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A probabilistic approach to air traffic complexity evaluation

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***Abstract*— Assessing air traffic complexity on a mid term horizon can help to timely identify those safety-critical encounter situations that would require many tactical resolution maneuvers to be resolved. This is particularly useful in advanced autonomous air traffic management systems, where aircraft are responsible for self-separation maintenance.**
In this paper, we propose a new method to evaluate mid term traffic complexity based on the aircraft intent information and current state. The key novelty of the approach is that uncertainty in the future aircraft positions is explicitly accounted for when evaluating complexity.

I. INTRODUCTION

An Air Traffic Management (ATM) system is a multi-agent system, where many aircraft are competing for a common, congestible resource, represented by airspace and runways space, while trying to optimize their own performance evaluated, e.g., in terms of travel distance, fuel consumption, passenger comfort. Coordination between different aircraft is needed to avoid conflicts where two or more aircraft get too close one to the other or even collide.

In the current, centralized ground-based ATM system, coordination is operated on two different time scales by the Air Traffic Control (ATC) and Traffic Flow Management (TFM) functions. The human-based ATC function operates on a mid term horizon with the goal of maintaining the appropriate separation between aircraft, thus avoiding that a conflict occurs. The TFM function operates on a long term horizon by defining the flow patterns so as to ensure a smooth and efficient organization of the overall air traffic, possibly reducing the need for the ATC intervention at a finer time-scale. The airspace is structured in sectors and a team of 2/3 air traffic controllers is in charge of each sector. The capacity of a sector is limited by the sustainable workload level of the air traffic controllers, and TFM accounts for this capacity constraint when performing traffic flow optimization.

The growth in air traffic demand is pushing to its limit the current ground-based ATM system. For example, in 2007 there was a 5.3% growth in the air traffic over Europe over 2006, with a disproportionate increase of 17.4% in the total delay [1]. This has fostered the development of new operational concepts in ATM, as witnessed by the SESAR (Single European Sky ATM Research), [2], and NextGen (Next Generation Air Transportation System) projects, [3].
Current initiatives for increasing the system capacity without compromising safety consist in introducing automated tools to support the air traffic controllers. On a longer time horizon perspective, a conceptually different innovation is foreseen with a significant transfer of separation responsibilities from ground controller to on board pilots. In advanced automated airborne ATM, aircraft entering the self-separation airspace will be allowed to modify their flight plan so as to optimize performance and improving the effectiveness of their flight. In turn, pilots will have to take over the ATC tasks for separation assurance, possibly relying on tools enabled by advanced technologies for sensing, communicating, and decision making. Ground control will assume a new role consisting in a higher level, possibly automated, supervisory function as opposed to lower level human-based control.

The objective of this paper is to develop a new method for *air traffic complexity* evaluation on a mid term horizon. The method explicitly takes into account the uncertainty affecting the future aircraft positions when evaluating complexity. Though representing a relevant factor in the assessment of complexity, this characteristic has been overlooked in the literature. Deterministic models for predicting the aircraft future positions along the reference time horizon have been in fact adopted in the literature for complexity evaluation and prediction.
In an autonomous ATM context, the proposed method could be useful to timely identify those safety-critical situations that could be over-demanding for the aircraft to solve autonomously.

The rest of the paper is organized as follows. In Section II, we briefly review the approaches in the literature to air traffic complexity evaluation. We then illustrate in Section III the novel notion of complexity proposed in this paper. A numerical example is reported in Section IV. Finally, some conclusions are drawn in Section V.

II. EXISTING APPROACHES TO COMPLEXITY EVALUATION

Most studies on air traffic complexity have been developed with reference to ground-based ATM, as it clearly appears from the literature reviews [4] and [5].

The concept of air traffic complexity has been originally introduced to evaluate the difficulty perceived by the air traffic controllers in handling safely a certain air traffic situation (*ATC workload*). The idea is that assessing the impact on the ATC workload of different air traffic configurations can help

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to evaluate how the current ground-based ATM system is operated, and can also provide guidelines on how to obtain more manageable sectors by reconfiguring the airspace and by modifying traffic patterns, see e.g. [6], [7], [8]. The work [9] was perhaps the first one to systematically examine the relationship between air traffic characteristics and ATC workload.

