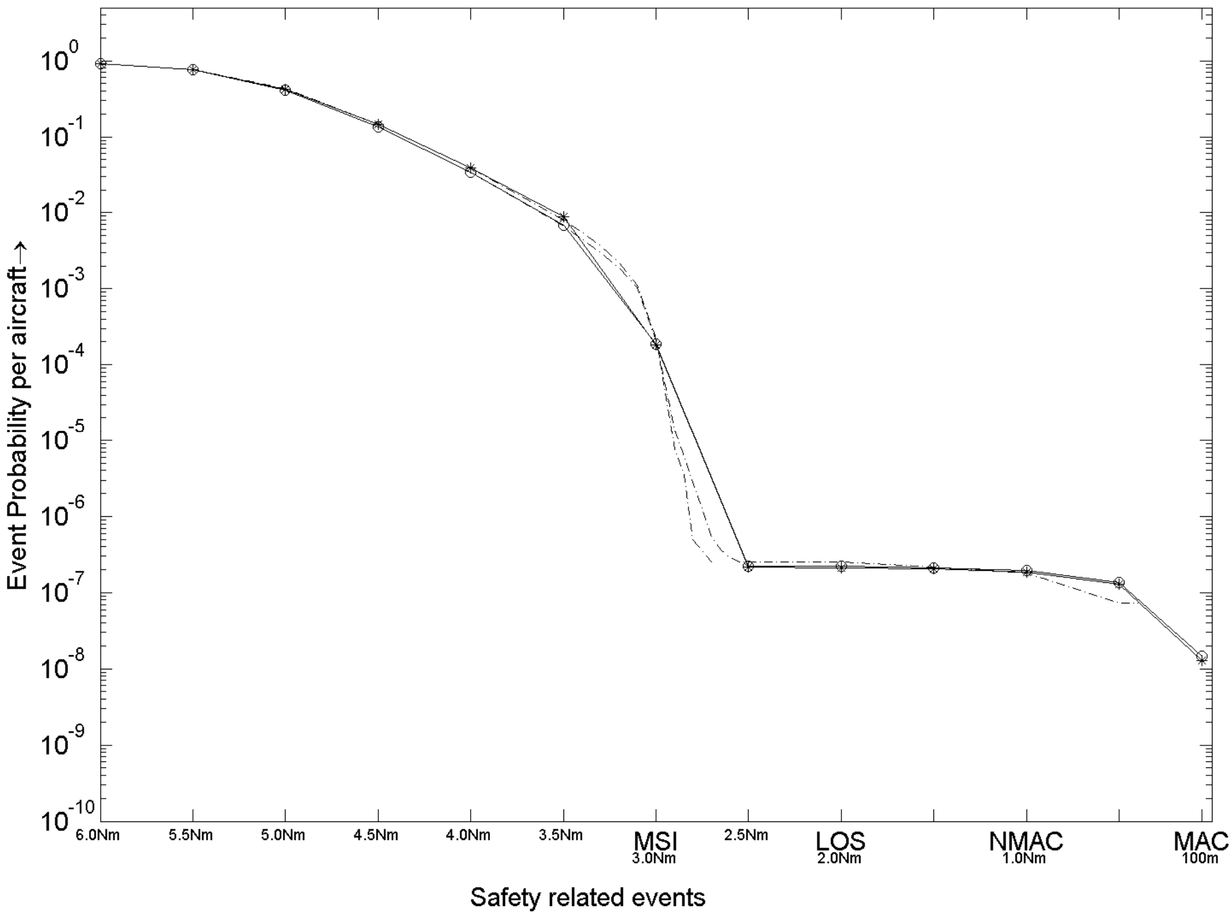

FIGURE 5: *Standard MC versus HHIPS based SMC for Varying Crew Response values. Estimated event probabilities for scenario 1.0 and 1.1. ○ = sequential MC results scenario 1.0 (baseline), * = sequential MC results scenario 1.1, -. = standard MC results.*

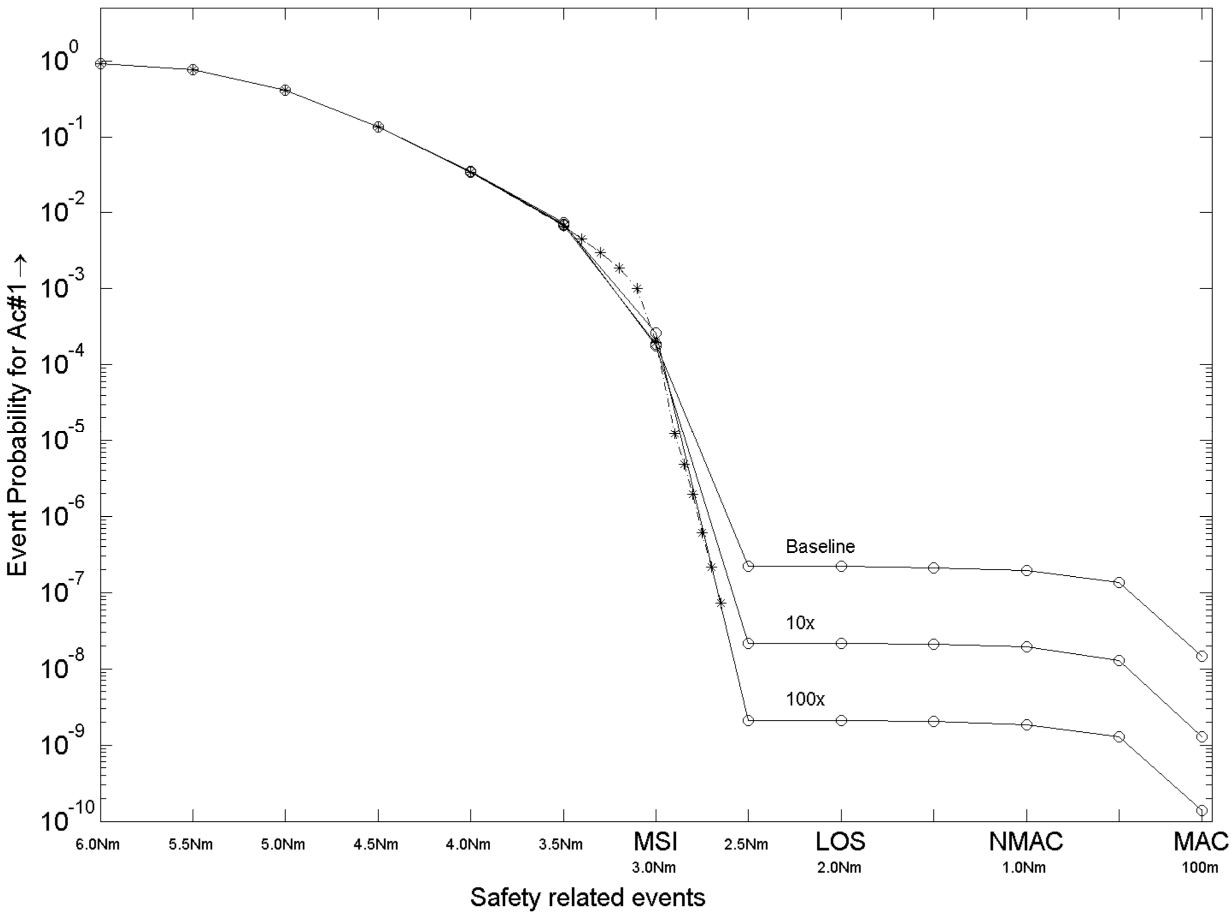

FIGURE 6*: Standard MC versus HHIPS based SMC for varying Dependability levels. The standard MC simulation results are for the 100x better dependability values only. Estimated event probabilities for scenario 1.0 and 1.2. ○ = sequential MC results, * = standard MC results.*

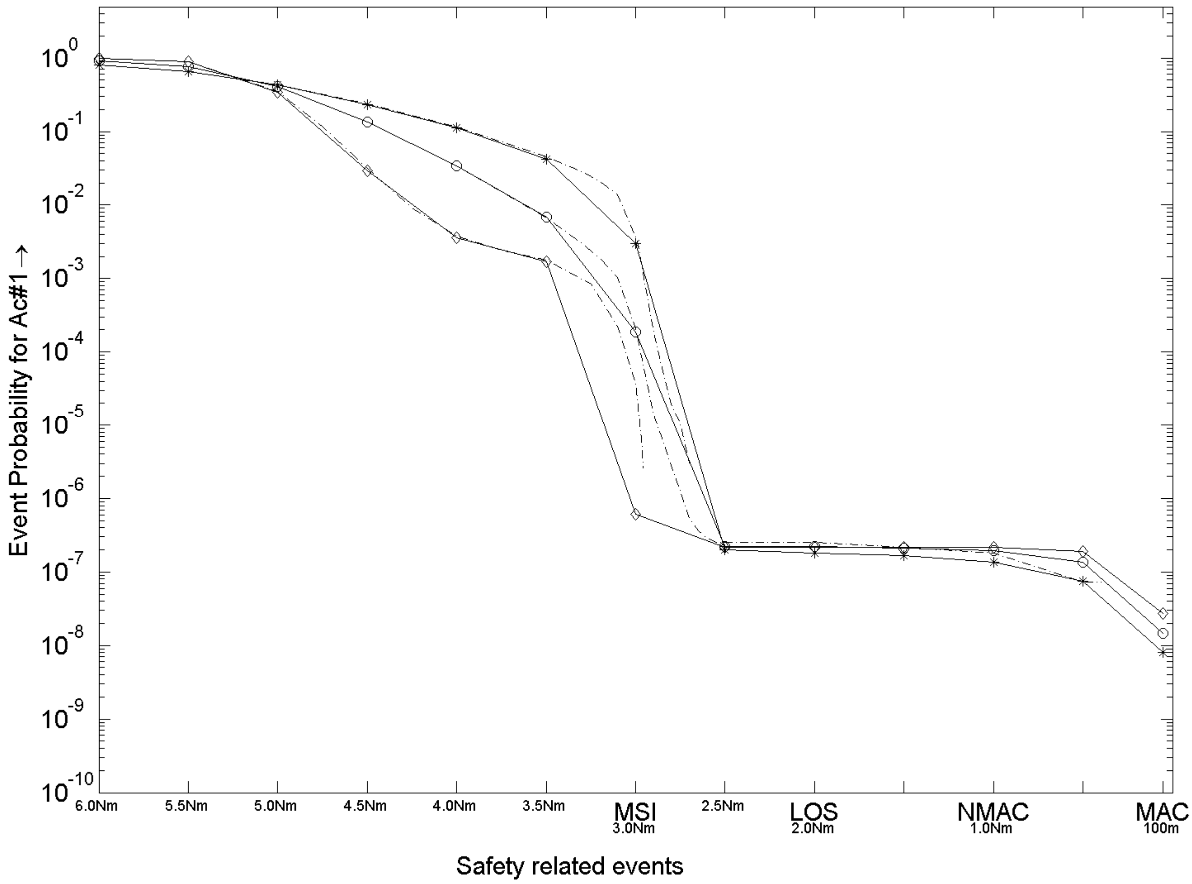

***FIGURE 7**: Standard MC versus HHIPS based SMC for varying ANP values. For ANP0.5 and ANP2, the number of MC runs has been much lower than for ANP1. ○ = sequential MC results scenario 1.0 (baseline), * = sequential MC results scenario 1.3 for ANP2, ◊ = sequential MC results scenario 1.3 for ANP0.5, -. = standard MC results for ANP0.5, ANP1 and ANP2 respectively.*