Among the proposed complexity measures, it is worth mentioning the *dynamic density* introduced in the pioneering work by NASA, [10], [11]. Dynamic density is a single aggregate indicator where traffic density and other controller workload contributors (such as the number of aircraft undergoing trajectory change and requiring close monitoring due to reduced separation) are combined linearly or though a neural network whose weights are tuned based on interviews to qualified air traffic controllers. Aircraft density on its own is not an adequate indicator of the ATC workload. Analysis of the traffic in a sector in fact indicates that sometimes controllers accept aircraft even when the assigned capacity is exceeded, while at some other time they will start rejecting entries even if capacity is below the threshold.

The difficulty in obtaining reliable workload measures has been one of the strongest motivations for investigating complexity metrics independent of the ATC workload, such as the *input-output approach* in [12], the *fractal dimension* in [13], and the *intrinsic complexity* measures in [14], [15] and [16]. These metrics are actually those that appear more portable to an autonomous ATM context.

Workload-independent metrics can be classified as control-dependent or control-independent, depending on the fact that they explicitly account for the controller in place or not. The fractal dimension and the intrinsic complexity measures are control-independent metrics and do not require the knowledge of the controller in place, which is accounted for only indirectly, through the effect of its action on the air traffic organization. The input-output approach provides a control-dependent metric, since complexity is evaluated in terms of control effort needed to accommodate an additional aircraft crossing the considered airspace region. Indeed, in [17] it is suggested that, to achieve the aggregate objective of avoiding excessive "air traffic complexity" in autonomous aircraft ATM, aircraft should plan their trajectory so as to preserve maneuvering flexibility to accommodate possible disturbances stemming from other traffic.
In principle, control-dependent metrics could be employed in the airborne self-separation framework. In practice, however, a control-independent measure of complexity appears to be better suited for an airborne self-separation ATM system where the controller has a decentralized time-varying structure, difficult to characterize for the purpose of control effort evaluation, and involving a human-in-the-loop component represented by pilots.

Those approaches providing a spatial complexity map, such as the input-output and the intrinsic complexity ones, can support decision making by isolating critical areas; whereas a scalar aggregate indicator of complexity can be useful as synthetic index to compare different air traffic situations. Fractal dimension, in particular, is an aggregate metric for measuring the geometrical complexity of a traffic pattern based on the trajectories observed on an infinite time period.

The time dependence aspect has been mostly neglected in the literature and should be better focused, introducing approaches to air traffic complexity evaluation tailored to the the specific time horizon. Complexity evaluation on a long term prediction horizon can help in identifying congested areas for strategic flight plan optimization, whereas complexity evaluation on a mid term horizon can help identifying encounter situations that are critical to solve.

To our knowledge, the uncertainty affecting the aircraft motion has been neglected in all approaches to air traffic complexity characterization. This is a critical aspect since the reliability of complexity prediction depends on that of the aircraft motion prediction, which is affected by different sources of uncertainty, primarily to wind, but also to errors in tracking, navigation, and control.

III. Proposed approach to complexity evaluation

The introduced notion of air traffic complexity aims at timely pointing out those safety-critical situations characterized by a limited inter-aircraft manoeuvrability space that could be difficult for the aircraft to resolve autonomously. As in mid term conflict resolution, mid term complexity evaluation is based on the aircraft state and intent information along a time horizon $[0, t_f]$ of the order of tens of minutes. The intent information allows to reconstruct the *nominal trajectory* of each aircraft over the look-ahead time horizon $[0, t_f]$.

Computations are performed in the level-flight case, under the assumption that a multi-legged approximation of the nominal trajectory can be adopted, with the aircraft flying at constant velocity in each leg.

A. Complexity from a global perspective

Consider N aircraft flying at the same constant altitude in the airspace region of interest $\mathcal{S} \subset \mathbb{R}^2$ during $[0, t_f]$. Fix $x \in \mathcal{S}$ and $t \in [0, t_f]$ and denote by $P_m^\rho(x,t)$ the probability that at least m aircraft will enter a ball of radius ρ centered at x within the time window $[t, t+\Delta]$, with $\Delta > 0$ denoting some short term look-ahead time horizon. We call this quantity as the *probabilistic occupancy of level* m *and size* ρ of the airspace at position $x \in \mathcal{S}$ and at time $t \in [0, t_f]$.

For $m = 1$, map $P_m^\rho(\cdot, t)$ is close to 1 along the nominal paths of the N aircraft and goes to zero if one gets far from the nominal paths, at a rate that depends on the uncertainty affecting the aircraft future positions. For $m \geq 2$, $P_m^\rho(\cdot, t)$ is instead close to 1 in those regions of the airspace $\mathcal{S}$ visited during the time interval $[t, t + \Delta]$ by at least $m \geq 2$ aircraft with high probability. For each $x \in \mathcal{S}$ and $t \in [0, t_f]$, $P_m^\rho(x,t)$ is decreasing as a function of $m \in \mathbb{N}$ and increasing as a function of $\rho \in \mathbb{R}_+$.

We introduce function $\rho_{\max} : [0, t_f] \to \mathbb{R}_+$ given by

$$\rho_{\max}(t) := \sup\{\rho \geq 0 : \sup_{x \in \mathcal{S}} P_2^\rho(x,t) \leq p_T\},$$

where p_T is some threshold value for the probability that two aircraft come close one to the other, and define

$$\rho^\star_{\max} := \sup_{t\in[0,t_f]} \rho_{\max}(t).$$

Radius $\rho^\star_{\max}$ is an index of robustness of the overall air traffic system to possible disturbances stemming from modifications of the flight plan of the aircraft and from additional aircraft entering the traffic. The larger is $\rho^\star_{\max}$, the more the aircraft are far one from the other, both in time and in space, with high ($> 1 - p_T$) probability, and, hence, the milder are the safety constraints on the admissible flight plans for the aircraft already present in the traffic and the easier is to safely accommodate an additional aircraft entering the traffic.

We propose to take

$$\xi := \frac{1}{\rho^\star_{\max}}$$

as a synthetic indicator of complexity of the traffic during the time horizon $[0, t_f]$, which will then depend on both the local aircraft density and the traffic organization through the aircraft flight plans.

ξ provides information on the possibility of future conflicts between the aircraft that are currently present in the traffic. Let $\bar\rho$ denotes the minimum safe distance between each aircraft pair. If $\xi \le \frac{1}{\bar\rho}$, then, all the aircraft keep at a distance larger than $\bar\rho$ during the whole time horizon $[0, t_f]$, with probability larger than $1 - p_T$. If $\xi > \frac{1}{\bar\rho}$, then, at least two aircraft will get close in space and time at some time instant $\bar t \in [0, t_f]$ with probability larger than or equal to p_T. Two aircraft will in fact visit the same circular are of radius $\rho_{\max}(\bar t) < \bar\rho$ within the time frame $[\bar t, \bar t+\Delta]$ with probability larger than or equal to p_T.

The airspace region with high *percentage of occupancy* over the time horizon $[0, t_f]$ can be identified through the complexity map $\Xi : \mathcal{S} \to [0, 1]$:

$$\Xi(x) = \frac{1}{t_f}\int_0^{t_f} P_2^{\bar\rho}(x, t)dt. \qquad (1)$$

$\Xi(x) = 0$ means that there will be at most a single aircraft within the ball of radius $\bar\rho$ centered at x during the whole interval $[0, t_f]$, that is, each aircraft passing through x at any time $t \in [0, t_f]$ will be at a safe distance from all the other aircraft. Aircraft passing through x such that $\Xi(x) > 0$ will be possibly involved in a conflict and the likelihood of this event grows with $\Xi(x)$.

1) Mathematical formulation: We suppose that each of the N aircraft A_i, $i = 1, \dots, N$, that are flying at the same constant altitude is following a flight plan given by a sequence of way-points with the associated arrival times $\{(O^{(i)}_h, t^{(i)}_h),\ h = 0, 1, \dots, n_i\}$ with $O^{(i)}_0$ representing the current position of the aircraft at time $t^{(i)}_0 = 0$. The flight plan of aircraft A_i determines a nominal, piecewise constant, velocity profile $u^{A_i} : [0, t_f] \to \mathbb{R}^2$ that the aircraft is trying to follow starting from $O^{(i)}_0$. The corresponding, piecewise constant, nominal heading function is denoted as $\gamma^{A_i} : [0, t_f] \to [0, 2\pi)$.