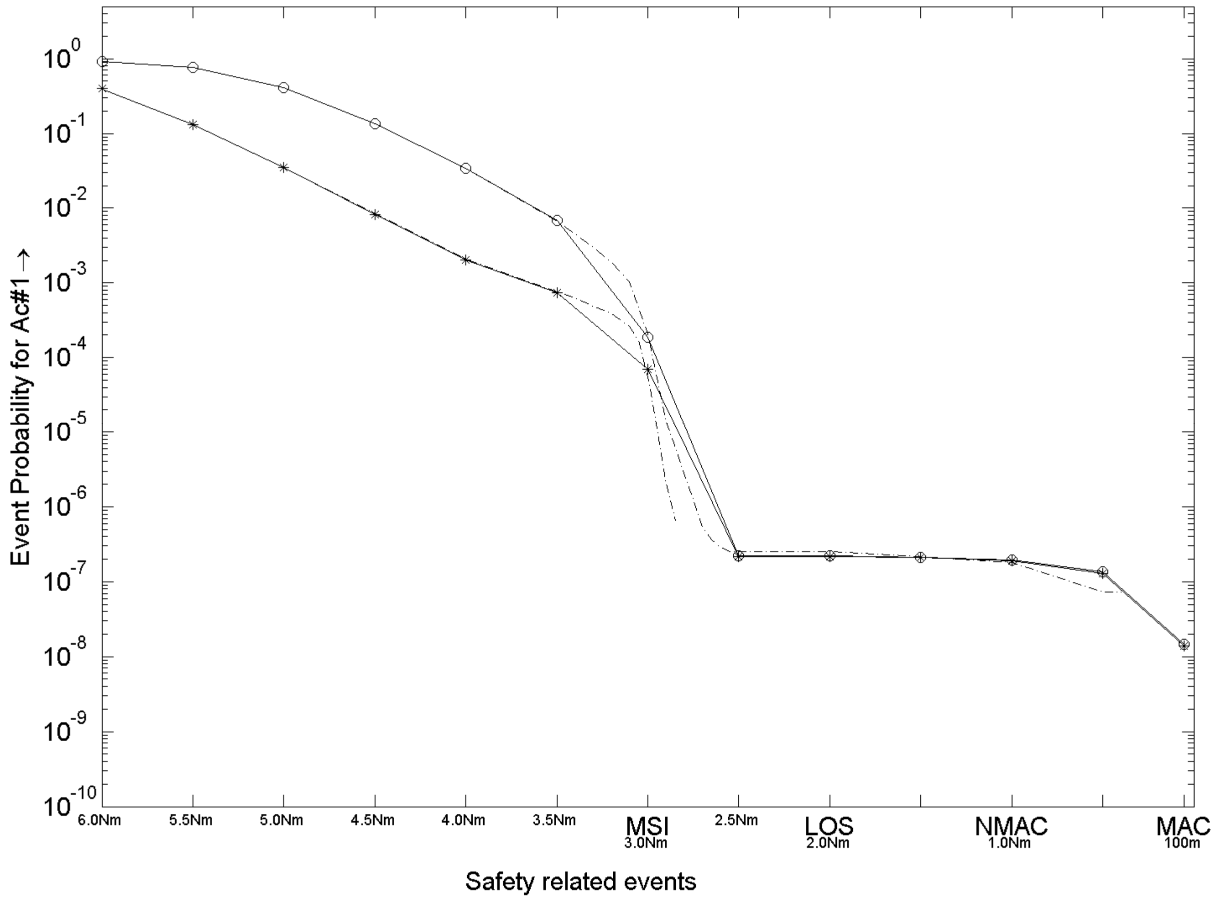

FIGURE 8: *Standard MC versus HHIPS based SMC for varying 4D trajectory separation values. Estimated event probabilities for scenario 1.0 and 1.4. ○ = sequential MC results scenario 1.0 (baseline), * = sequential MC results scenario 1.4, -. = standard MC results.*

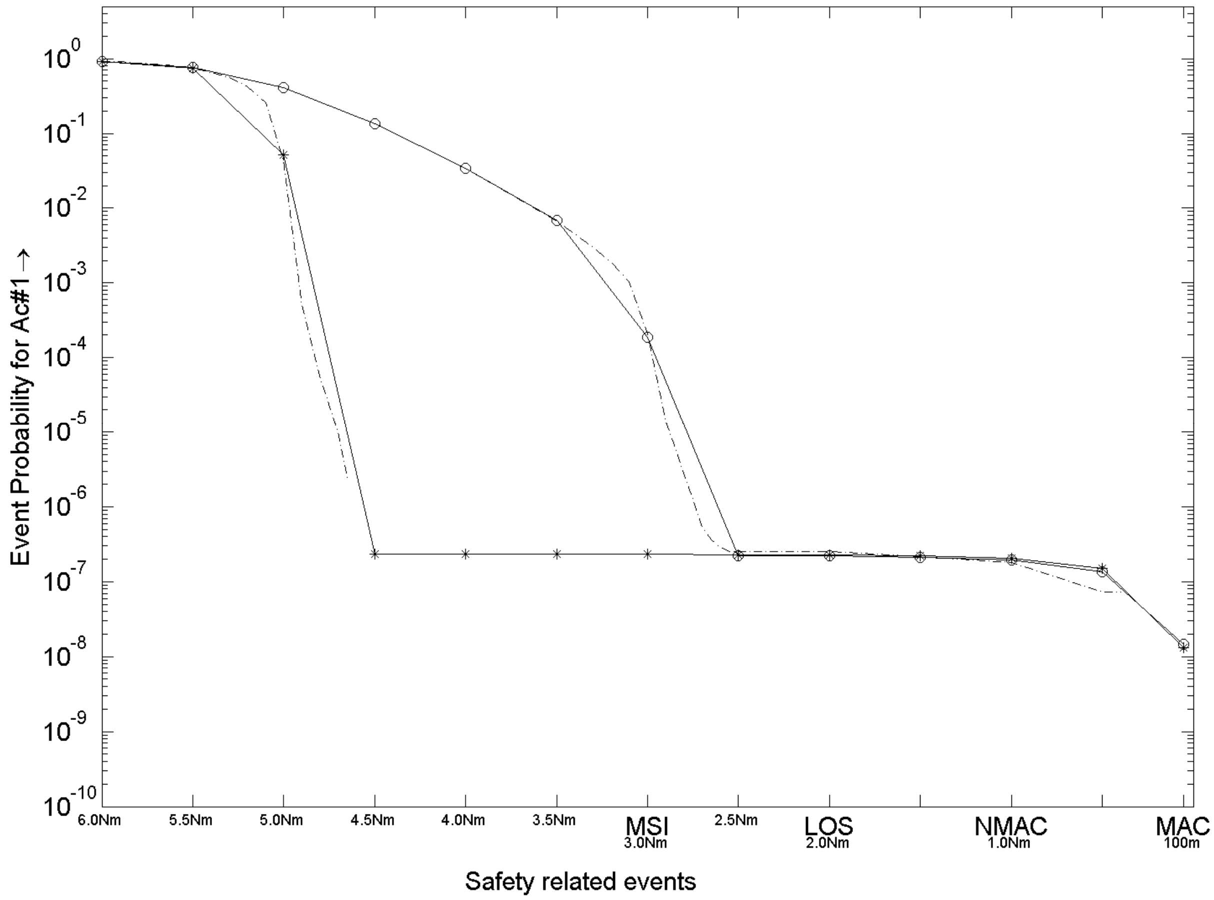

FIGURE 9: *Standard MC versus HHIPS based SMC for varying tactical separation values. ○ = sequential MC results scenario 1.0 (baseline), * = sequential MC results scenario 1.5, -. = standard MC results.*

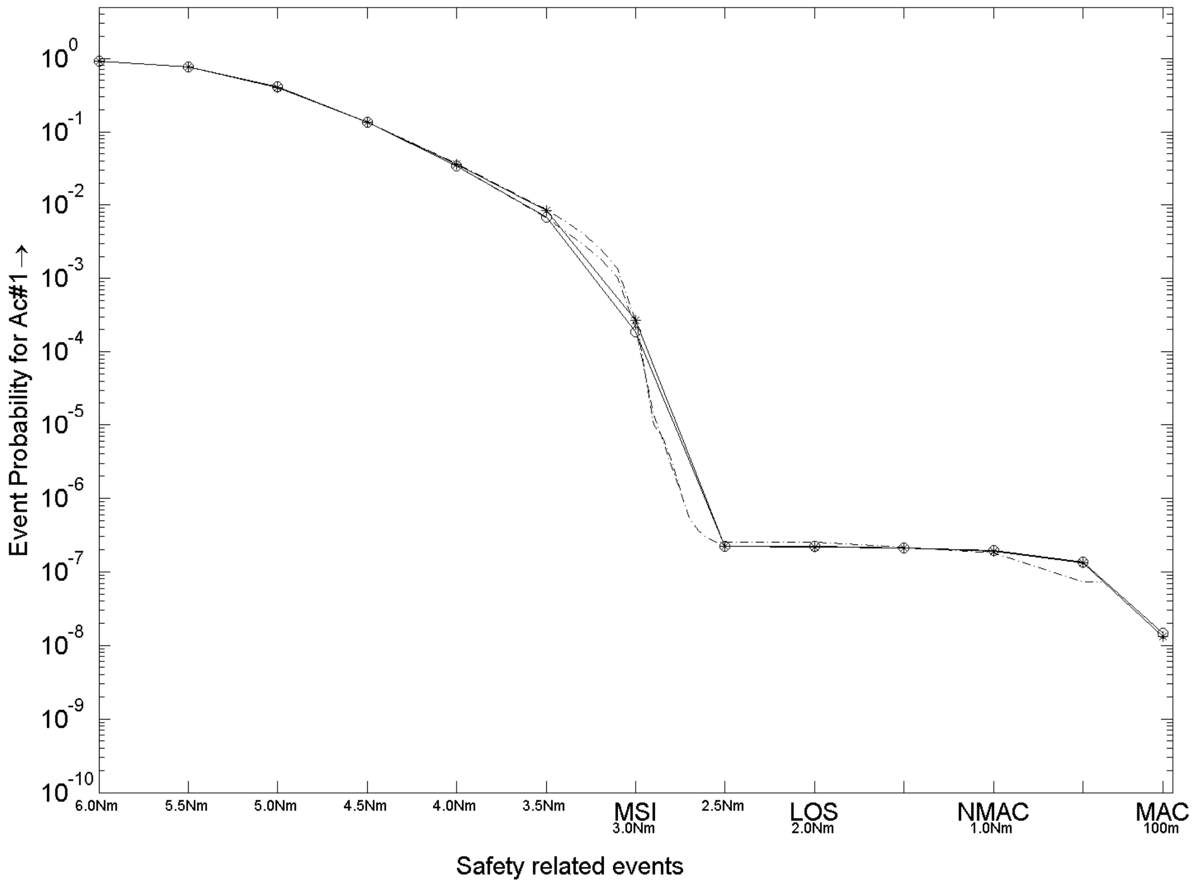

FIGURE 10: *Standard MC versus HHIPS based SMC for varying Groundspeed. ○ = sequential MC results scenario 1.0 (baseline), * = sequential MC results scenario 1.6, -. = standard MC results.*

6 Rare Event Monte Carlo Simulation Results for Eight-Aircraft

6.1 Eight-aircraft encounter

Next we consider the eight-aircraft encounter scenario pictured in Figure 11. Each aircraft starts at the same flight level and from a circle of about 320 km (172 Nm) in diameter. The initial 3-dimensional position has standard deviations of 20m along the RBT centerline, 0.5Nm in the lateral direction (RNP1) and 20m in the height. Each aircraft has a ground speed of 250 m/s and is heading to the opposite point on the circle.