The actual future position x^{A_i} of aircraft A_i along the look-ahead time horizon $[0, t_f]$, however, is not precisely known and we assume that it is given by

$$x^{A_i}(t) = x^{A_i}_0 + \int_0^t u^{A_i}(s)ds + R(\gamma^{A_i}(t))\Sigma W^{A_i}(t). \quad (2)$$

$W^{A_i}(t)$ in this equation is a standard 2-D Brownian Motion (BM) whose variance is modulated by $\Sigma = \text{diag}(\nu_a, \nu_c)$, ν_a^2, ν_c^2 being the power spectral densities of the perturbations affecting the position in the along-track and cross-track directions. The initial position is given by $x^{A_i}_0 = O^{(i)}_0$. $R(\gamma)$ is the rotation matrix associated with $\gamma \in [0, 2\pi)$ given by

$$R(\gamma) = \begin{bmatrix}\cos\gamma & -\sin\gamma\\ \sin\gamma & \cos\gamma\end{bmatrix}.$$

The variance of the BM $W^{A_i}(t)$ grows linearly with time, thus modeling the fact that the uncertainty in the position of aircraft A_i becomes larger as the prediction horizon gets more extended. Similar models for predicting the aircraft future positions have been proposed in [18], [19], [20] and motivated based on the different sources of uncertainty affecting the along-track and cross-track tracking errors.

For each aircraft A_i we can define as $\pi^\rho_i(x, t)$ the probability that aircraft A_i will enter the ball of radius ρ centered at $x \in \mathcal{S}$ within the time frame $[t, t + \Delta]$.

If the BMs affecting the future positions of the N aircraft are independent, then the probabilistic occupancies P^ρ_1 and P^ρ_2 can be computed in terms of $\pi^\rho_i(x, t)$, $i = 1, 2, \dots, N$, as follows

$$P^\rho_1(x, t) = 1 - \prod_{j=1}^{N}\left(1 - \pi^\rho_j(x, t)\right)$$

$$P^\rho_2(x, t) = P^\rho_1(x, t) - \sum_{h=1}^{N}\left(\pi^\rho_h(x, t)\prod_{j=1,\, j\neq h}^{N}\left(1 - \pi^\rho_j(x, t)\right)\right).$$

The independence assumption is actually reasonable if the N aircraft are not flying too close one to the other, so that the correlation introduced by the wind affecting the aircraft motion is negligible, [21], [22]. If this is not the case, the expressions above can be considered as estimates.

We now address the problem of determining the probability $\pi^\rho_i(x, t)$. Analytic – though approximate – expressions for $\pi^\rho_i(x, t)$ as a function of $x \in \mathcal{S}$ and $t \in [0, t_f]$ will be derived, starting from the case when aircraft A_i is following a one-leg nominal trajectory and then extending the approach to the multi-legged case. The approximation scheme in the one-leg case is based on the approach in [20] for estimating the probability of conflict. For ease of notation, we shall refer to aircraft A_i as aircraft A, omitting the subscript i.

a) One-leg nominal trajectory: Consider aircraft A flying with constant velocity $u^A \in \mathbb{R}^2$ and heading $\gamma^A \in [0, 2\pi)$ starting from $x^A_0 \in \mathcal{S}$. We address the problem of evaluating the probability $\pi^\rho(x, t)$ that aircraft A enters a circle of radius ρ centered at $x \in \mathcal{S}$ within the time frame $[t, t + \Delta]$.

By (2), the relative position of aircraft A with respect to x is governed by:

$$\Delta x(t) = \Delta x_0 + \Delta u t - n(t), \tag{3}$$

where we set $\Delta x(t) := x - x^A(t)$, $\Delta x_0 := x - x_0^A$, $\Delta u := -u^A$, and $n(t) := R(\gamma^A)\Sigma W^A(t)$.