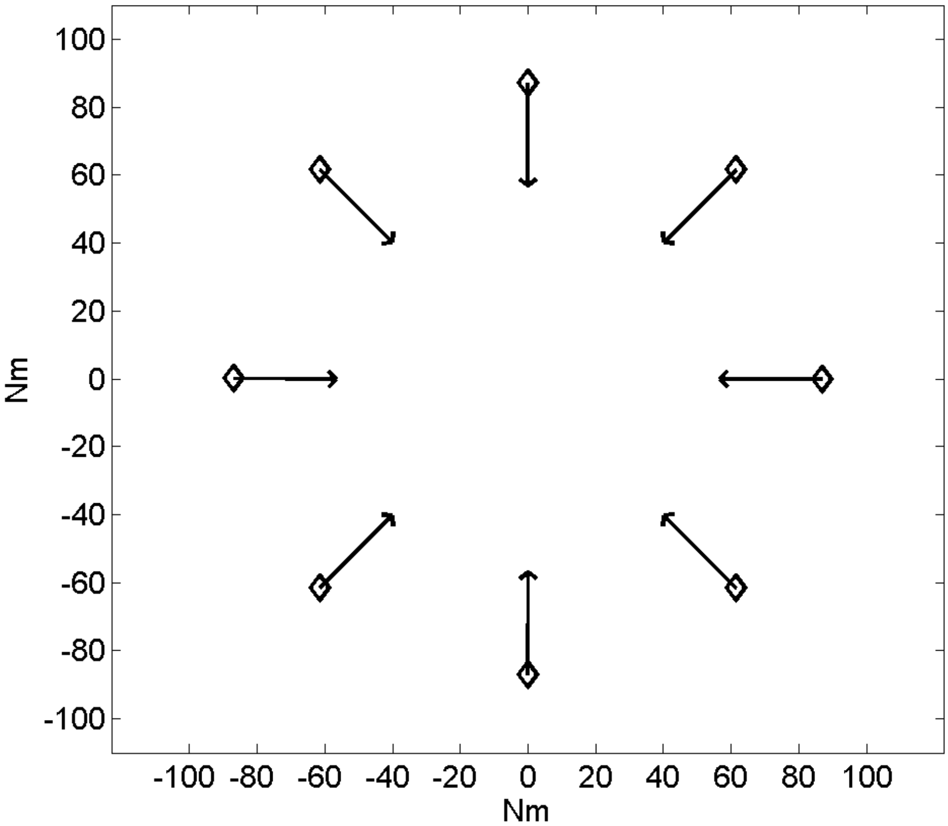

FIGURE 11: Eight aircraft encounter scenario at same flight level

Because of random initial conditions and random disturbances, each MC simulated eight aircraft encounter generates trajectories that differ from those generated before. Figure 12 shows a top view of an example of trajectories that are generated for the eight-aircraft encounter scenario under the A^3 concept of operation.

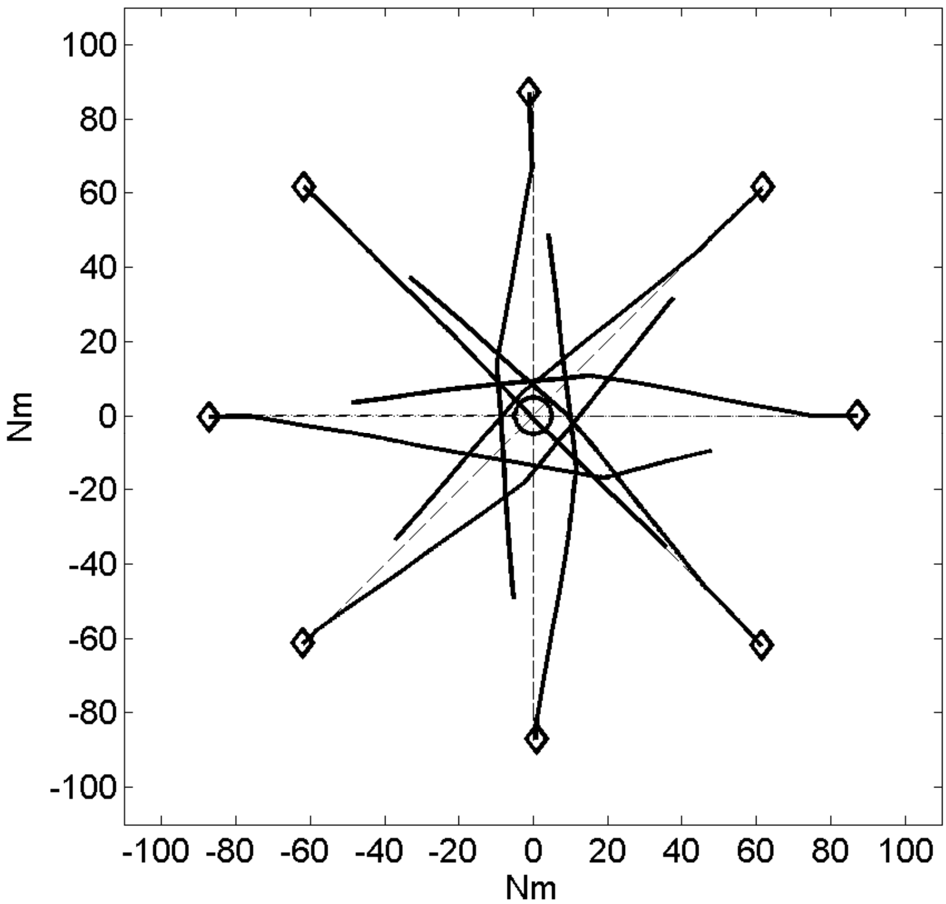

FIGURE 12: A^3 *generated conflict resolutions example for eight aircraft encounter scenario; ◊ = start of simulated trajectory. The circle in the centre has a 10Nm diameter.*

6.2 MC and IPS based SMC scenarios considered

The scenarios considered are specified in Table IX; the identity number refer to the parameter settings adopted in Table VII. For each parameter setting a standard MC is conducted. For the baseline scenario also an IPS based SMC is conducted. The number of MC runs or the number of particles used is also given in Table IX. In addition, Table IX shows the time-duration of the simulation. Because the standard MC simulation is so time demanding, for the baseline scenarios only a large number of MC runs has been simulated.

TABLE IX. Parameter value scenarios simulated, MC types and time durations of the computations on two Dell precision T7500

Parameter value scenario	**Id**	**Figure**	**Standard MC**		**IPS**	
			# of runs	**Duration**	**# of particles**	**Duration**
Baseline	0	13	14 million	207 hours	$12\times$ 15 thousand	7.5 hours
Crew response	1	14	1.2 million	19 hours	-	-
Dependability	2	15 *)	-	-	-	-
ANP	3	16	0.6 million	10 hours	-	-
MTCD&R	4	17	0.64 million	11 hours	-	-
STCD&R	5	18	1.2 million	19 hours	-	-
Groundspeed	6	19	0.72 million	12 hours	-	-

*) Figure 15 is obtained by performing a systematic analysis of the standard MC simulation results obtained for the baseline scenario.

In Figure 13, the frequencies estimated by a standard MC simulation are quite accurate thanks to conducting 14 million runs. This kept the two Dell T7500 computers busy for 207 hours. For each IPS based SMC simulation we used 15 thousand particles. For each particle we counted the event occurrences for aircraft #1. Running such an IPS 12 times required about 7.5 hours on two Dell Precision T7500 with a computer memory load of 40 Gigabyte for each Dell. Figure 13 shows that the MC results outperform those of the IPS approach. The problem is that due to dynamic memory limitation, the possibility to increase the number of particles used by an IPS is much more limited than it is for increasing the number of runs of a standard MC simulation. Without any control, the estimated probability of MSI, NMAC and LOS for an individual aircraft are all equal to 1.0 while for MAC the probability is approximately 0.33. These figures are obtained using standard MC, and also analytically.

The total duration of using both Dell machines for the running of simulations for eight aircraft encounter scenarios amounts 285 hours, which comes down to running the two Dell computers 12 days full time. In practice, there also are a similar number of days needed for the preparation of the simulations (including testing of software adaptations), and for the evaluation and documentation of the simulation results obtained.

6.3 Simulation results: MC versus IPS based SMC

The simulation results are shown in Figures 13 through 19. In this subsection we only address the differences observed in these Figures between MC and IPS based Sequential MC (SMC). In [iFly D7.4] the behavior of the A3 model itself will be discussed on the basis of the simulation results obtained.

In the standard MC simulations we counted the event occurrences for each aircraft and also the miss distances. This resulted in the event probabilities for each of the 8 aircraft separately. In Figures 13-19 we show the mean of the eight estimated probabilities.

Comparison of the standard MC simulation results with the IPS based SMC results show that standard MC has an advantage over IPS.

A special situation applies for the MC based curve for 100x better dependability in Figure 15. Here the standard MC simulation results are obtained by conducting an evaluation of the standard MC results obtained for the baseline scenario. In this evaluation, the rare events counted during the baseline setting have been classified according to their cause. Rare events caused by a non-nominal behavior of ASAS supporting systems are counted at a factor 100x reduced weight to imitate the factor 100x better dependability. This explains how we arrived in Figure 15 at the MC based curve for 100x better dependability without conducting another MC simulation that takes so long (207 hours).