Process $n(t)$ can be reduced to the standard 2-D BM $W^A(t)$ by using the coordinate transformation with matrix $T = \Sigma^{-1}R(\gamma^A)^{-1}$:

$$\Delta s(t) = \Delta s_0 + u\,t - W^A(t),$$

where $\Delta s(t) := T\Delta x(t)$ is the relative position of the aircraft in the new coordinates, $\Delta s_0 := T\Delta x_0$ and $u := T\Delta u$. In the new coordinate system, the circular zone of radius ρ centered at x is transformed into an ellipse with boundary described by:

$$\nu_a^2(x_1 - c_1(t))^2 + \nu_c^2(x_2 - c_2(t))^2 = \rho^2, \tag{4}$$

whose center $c(t) = (c_1(t), c_2(t))$ moves according to $c(t) = \Delta s_0 + u\,t$ (see Figure 1).

Aircraft A then gets within a distance ρ from x within $[t, t+\Delta]$ if the 2-D standard BM $W^A(t)$ starting at the origin wanders into this moving ellipse within $[t, t+\Delta]$. Denote this event by F_t. Then, $\pi^\rho(x,t)$ is the probability of F_t.

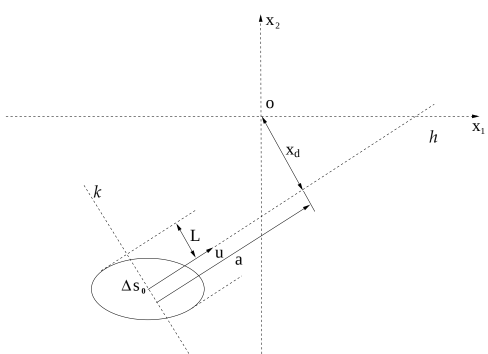

Fig. 1. Transformed protection zone.

Let x_d be the distance of the origin from the line h along which the center of the ellipse is moving, and a be the distance from the position Δs_0 of the center at $t = 0$ to the projection of the origin on h, as indicated in Figure 1 representing the new coordinate system. Then, x_d and a can be computed as follows:

$$x_d = \frac{|\Delta s_0^T R(\frac{\pi}{2})u|}{\|u\|}, \qquad a = -\frac{\Delta s_0^T u}{\|u\|}. \tag{5}$$

Observe that a positive value for a indicates that aircraft A is approaching x, whereas a negative value for a indicates that it is flying away from x.

The probability $P(F_t)$ of F_t does not admit a closed-form formula. However, we can approximate it by a "decoupled" event. Let k be the line passing through the center of the ellipse and orthogonal to u which moves along with the ellipse at the velocity u (see Figure 1). The projected width $2L$ of the ellipse along line k can be computed as follows:

$$L = \frac{\rho}{\nu_a \nu_c}\frac{\sqrt{u_1^2\nu_a^2 + u_2^2\nu_c^2}}{\|u\|}. \tag{6}$$

Denote by τ the first time $W^A(t)$ hits k and define F_t' to be the event that the first time $W^A(t)$ hits line k during the time horizon $[t, t+\Delta]$, it is within a distance of L from the center of the ellipse.

We consider $P(F_t')$ as an estimate of $P(F_t)$. This approximation is actually fairly accurate if the aircraft velocity is much larger than the variance growth rate of the BM. The intuition for this is that when the velocity of the moving ellipse is high, the event F_t is largely determined by the width of ellipse viewed in the direction of u.

Without loss of generality, to compute $P(F_t')$ we assume that u is aligned with the positive x_1 axis. Indeed, the axes rotation eventually necessary to make u aligned with the positive x_1 axis can be incorporated into the transformation matrix T, and still $W^A(t)$ remains a standard BM, since BM is invariant with respect to rotations.

When the aircraft A is approaching x, a given by equation (5) is positive, and, if we ignore the effect of the noise, in the new coordinate system the minimal separation distance is given by x_d in (5). Moreover, time τ for the BM $W^A(t)$ to reach line k has evidently the distribution $p_\tau(\cdot)$ given by the following Lemma 1 with $\mu = \|u\|$.

Lemma 1 (Bachelier-Levy, [23]): Let $b(t)$ be a standard one dimensional BM starting at the origin. Fix $\mu \in \mathbb{R}$ and define $\tau := \inf\{t \geq 0 : b(t) = a - \mu t\}$ to be the first time $b(t)$ reaches a point which is moving with speed μ towards the origin starting at position $a > 0$. Then, τ has probability density function:

$$p_\tau(t) = \frac{a}{\sqrt{2\pi t^3}}\exp[-\frac{(a-\mu t)^2}{2t}], \quad t \geq 0.$$

The approximate probability $P(F_t')$ can then be written as:

$$\begin{aligned} P(F_t') &= \int_t^{t+\Delta} p_\tau(t)\int_{|y-x_d|<L}\frac{1}{2\pi t}\exp(-\frac{y^2}{2t})\,dy\,dt \\ &= \int_t^{t+\Delta} p_\tau(t)\big[Q(\frac{x_d - L}{\sqrt{t}}) - Q(\frac{x_d + L}{\sqrt{t}})\big]\,dt, \end{aligned} \tag{7}$$

where we set $Q(y) := \int_y^\infty \frac{1}{\sqrt{2\pi}}\exp(-z^2/2)\,dz$.

It can be shown that $E[\tau] = a/\|u\|$ and $var[\tau] = a/\|u\|^3$. If we use a 0-th order expansions of $Q(\frac{x_d-L}{\sqrt{t}})$ and $Q(\frac{x_d+L}{\sqrt{t}})$ in (7), we get the following result.

Result 1: Suppose that aircraft A is approaching position $x \in \mathcal{S}$ ($a > 0$). Then, the probability $\pi^\rho(x,t)$ that aircraft A enters the ball of radius ρ centered at x within the time frame $[t, t+\Delta]$ can be approximated by:

$$\hat{P}(F_t) := \big(V(t+\Delta) - V(t)\big)\Big(Q\big(\frac{x_d - L}{\sqrt{t_0}}\big) - Q\big(\frac{x_d + L}{\sqrt{t_0}}\big)\Big)$$

where $V(s) = Q\big(\frac{a-\mu s}{\sqrt{s}}\big) + e^{2a\mu}Q\big(\frac{a+\mu s}{\sqrt{s}}\big)$, $t_0 = \frac{a}{\mu}$, if $\frac{a}{\mu} \in [t, t+\Delta]$; $t + \frac{\Delta}{2}$, otherwise, and $\mu = \|u\|$. □

For the purpose of complexity computations, we set $\hat{P}(F_t) = 0$ when aircraft A is flying away from x ($a < 0$).

b) Multi-legged nominal trajectory: Consider aircraft A flying with piecewise constant velocity $u^A : [0, t_f] \to \mathbb{R}^2$ and heading $\gamma^A : [0, t_f] \to [0, 2\pi)$ starting from $x_0^A \in \mathcal{S}$. The future position x^A of aircraft A is given by

$$x^A(t) = x_0^A + \int_0^t u^A(s)ds + R(\gamma^A(t))\Sigma W^A(t).$$

The relative position of aircraft A with respect to x evolves according to the equation

$$\Delta x(t) = \Delta x_0 + \int_0^t \Delta u(s)ds - R(\gamma^A(t))\Sigma W^A(t), \quad (8)$$

where $\Delta x_0 := x - x_0^A$ is the aircraft relative position at time $t = 0$ and $\Delta u(s) := -u^A(s)$.

At time t the aircraft is tracking some leg h of its nominal trajectory, associated with the deterministic time interval $[t_h, t_{h+1})$, the (constant) heading angle $\gamma_h^A = \gamma^A(t_h)$, and the (constant) relative velocity $\Delta u_h = u^B - u^A(t_h)$. With reference to $[t_h, t_{h+1})$, equation (8) can then be rewritten as follows:

$$\Delta x(t) = \Delta x_{h,0} + \Delta u_h t - n(t), \qquad t \in [t_h, t_{h+1}), \qquad (9)$$

where we set $\Delta x_{h,0} := \Delta x_0 + \int_0^{t_h} \Delta u(s)ds - \Delta u_h t_h$ and $n(t) := R(\gamma_h^A)\Sigma W^A(t)$.