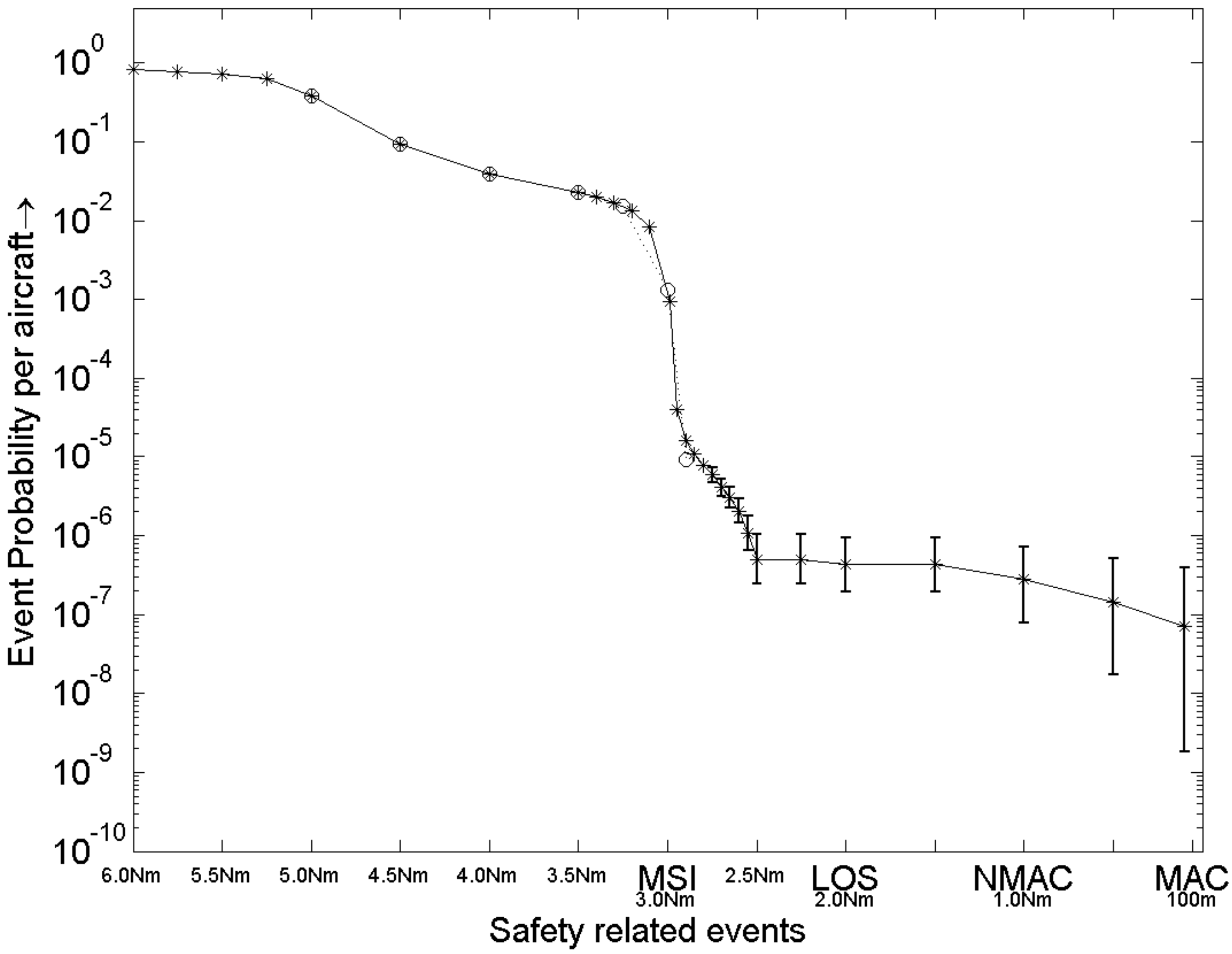

FIGURE 13: *Standard MC versus IPS based SMC for eight aircraft encounter under A3 and baseline parameter values. Estimated probabilities of safety related events per aircraft in the eight-aircraft encounter. * = standard MC result estimate using data of Ac1, ○ = sequential MC results. The vertical line segments show the 95% uncertainty interval of the standard MC results.*

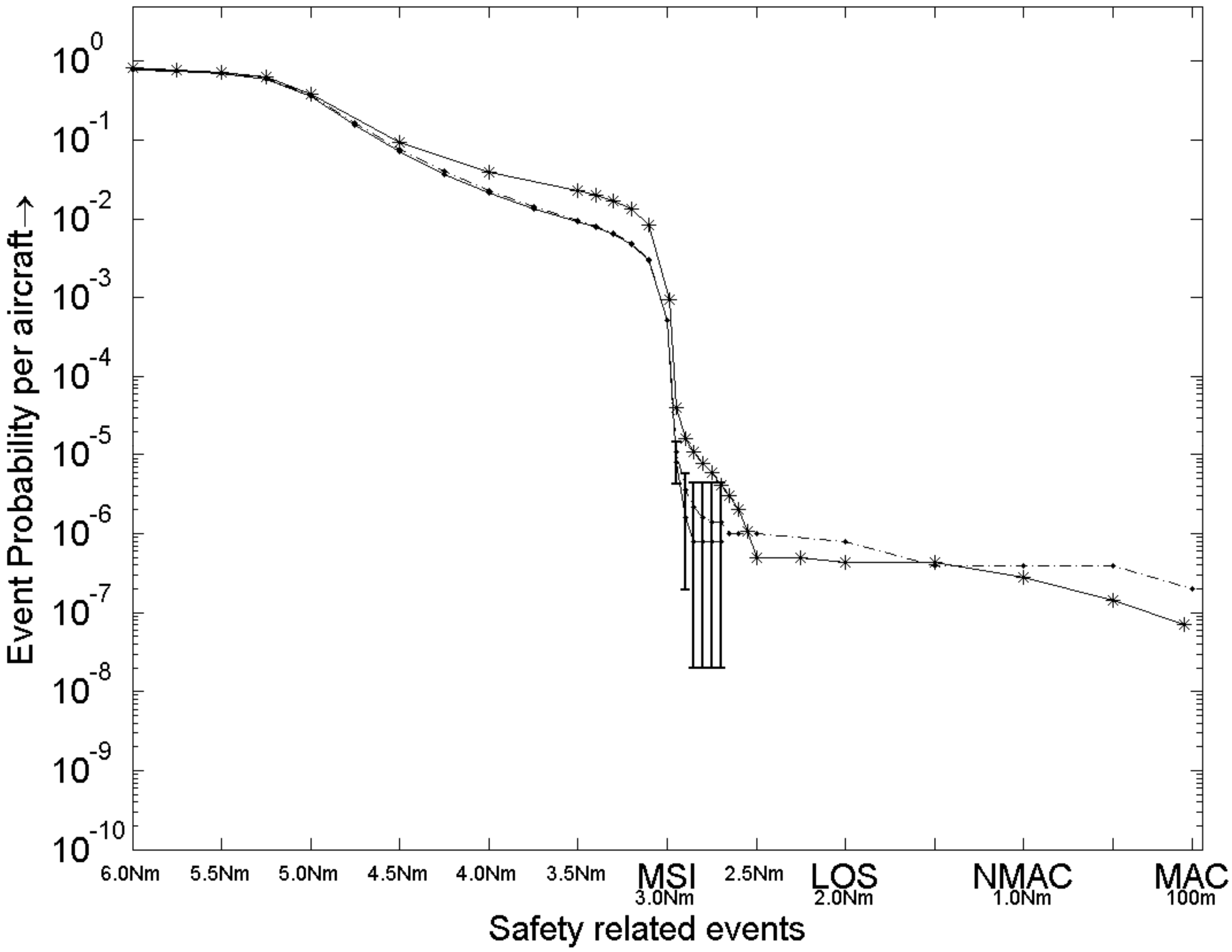

FIGURE 14: *Standard MC for varying crew response values. * = standard MC results scenario 0 (baseline), . = standard MC results scenario 1 with 95% uncertainty interval with solid line is estimate using data of Ac1, dashed line is estimate using data of all aircraft.*

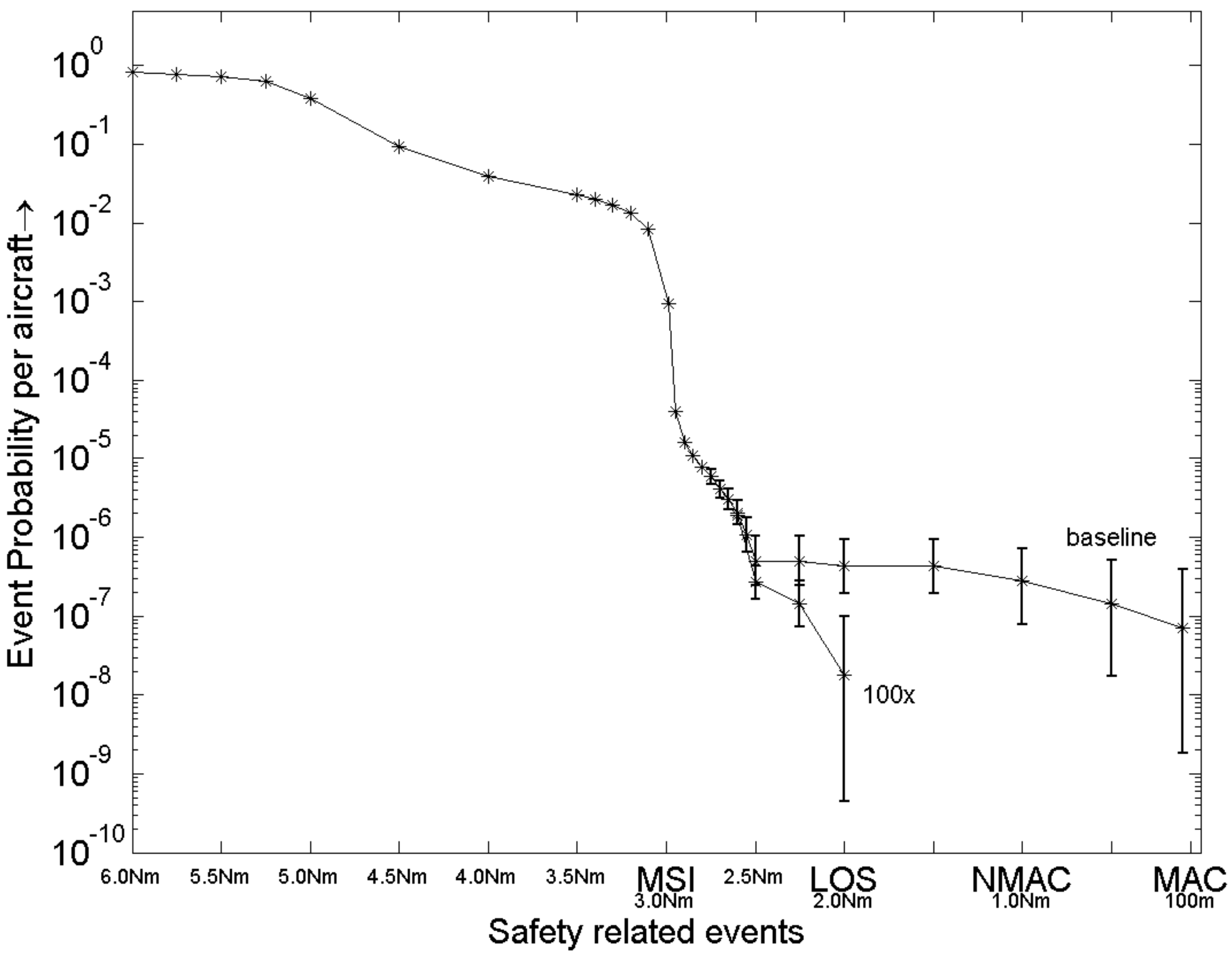

FIGURE 15: *Standard MC for varying Dependability values. Estimated probabilities of safety related events per aircraft in the eight-aircraft encounter with baseline and 100x improved dependability of GNSS, ADS-B/SWIM and ASAS systems, * = standard MC results with 95% uncertainty interval for baseline and for 100x better dependability. The standard MC results for 100x better dependability are obtained through a systematic analysis of the standard MC simulation results obtained for the baseline parameter scenario using data of all aircraft.*

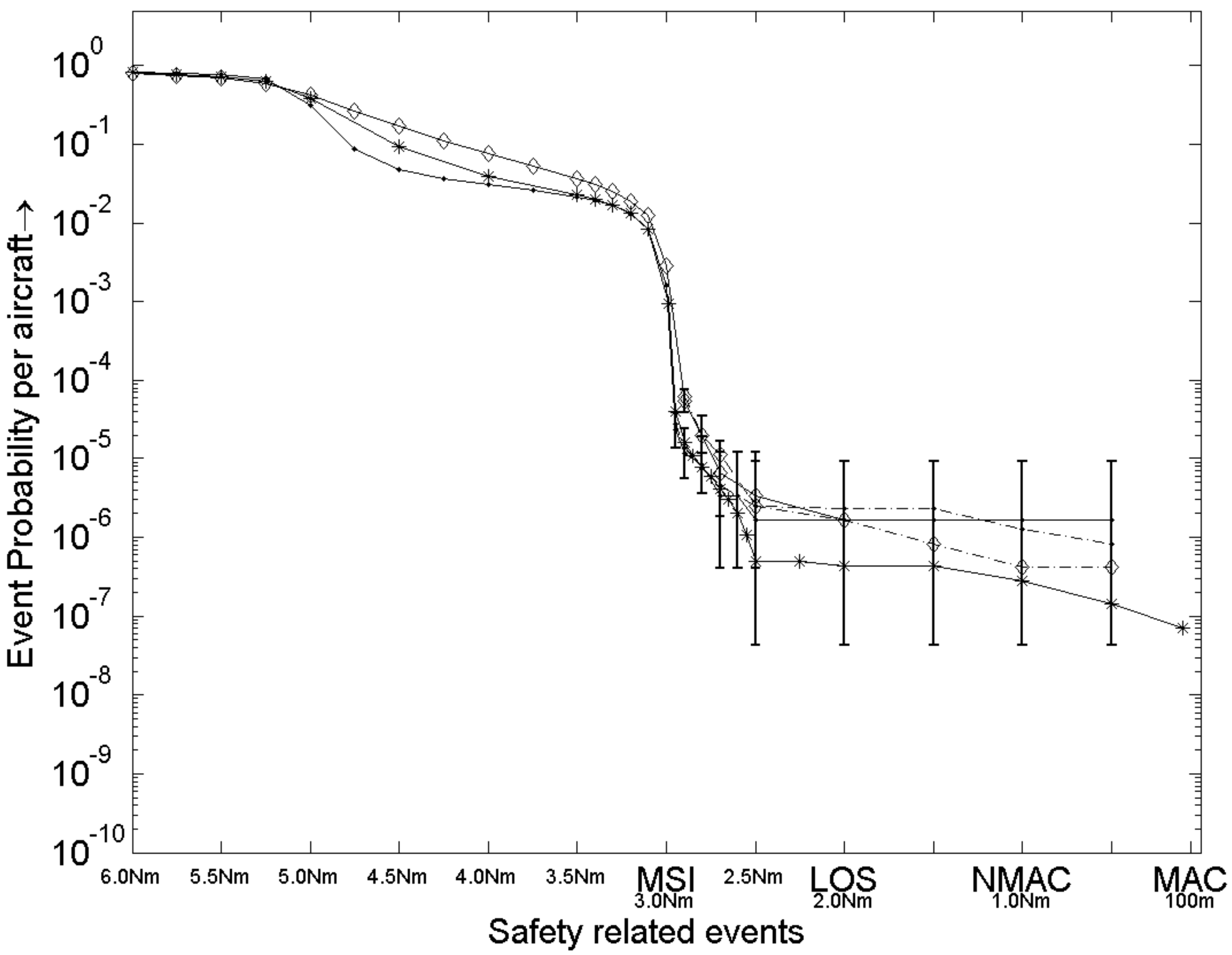

FIGURE 16: *Standard MC for varying ANP values. * = standard MC results for ANP1 (baseline), ◊ = standard MC results scenario 3 for ANP2, . = standard MC results scenario 3 for ANP0.5. Solid line is estimate using data of Ac1, dashed line is estimate using data of all aircraft. The vertical line segments show the 95% uncertainty interval for the Ac 1 estimates.*

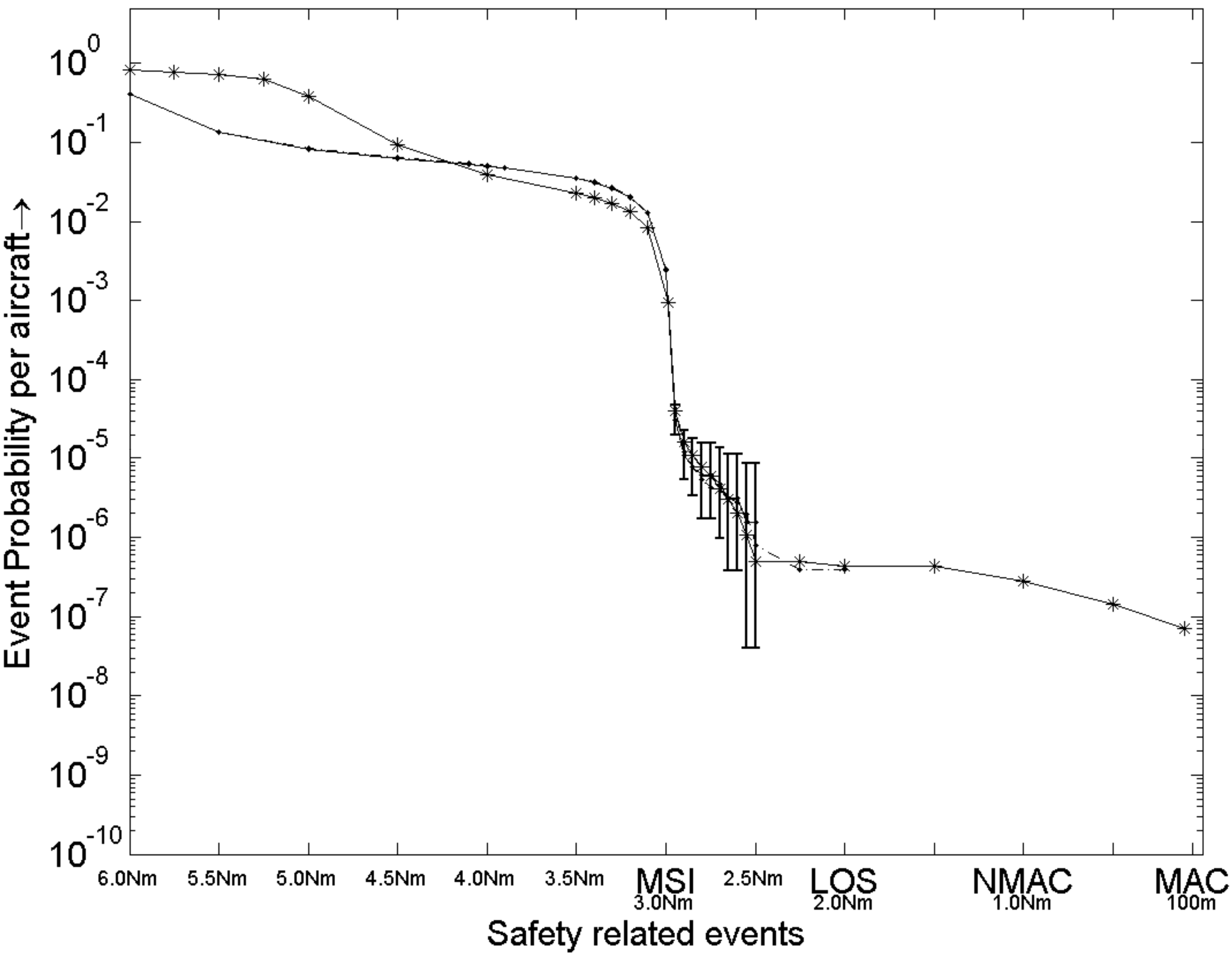

FIGURE 17: *Standard MC for varying 4D trajectory separation values. * = standard MC results scenario 0 (baseline), . = standard MC results scenario 4 with 95% uncertainty interval. Solid line is estimate using data of Ac1, dashed line is estimate using data of all aircraft.*

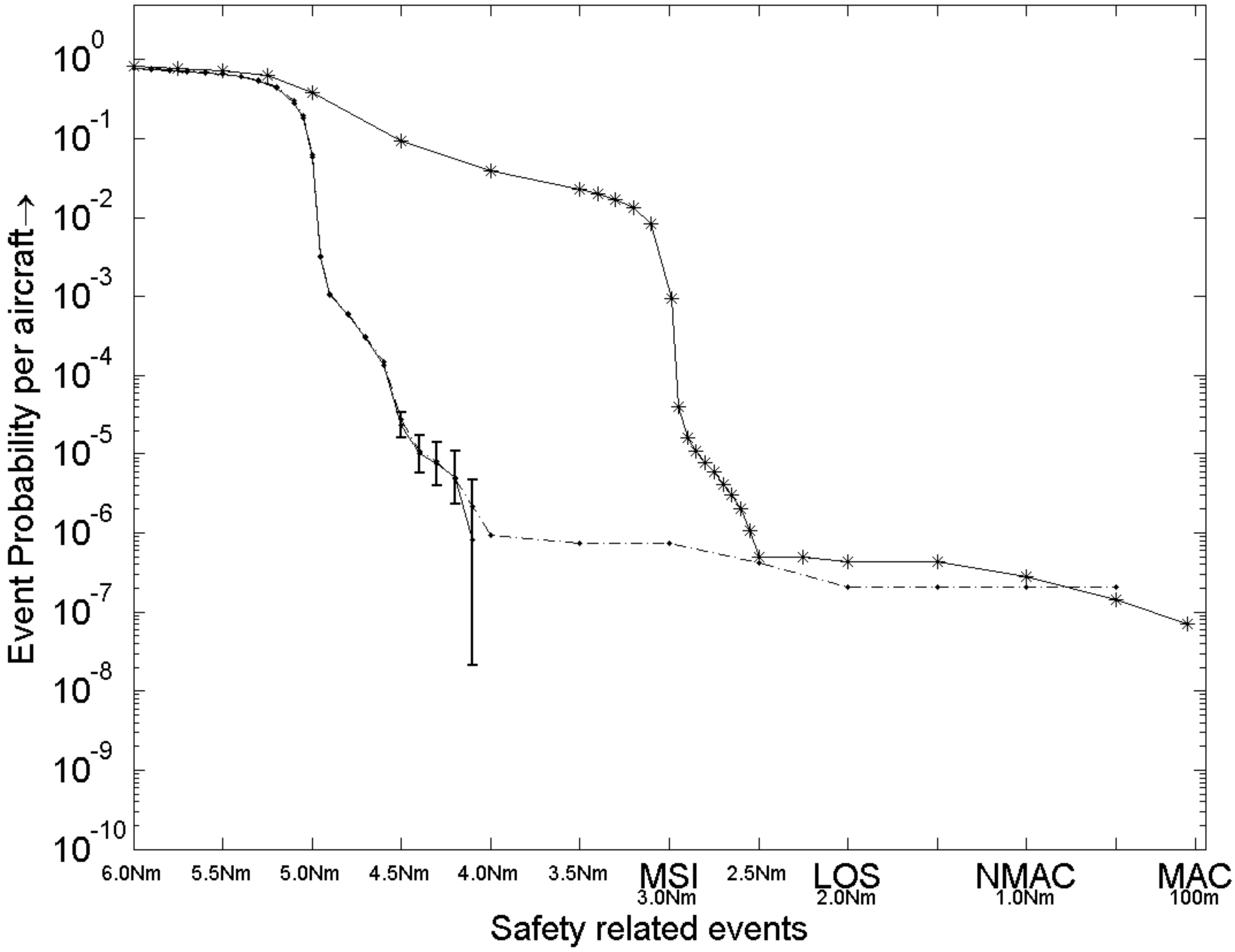

FIGURE 18: *Standard MC for varying tactical separation values. * = standard MC results scenario 0 (baseline), . = standard MC results scenario 5 with 95% uncertainty interval with solid line is estimate using data of Ac1, dashed line is estimate using data of all aircraft.*