Consider first the case when $[t, t+\Delta] \subseteq [t_h, t_{h+1})$. By (9), it is easily seen that to the purpose of computing $\pi^\rho(x, t)$, one can evaluate the probability that the perturbation $n(t)$ enters the ball of radius ρ whose center is moving at constant velocity Δu_h starting from $\Delta x_{h,0}$ at time $t = 0$. Similarly to the one-leg case, by applying the transformation matrix $T_h = \Sigma^{-1} R(\gamma_h^A)^{-1}$, equation (9) can be rewritten as

$$\Delta s(t) = \Delta s_{h,0} + u_h t - W^A(t), \qquad t \in [t_h, t_{h+1}),$$

where $\Delta s(t) := T_h \Delta x(t)$, $\Delta s_{h,0} := T_h \Delta x_{h,0}$ and $u_h := T_h \Delta u_h$. The problem then becomes that of evaluating the probability that during the time horizon $[t, t+\Delta]$ the standard BM $W^A(t)$ enters the ellipse with boundary given by (4) and center $c(t) = (c_1(t), c_2(t))$ moving according to equation $c(t) = \Delta s_{0,h} + u_h\, t$. An estimate of this probability can then be derived by the same approximation scheme as in the one-leg case.

If $[t, t+\Delta] \subseteq [t_h, t_{h+1})$ is not satisfied, we can partition $[t, t+\Delta]$ in sub-intervals, each one corresponding to a leg of the nominal trajectory. For each element of the partition we can apply the procedure above to determine an estimate of the corresponding probability $P(F_{t,h})$. By considering the events $F_{t,h}$ as if they were independent, $\pi^\rho(x, t)$ can be approximated by

$$\hat{P}(F_t) = 1 - \prod_{h=1}^{m_t} [1 - \hat{P}(F_{t,h})],$$

where m_t is the number of legs of the trajectory of aircraft A within the time interval $[t, t+\Delta]$.

B. Complexity from a single aircraft perspective

Each single aircraft, say aircraft A_i, is flying in some specific region of $\mathcal{S}$ and is interested in predicting the level of complexity encountered along its own intended nominal trajectory. To this purpose, we consider the set of all the other $N - 1$ aircraft, excluding aircraft A_i, and evaluate the probabilistic occupancy of level m and size ρ within $[t, t+\Delta]$ with reference to such a set. We denote this quantity as $P_{m,i}^\rho(x, t)$.

According to a reasoning similar to that in Section III-A, we introduce function $\rho_{\max,i} : [0, t_f] \to \mathbb{R}_+$ given by

$$\rho_{\max,i}(t) := \sup\{\rho \geq 0 : P_{1,i}^\rho(\bar{x}^{A_i}(t), t) \leq p_T\},$$

where $\bar{x}^{A_i}(t)$ is the nominal position of aircraft A_i at time $t \in [0, t_f]$, and define

$$\rho_{\max,i}^\star := \sup_{t \in [0, t_f]} \rho_{\max}(t).$$

Radius $\rho_{\max,i}^\star$ is an index of robustness of the air traffic encountered by aircraft A_i along its nominal trajectory. The larger is $\rho_{\max,i}^\star$, the more aircraft A_i is far from the other aircraft, both in time and in space, with high ($> 1 - p_T$) probability, and, hence, the larger is the robustness of its trajectory to possible disturbances due to possible deviations of the other aircraft from their intent and additional aircraft entering the traffic.

The quantity $\xi_i := \frac{1}{\rho_{\max,i}^\star}$ can then be taken as synthetic indicator of the air traffic complexity from the perspective of aircraft A_i during the time horizon $[0, t_f]$. If $\xi_i > \frac{1}{\bar{\rho}}$, then, some conflict can occur with probability $\geq p_T$ and the criticality of this conflict can be better assessed by computing the earliest *conflict time* $t_i^\star = \min\{t \geq 0 : \rho_{\max,i}(t) < \bar{\rho}\}$, and the *likelihood of a multiple* ($m > 2$) *aircraft conflict* $P_m^{\bar{\rho}}(\bar{x}^{A_i}(t_i^\star), t_i^\star)$.
Depending on the specific performance of the solver in place and on its capability of solving conflicts that are close in time and possibly involve multiple aircraft, one can define a critical value for $t_i^\star$ and take a value for m larger than 2 when assessing the likelihood of a multiple aircraft conflict.