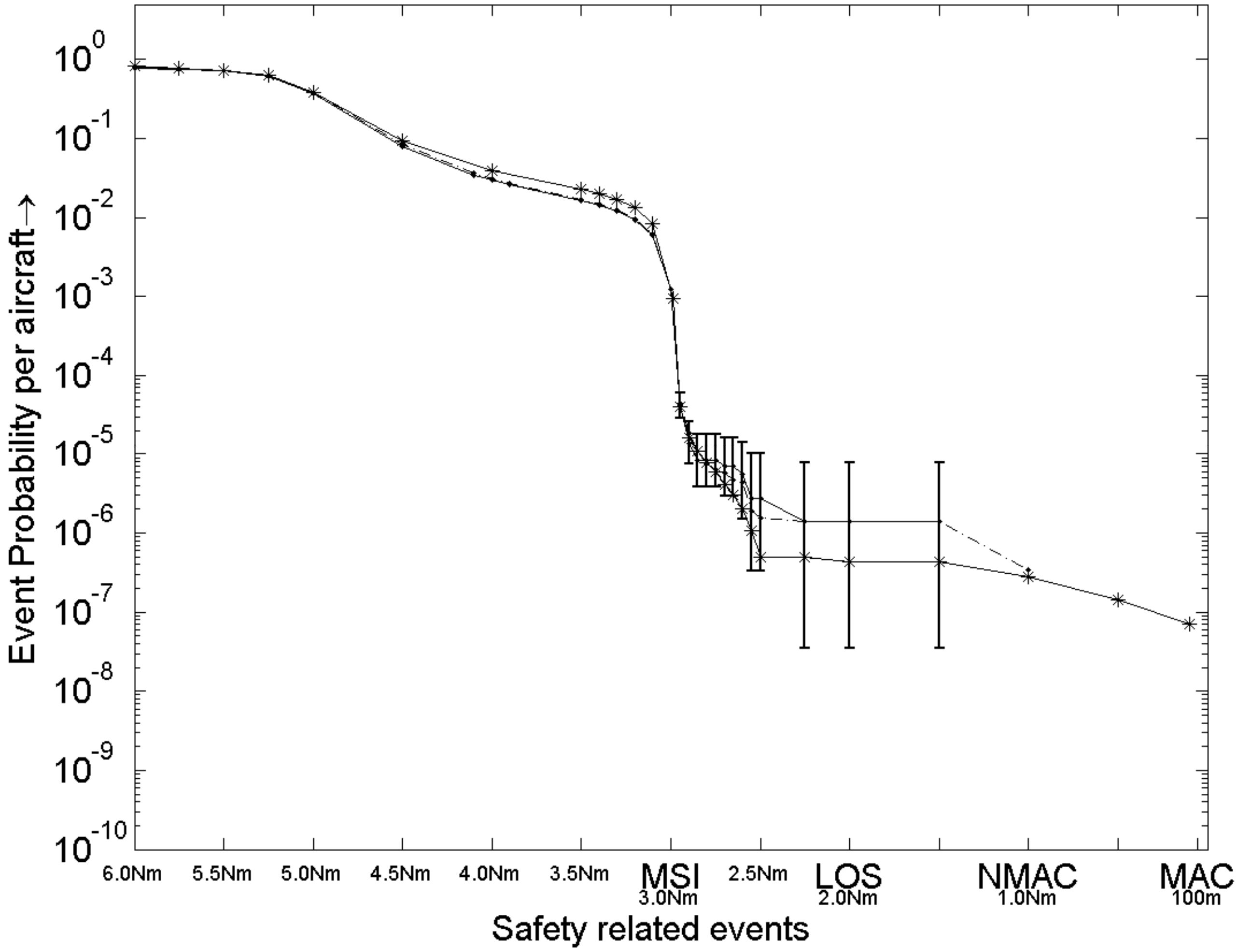

FIGURE 19: *Standard MC for varying Groundspeed values. * = standard MC results scenario 0 (baseline), . = standard MC results scenario 6 with 95% uncertainty interval with solid line is estimate using data of Ac1, dashed line is estimate using data of all aircraft.*

7 Rare Event Monte Carlo Simulation Results for Dense traffic

7.1 Dense random traffic

The third encounter scenario simulates A^3 equipped aircraft flying randomly through a virtually unlimited airspace. In order to accomplish this, the virtually unlimited airspace is filled up with rectangular boxes. Within each box there are a fixed number of eight aircraft flying. With the exception of aircraft one, seven aircraft (i = 2, .., 8) fly at arbitrary position and in arbitrary direction at a ground speed of 250 m/s. Aircraft number one (i = 1) aims to fly straight through a sequence of connected boxes, at the same speed, and the aim is to estimate its probability of collision with any of the other aircraft per unit time of flying. Per box, the aircraft within it behave the same, and for aircraft that pass the boundary of a box the Periodic Boundary Condition (PBC) approach is applied (see section 3). This implies that we have to simulate the eight aircraft in one container only, as long as we apply the ASAS conflict prediction and resolution also to virtual aircraft copies in any of the other boxes. By changing the size of the boxes the traffic density can be varied as long as this does not lead to a too small box [iFly D7.2d].

Our baseline traffic density value is selected to be 4.0 times the level of one of the busiest en-route sectors in Europe in 1999. Based on a data set of European air traffic that has been collected for a busy day in July 1999, the highest aircraft density reference point is a number of 17 aircraft counted at 23rd July 1999 in an en-route area near Frankfurt of size 1 degree x 1 degree x FL290-FL420. This comes down to 0.0032 aircraft per Nm^3. Multiplied by 4.0 yields our baseline traffic density of 0.0128 aircraft per Nm^3. The latter is 12.8 times the traffic density that has been considered in the example of [Andrews et al., 2005, 2006] and 1.6 times the baseline traffic density considered for AMFF [Blom, ATC-Q2009].

For the MC simulation of baseline traffic density, i.e. 0.0128 aircraft per Nm^3, we assume for the MC simulations that all 8 aircraft fly on the same FL within the box. For the baseline traffic density, this yields 8 aircraft per 62Nm×62Nm×1000ft. Hence in a MC simulation (both standard MC as well as Sequential MC), we use the 62Nm×62Nm horizontal box size.

Because the initial conditions of seven of the eight aircraft are random, there will be serious short term as well as medium term conflicts in the beginning. Hence for each initial condition, we give the A3 ConOps a time period of 10 minutes to organize the given traffic situation in line with its concept of operation. Only after this 10 minutes convergence time, we start to measure safety related events, during a period of 10 minutes.

7.2 Dense random traffic scenarios simulated

The scenarios that are simulated are specified in Table X. This Table also shows that the random traffic density is set at a value 3x as high as a busy sector in 2005, and at a value 6x as high. The latter we simulated by reducing the size of the Periodic Boundary Condition (PBC) by a factor $\sqrt{2}$ in each horizontal direction. As before, the column Id refers to the parameter scenario number in Table VII. For each parameter setting, both light standard MC and IPS [Blom, CDC2006, CRC2007] are conducted. The choice for IPS is because HHIPS remains to be developed for handling multiple aircraft scenarios (see Section 3).

TABLE X. Parameter value scenarios simulated, MC types and time durations of the computations on two Dell precision T7500

Parameter value scenario	**Id**	**Figure**	**Standard MC**		**IPS based SMC**	
			# of runs	**Duration**	**# of particles**	**Duration**
Baseline & 3x high 2005	0	20, 21	3.56 thousand	1 hour	120 thousand + 45 x 10 thousand	42 hours + 138 hours
Baseline & 6x high 2005	0	20	0.5 thousand	< 1 hours	24 thousand + 20 x 2 thousand	44 hours + 108 hours
STCD&R & 3x high 2005	5	21	110 thousand	31 hours	24 x 10 thousand	64 hours

The total duration of using both Dell machines for the running of simulations for dense random traffic scenarios amounts 430 hours, which comes down to running the two Dell computers 18 days full time. A similar amount of days was needed for the preparation of the simulations (including testing of software adaptations) and for the evaluation and documentation of the simulation results obtained.

7.3 MC results simulation results: MC versus SMC

The simulation results are shown in Figures 20 and 21. For the random traffic scenario, the estimated mean probabilities have been obtained from 10 minutes sequential MC simulation of random traffic. For the scaling of the event probabilities per 10 minutes to event probabilities per hour, the following equation is used: $p = 1-(1-p_{SMC})^{\frac{T}{T_{SMC}}}$ with T = 60 minutes, T_{SMC} the time period used in the sequential MC simulation (convergence period is not included) and with p_{SMC} the estimated probability per T_{SMC}. For small p_{SMC} values, this yields $p \approx \frac{T}{T_{SMC}} \cdot p_{SMC}$.

The standard MC simulations have mainly been conducted in order to identify a good sequence of early level sets. The resulting sequence of level set values is depicted in Table XI. In addition the results of these standard MC simulations made it possible to improve low frequency details in the curves of Figure 20.

TABLE XI. IPS conflict severity levels used for evaluating random scenario 5 under A3 ConOps

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d_k Nm	6.0	5.5	5.25	5.0	4.8	4.5	4.3	4.0	3.5	3.0	2.5	2.0	1.0	0.5	.054
h_k ft	900	900	900	900	900	900	900	900	900	900	750	600	400	300	131
Δ_k min	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

As shown in Table X, multiple CPU's have been exploited by IPS runs in two ways:

1) In running multiple IPS's, each on one CPU.
2) In running one IPS on multiple CPU's

The latter approach has been explained in Section 3. This approach allows to run one IPS with an order in magnitude more particles, which in theory is expected to lead to a possible improvement in the estimation of rare events. However, in the IPS results obtained for the A3 ConOps no difference has been found between the two IPS approaches. For this reason the scenario with parameter scenario 5 (i.e. 5Nm tactical separation minima) has been conducted through running one IPS on each CPU.

It is remarkable that in none of these IPS simulations a single event has been counted in which the miss distance was lower than 2.0 Nm. And the 2.0 Nm value has been counted only once, and this was for the 6x high 2005 scenario. Because we used only 44 thousand particles for the evaluation of this scenario, this means that the speed-up of the IPS approach has been working well. Although for the 3x high 2005 scenario we used an order in magnitude more particles this 2.0 Nm level has even not been reached. However, it should be expected that also for random traffic scenarios there will be some level at which the ASAS dependability values will start to play a role. Also for two aircraft encounters we have seen that this level can be assessed using HHIPS, but not by IPS. Because HHIPS remains to be extended for its application to multiple a/c encounters, this could not yet be assessed through simulations.

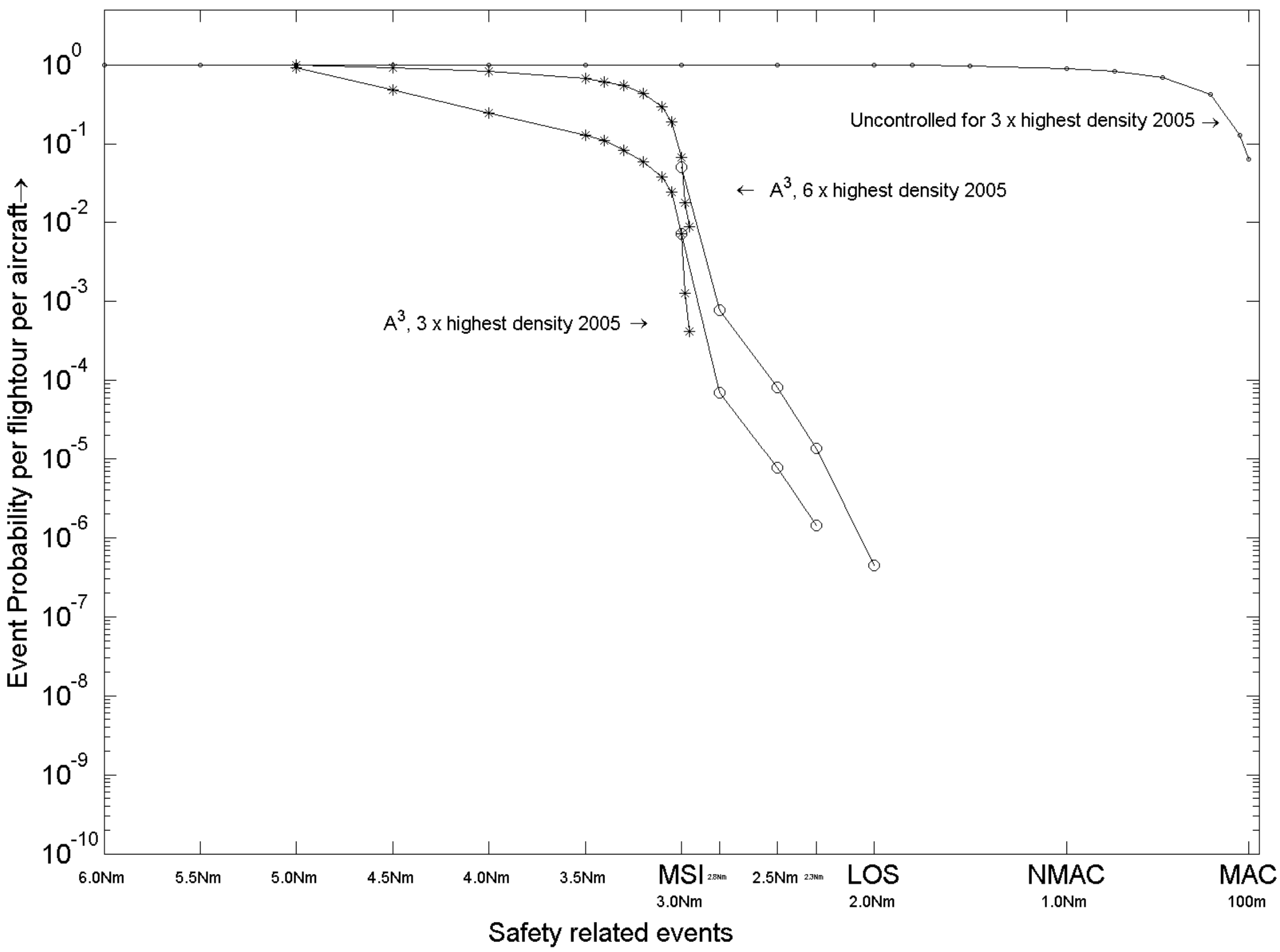

FIGURE 20: Standard MC and IPS based SMC estimated event probability per aircraft per flighthour for random traffic, both uncontrolled and under A^3 *model control (baseline parameter values). Traffic densities are 3x and 6x high en-route traffic density in 2005. * = standard MC result estimate, ○ = SMC results.*

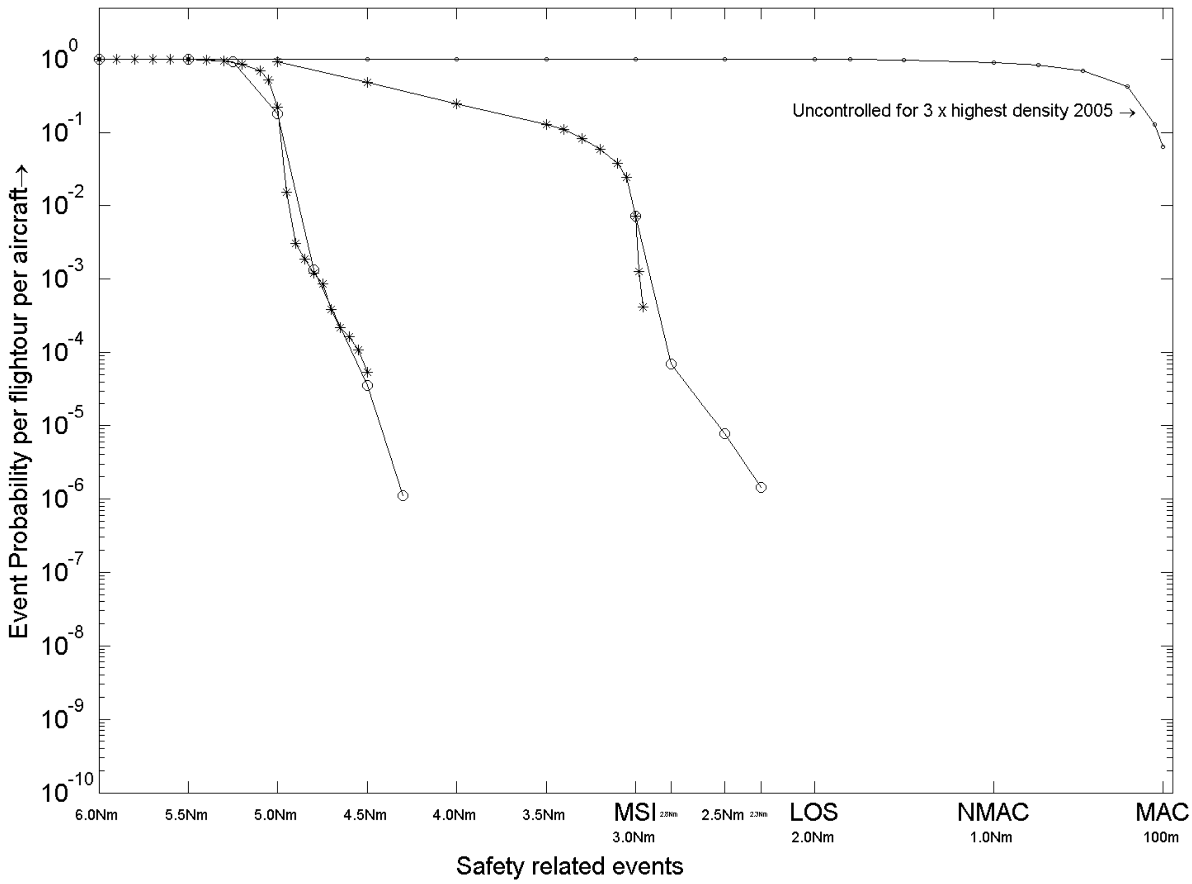

*FIGURE 21: Estimated event probability per aircraft per flighthour for random traffic under A^3 model control and uncontrolled. Traffic densities is 3x high en-route traffic density in 2005. * = standard MC, ○ = sequential MC results. Left curve is for tactical separation criterion of 5Nm, right curve for tactical separation of 3 Nm (baseline).*

In spite of this limitation of IPS, a comparison of the simulation results obtained by standard MC versus those obtained by the IPS based SMC shows that the latter has a significant advantage over the former. For scenario 5, about 2x more particles have been applied than number of MC runs. However, the estimated frequency goes a factor 50 lower for IPS than it goes for standard MC.

8 Concluding remarks

Within WP7.2 of the iFLY project, several studies have been performed on the development of various complementary methods that aim to improve the speed-up performance of rare event Monte Carlo simulation of advanced ATM concept of operations. This report has provided an overview of these complementary speed-up results, and has shown how this has been exploited, within the iFly project, in the rare event Monte Carlo simulation of the A3 ConOps.

Section 2 has given an overview of the various iFly studies to improve the Interacting Particle System (IPS) method that have been used for the rare event evaluation of the AMFF ConOps [Blom, ATC-Q2009]. An overview of the background of this IPS approach has been given in [iFly D7.2a]. The complementary methods studied within iFly WP7.2 are:

- Markov Chain Monte Carlo (MCMC) [iFly D7.2b]
- Complexity measures [iFly D7.2c], [Prandini, 2011]
- Periodic Boundary Condition (PBC) [iFly D7.2d]
- Hierarchical Hybrid IPS (HHIPS) [iFly D7.2e]
- Regression analysis [iFly D7.2f]

Section 3 has explained how the results of these iFly studies on further speed-up of IPS have been used (or not) within the iFly project for the rare event evaluation of the A3 ConOps. All five complementary methods have been carefully considered for their exploitation within the iFly project. For two (MCMC & Regression) of the five, it has been identified that these novel approaches were promising, but at the same time were in need of further development prior to their application to the rare event evaluation of the A3 ConOps. For one (Complexity) of the five, no practical way has been found yet to extend its importance sampling effectiveness realized for the AMFF ConOps to the A3 ConOps. For two (PBC & HHIPS), the new results have proven to be of use for the rare event evaluation of the A3 ConOps.

Section 4 has provided an overview of the traffic scenarios evaluated using rare event MC simulations, and which of the specific methods have been used for the evaluation of which scenario.

Section 5 has shown the rare event MC simulation results obtained for a two aircraft encounter scenario. Here the focus is on comparing standard MC simulation against Sequential MC simulation. The evaluation of what the MC simulation results obtained imply for the A3 ConOps is considered in [iFly D7.4].

Sections 6 and 7 have done similar as Section 5, but now for an eight aircraft encounter scenario and a random traffic scenario respectively.

The main finding regarding the speed-up methods studied is that standard MC simulation has an advantage over Sequential MC (SMC) in the sense that it provides more detailed results for events that happen regularly. However for rare events, properly tuned SMC allowed to evaluate for the A3 ConOps up to four orders of magnitude less frequent events. This means that in practice it is best to make a combined use of standard MC simulations and SMC simulations. This allows for example to define the level sets in an IPS or HHIPS on the basis of the standard MC results.