IV. A NUMERICAL EXAMPLE

Consider a rectangular airspace region $\mathcal{S}$ where 6 aircraft are following a one-leg nominal trajectory from some starting to some destination position during the look-ahead time horizon $[0, t_f]$ with $t_f = 15$ minutes (min), while trying to keep at a minimum safe distance $\bar{\rho} = 3$ nautical miles (nmi). The configuration of the aircraft nominal trajectories is shown in Figure 2, where starting positions are marked with $*$ and destination positions with $\diamond$.

The trajectories in this figure are obtained by implementing the decentralized resolution strategy introduced in [20], which accounts for the uncertainty affecting the aircraft motion according to a similar model for the aircraft predicted motion. According to this strategy, resolution manoeuvres involve only heading changes.

The global complexity of the considered air traffic system obtained with $p_T = 0.2$ is $\xi \simeq 3$, which means that aircraft

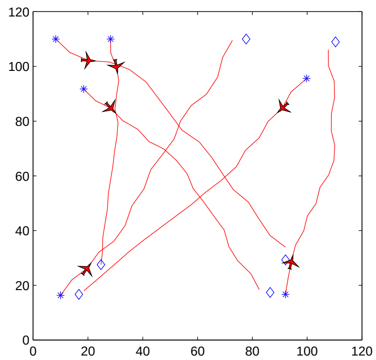

Fig. 2. Snapshot at time $t = 2$ min of the resolution manoeuvres of a 6 aircraft system. The aircraft are moving from starting ($*$) to destination ($\diamond$) positions, while trying to keep at a distance $\bar{\rho} = 3$ nmi.

are only guaranteed to keep at a distance of about 0.33 nmi, with probability greater than 0.8.

The complexity map $\Xi : \mathcal{S} \rightarrow [0, 1]$ plotted in Figure 3 shows that there are two main regions with some significant percentage of occupancy (larger than 10%): one in the upper left-hand-side, and the other close to the center of the airspace area $\mathcal{S}$.

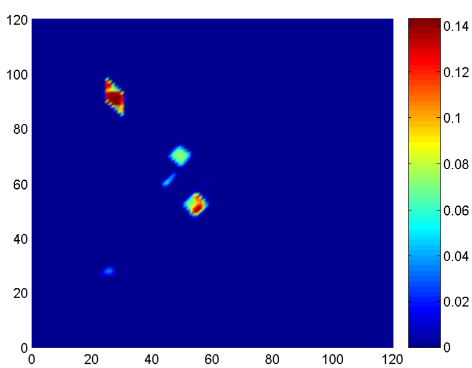

Fig. 3. Complexity map $\Xi : \mathcal{S} \rightarrow [0, 1]$ obtained for $\bar{\rho} = 3$ nmi.

The earliest conflict time for both the two aircraft in the upper left-hand-side of the airspace area $\mathcal{S}$ is $t_i^\star = 2$ min. Indeed, the snapshot of the resolution manoeuvres taken at time $t = 2$ min shows that this is the earliest time that a significant deviation action is taken by the decentralized solver and that it involves the two aircraft in the upper left-hand-side (Figure 2).

In this example, the complexity map $\Xi : \mathcal{S} \rightarrow [0, 1]$ defined in (1) has been evaluated at uniformly sampled grid points $x \in \mathcal{S} = [0, 120] \times [0, 120]$ with a grid size $\delta_{x_1} = \delta_{x_2} = 1$. Adopting a variable grid resolution, with a larger grid size far from the aircraft and a finer one close to the aircraft, would reduce the computational load.
In the numerical evaluation of the integral over $[0, t_f]$ involved in (1), $[0, t_f]$ has been uniformly sampled with $\delta_t = 1$. The short term look-ahead time horizon Δ has been set equal to 2 min, and $\nu_a = 0.25$ and $\nu_c = 0.2$ with the power spectral densities ν_a^2 and ν_c^2 measured in nmi^2/min.

V. Conclusions

In this paper, we have presented a novel method to evaluate air traffic complexity on a mid term horizon, which accounts for the uncertainty in the prediction of the aircraft future positions. The computational issues have been addressed in the 2D airspace case. Extensions to the 3D case are currently being carried out.

A simple numerical example has been reported to illustrate the approach. Further work is needed to assess the performance of the method on air traffic data and its impact on conflict resolution.

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