What does this mean for follow-up studies of speeding-up rare event simulation of advanced ATM ConOps? Regarding this question our view is as follows:

- HHIPS [iFly D7.2e] has proven to be able to assess very infrequent rare events for two aircraft encounters. Hence, in order to do the same for random traffic scenarios, HHIPS should be extended for its application to multiple aircraft scenarios.

- In [Krystul et al., 2011] the convergence proof of IPS has been extended to the Hybrid IPS version of [Krystul&Blom, 2005]. A further extension of this convergence proof to HHIPS remains to be done. Such a proof should deliver the exact mathematical conditions under which convergence behavior is as expected, and when not.
- MCMC is a promising approach for the further improvement of IPS [iFly D7.2b]. As has been explained in Section 3, this asks a proper handling of several ATM relevant aspects, the most critical of which is the development of an effective MCMC operator step for use in an advanced ATM directed IPS.
- Complexity prediction methods [iFly D3.2] as an effective importance sampling mechanism for rare event MC simulation of the A^3 ConOps, deserves further attention.
- Regression analysis [iFly D7.2f] is another promising approach to be properly combined with the IPS based SMC approach. Because of the huge size of ATM safety models, a prerequisite for making this feasible is that for the IPS approach an an order in magnitude extra speed-up is being developed. Otherwise regression analysis does not form a realistic alternative for the One At-a Time (AOT) approach.
- Relative to the speeding-up studies performed within iFly, follow-up studies have a better reference point (i.e. the A3 ConOps) in the design space for advanced ATM ConOps that is able to safely accommodate very high traffic demand levels.

References

[Alexander, 1970] B. Alexander (1970), Aircraft density and midair collision, Proceedings of the IEEE, Vol. 58, 1970, pp. 377-381.

[Andrews et al., 2005] J.W. Andrews, J.D. Welch, H. Erzberger (2005). Safety analysis for advanced separation concepts. Proceedings of USA/Europe ATM R&D Seminar, Baltimore, USA, 27–30 June 2005.

[Andrews et al., 2006] J.W. Andrews, H. Erzberger, J.D. Welch (2006). Safety analysis for advanced separation concepts. ATC Quarterly, Vol. 14, 2006, pp. 5-24.

[Blom, CDC2006[H.A.P. Blom, J. Krystul, G.J. Bakker, (CDC2006), A particle system for safety verification of free flight in air traffic, Proc. IEEE Conf. Decision and Control, San Diego, CA, 13-15 December 2006.

[Blom, CRC2007] H.A.P. Blom, J. Krystul, G.J. Bakker, M.B. Klompstra, B. Klein Obbink, Free flight collision risk estimation by sequential Monte Carlo simulation, Eds : C.G. Cassandras and J. Lygeros, Stochastic hybrid systems ; recent developments and research trends, Taylor & Francis/CRC Press, 2007, pp. 249-281.

[Blom, CDC2007] H.A.P. Blom, Bakker G.J., Krystul J. (CDC2007), Reachability analysis for large scale stochastic hybrid systems, Proc. IEEE Conference on Decision and Control, December 12-14, 2007, New Orleans, USA.

[Blom, Wiley2009] H.A.P. Blom, G.J. Bakker and J. Krystul (2009), Rare event estimation for a large scale stochastic hybrid system with air traffic application, Chapter in: G. Rubino and B. Tuffin (editors), Rare event simulation using Monte Carlo methods, Wiley, 2009.

[Blom, ATC-Q2009] H.A.P. Blom, B. Klein Obbink, G.J. Bakker, Simulated Safety Risk of an Uncoordinated Airborne Self Separation Concept of Operation, Air Traffic Control Quarterly, Volume 17 (2009) Number 1, pp. 63-93.

[Blom&Everdij, 2010] H.A.P. Blom and M.H.C. Everdij, General Stochastic Hybrid Processes as pathwise unique solutions of Hybrid Stochastic Differential Equations on a Hybrid State Space, iFly report D7.2x, 2010.

[Cerou et al., 2002] F. Cérou, P. Del Moral, F. Le Gland and P. Lezaud (2002). Genetic genealogical models in rare event analysis, Publications du Laboratoire de Statistiques et Probabilites, Toulouse III, 2002.

[Cerou et al., 2005] F. Cérou, Del Moral P., Le Gland F., Lezaud P. (2005), Limit theorems for the multilevel splitting algorithms in the simulation of rare events. Proc. Winter Simulation Conference, Orlando, USA, 2005.

[Cerou et al., 2006] F. Cérou, Del Moral P., Le Gland F., Lezaud P. (2006). Genetic genealogical models in rare event analysis, in: ALEA, Latin American Journal of Probability and Mathematical Statistics, Paper 01–08, 2006, vol. 1, p. 181–203.

[Endoh & Odoni, 1983]	S. Endoh and A.R. Odoni (1983), A generalized model for predicting the frequency of air conflicts, Proc. Conf. on Safety Issues in ATM Systems Planning and Design, Princeton, New Jersey, September 1983.
[Everdij & Blom, 2005]	M.H.C. Everdij, H.A.P. Blom. Piecewise deterministic Markov processes represented by Dynamically Coloured Petri Nets, Stochastics, Vol. 77, pp.. 1–29, 2005.
[Everdij & Blom, 2006]	M.H.C. Everdij, H.A.P. Blom. Hybrid Petri nets with diffusion that have into mappings with generalised stochastic hybrid processes. . Eds: H.A.P. Blom, J. Lygeros. Stochastic Hybrid Systems: Theory and Safety Critical Applications, LNCIS series, Springer, Berlin, July 2006, pp. 31–64.
[Everdij, 2010]	M.H.C. Everdij, Compositional modelling using Petri nets with the analysis power of stochastic hybrid processes, PhD Thesis, Twente Univ. June 2010.
[iFly D1.3]	iFly Deliverable D1.3, Autonomous Aircraft Advanced (A^3) ConOps, written by Isdefe (Gustavo Cuevas, Ignacio Echegoyen, José García García), Honeywell (Petr Cásek, Claudia Keinrath, …,…), NLR (Frank Bussink), and Utartu (Aavo Luuk), January 2010. Available on iFly website http://ifly.nlr.nl/
[iFly D2.2]	iFly Deliverable D2.2, Situation Awareness, Information, Communication and Pilot Tasks of under autonomous aircraft operations, John Wise, Claudia Keinrath, Fleur Pouw, Amel Sedaoui, Vincent Gauthereau and Aavo Luuk, Version 1.3, 8 April 2009. Available on iFly website http://ifly.nlr.nl/
[iFly D3.2]	iFly Deliverable D3.2, Report on timely predction of complex conditions for en route aircraft, M. Prandini, L. Piroddi, S. Puechmorel, P. Casek and S.L. Bradzilova, May 2011.
[iFly D7.1a]	iFly Deliverable D7.1a, Accident risk and flight efficiency of A^3 operation -Scoping and safety target - by H.A.P. Blom, Version 1.1 of 3 February 2009. Available on iFly website http://ifly.nlr.nl/
[iFly D7.1b]	iFly Deliverable D7.1.b, Hazard Identification and Initial Hazard Analysis of A^3 ConOps based operation, H.A.P. Blom, G.J. Bakker, M.B. Klompstra and F.J.L. Bussink, Version 0.8, September 2009. Available on iFly website http://ifly.nlr.nl/
[iFly D7.1c]	iFly Deliverable D7.1c, Report on Petri Net modelling of the A^3 operation by G.J. Bakker and H.A.P. Blom. Version 0.6 of 28 Sep 2010
[iFly D7.2a]	iFly Deliverable D7.2a, Review of risk assessment status for air traffic. Editors: H.A.P. Blom, J. Krystul, P. Lezaud and M.B. Klompstra, Version 1.0 of 14 Jan 2009. Available on iFly website http://ifly.nlr.nl/
[iFly D7.2b]	iFly Deliverable D7.2b, Trans-dimensional simulation for rare-events estimation on stochastic hybrid systems by N. Kantas and J.M. Maciejowski, Version 1.0 of 1 May 2009. Available on iFly website http://ifly.nlr.nl/
[iFly D7.2c]	iFly Deliverable D7.2c, Interim report on importance sampling of

	multi aircraft encounter geometries - Air traffic complexity and the interacting particle system method: an integrated approach for collision risk estimation by Maria Prandini, Henk A.P. Blom, Bert G.J. Bakker. Version 0.5 of 8 Apr 2011
[iFly D7.2d]	iFly Deliverable D7.2d, Periodic Boundary Condition in Simulating Large Scale Airborne Self Separation Airspace by A. Goswami, G.J. Bakker, H.A.P. Blom, Version 1.0 of 11 April 2010
[iFly D7.2e]	iFly Deliverable D7.2e, Rare event estimation for a large scale stochastic hybrid system with air traffic application - IPS extension to large hybrid systems - by H.A.P. Blom, G.J. Bakker and J. Krystul Version 0.6 of 28 Jan 2009. Available on iFly website http://ifly.nlr.nl/
[iFly D7.2f]	iFly Deliverable D7.2f, Sensitivity analysis in Monte Carlo simulation based rare event estimation, by M.B. Klompstra, G.J. Bakker and H.A.P. Blom, Version 1.3 dated 22 March 2011
[iFly D7.4]	iFly Deliverable D7.4, Final Report on Accident Risk Assessment of the A3 operation, by H.A.P. Blom and G.J. Bakker, September 2011.
[Krystul, 2006]	J. Krystul, Modelling of stochastic hybrid systems with applications to accident risk assessment, PhD Thesis, Twente University, 2006.
[Krystul & Blom, 2005]	J. Krystul, H.A.P. Blom, Sequential Monte Carlo simulation of rare event probability in stochastic hybrid systems, Proc. 16th IFAC World congress, July 4-8, 2005.
[Krystul et al., 2007]	J. Krystul, H.A.P. Blom and A. Bagchi, Stochastic hybrid processes as solutions to stochastic differential equations, Eds: C.G. Cassandras and J. Lygeros, Stochastic hybrid systems: Recent developments and research trends, CRC Press, 2007, pp. 15-45.
[Krystul et al., 2011]	J. Krystul, F. Le Gland and P. Lezaud, Sampling per mode for rare event simulation in switching diffusions, INRIA Report No. 7499, December 2010.
[NATS, 2011]	NATS presentation at FAA/Eurocontrol AP15 meeting, Washington DC, March 8-10, 2011.
[Prandini et al., 2011]	M. Prandini, H.A.P. Blom, G.J. Bakker, Air traffic complexity and the interacting particle system method: An integrated approach for collision risk estimation, Proc. American Control Conf., ACC 2011, June 2011.
[Rapaport, 2004]	D.C. Rapaport, The art of molecular dynamics simulation, Cambridge University Press, 2004.
[RTCA, 2002]	RTCA (2002), DO-242a, Minimum aviation system performance standards for ADS-B, issued 25th June 2002
[Scholte & KleinObbink, 2005]	J.J. Scholte, B. Klein Obbink (2005), Self separation assurance ASOR, MFF report R734L, edition 0.9, June 2005.