A geometric approach to air traffic complexity evaluation for strategic trajectory management

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Abstract— **In the perspective next generation Air Traffic Management (ATM) systems, aircraft will be endowed with part of the responsibility for separation maintenance, and traffic complexity reduction functions are envisaged to play an important role in avoiding to overload the on-board conflict resolution system. In this work, we introduce a method to assess air traffic complexity in the long term, with the goal of pointing out the presence of critical situations with limited aircraft manoeuvrability in the time/space coordinates, and provide support to the flow/strategic trajectory management functions. The method is based on the concept of "influence zone" of an aircraft, which accounts for both the direction and velocity of the aircraft, and can be regarded as the set of possible locations reachable by an aircraft through local deviations from its intended trajectory. Complexity is then related to the presence and magnitude of intersections between influence zones of different aircraft. From a computational point of view, the influence zones (as well as other regions of the airspace to be avoided) can be approximated with polyhedra, and their interaction analyzed using efficient tools borrowed from computational geometry. This allows to handle hundreds of aircraft without an excessive computational load, which is a crucial factor in this application.**

I. INTRODUCTION

An Air Traffic Management (ATM) system is a multi-agent system where many aircraft compete for a common, congestible resource represented by airspace and runways space, while trying to optimize their own cost, evaluated in terms of, *e.g.*, travel distance, fuel consumption, and passenger comfort. Coordination between aircraft is needed to avoid conflict situations where two or more aircraft get too close to one another.

In the current, centralized ground-based ATM system, coordination is operated on two different time scales by the Air Traffic Control (ATC) and Traffic Flow Management (TFM) functions. The human-based ATC function operates on a mid/short term horizon with the goal of maintaining the appropriate separation between aircraft in the different stages of their flights from departure to destination. The TFM function operates on a long-term horizon by defining the flow patterns so as to ensure a smooth and efficient organization of the overall air traffic, possibly reducing the need for the ATC intervention at a finer time-scale. The airspace is structured in sectors and a team of 2/3 air traffic controllers is in charge of each sector. The capacity of a sector is limited by the sustainable workload level of the air traffic controllers, and TFM accounts for this capacity constraint when performing traffic flow optimization.

The growth in air traffic demand is pushing the limits of the current ground-based ATM system. As reported in [1], the average daily traffic above Europe in 2006 was 26286 flights per day, with an increase of 4.1% over 2005, whereas the total delay increased by 4.6%, much more than expected based on the 4.1% of air traffic growth. This has motivated research aimed both at developing methods for the dynamic allocation of the resources involved in the current ATM system, and at conceiving new operational concepts in ATM.

The characterization of the *air traffic complexity* turns out to be a key aspect within both these directions of research. In general terms, air traffic complexity is a concept introduced to measure the difficulty and effort required to safely manage air traffic. In the current ATM system complexity is ultimately used to redistribute and reassign human resources and to reconfigure sectors in order to adapt the capacity of the ATM system to the air traffic demand. Complexity metrics are also used for evaluating ATM productivity and assessing the impact of new tools and procedures, [2].

New generation ATM systems are currently being developed within the SESAR (Single European Sky ATM Research, [3]) and NextGen (Next Generation Air Transportation System, [4]) projects. A common perspective of both these projects is that an increased air traffic volume can be efficiently managed allowing for a (partial) delegation of the ATM effort to the involved aircraft. On this line, the iFly European project "Safety, Complexity and Responsibility based design and validation of highly automated Air Traffic Management" is studying an advanced airborne self-separation design for enroute autonomous aircraft ATM (A^3TM). Indeed, ASAS (Airborne Separation Assistance System) has by now become a keyword in aeronautics.

A notion of air traffic complexity could be particularly useful in the new ATM systems to assess and predict traffic conditions that may be over-demanding to the autonomous aircraft design. This is a crucial task for avoiding encounters that appear safe from the individual aircraft perspective, but are actually safety-critical from a global perspective. In addition, complexity reduction functions could play an important role in avoiding excessive tactical manoeuvring of the aircraft, and be used to support the flow/strategic trajectory management functions.

In this work, a method is introduced to assess air traffic complexity in the long term, that allows to point out critical situations with limited aircraft manoeuvrability in the time/space coordinates at a limited and affordable computa-

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tional cost. The method relies on the concept of "influence zone" of an aircraft, which accounts for both the direction and velocity of the aircraft, and can be regarded as the set of possible locations reachable by an aircraft through local deviations from its intended trajectory. Intersections of influence zones pertaining to different aircraft indicate a source of complexity, the level of which is related to the number of the involved aircraft and the time and space extension of the intersection.

From a computational point of view, the influence zone is approximated with a polyhedron, which allows to borrow tools from computational geometry to assess complexity. This allows to handle hundreds of aircraft without an excessive computational load, which is a crucial factor in this application. Notice also that forbidden zones of any type (due, *e.g.*, to bad weather formations) can also be approximated by time varying polyhedra, and seamlessly included in the complexity assessment, although this issue is not addressed here for brevity.

II. PRELIMINARIES ON AIR TRAFFIC COMPLEXITY

A. Air traffic complexity within the current ATM systems

Most studies on air traffic complexity have been developed with reference to ground-based ATM, [5], [2]. The concept of air traffic complexity has been originally introduced to evaluate the difficulty perceived by the air traffic controllers in handling safely a certain air traffic situation (ATC workload), [6]. The idea is that assessing the impact on the ATC workload of different air traffic configurations can help evaluating how the current ground-based ATM system is operated, and provide guidelines as to how reconfigure the airspace and sectors, [7], [8], [9], [10]. Complexity is tightly related to air traffic density, but it is widely recognized that traffic organization is also an essential factor.

Among the proposed complexity measures, it is worth mentioning the *dynamic density* introduced in the pioneering work by NASA, [11], [12]. Dynamic density is a single aggregate indicator where traffic density and other controller workload contributors (such as the number of aircraft undergoing trajectory change and requiring close monitoring due to reduced separation) are combined linearly or through a neural network whose weights are tuned based on interviews to qualified air traffic controllers. The major drawback of dynamic density is the dependence of the tuning on the particular traffic management policy, as well as many subjective parameters, such as sector characteristics, controllers' skill and conditions, etc. There is therefore little hope to turn the dynamic density into a versatile, all-purpose complexity indicator.

The difficulty in obtaining reliable workload measures has been one of the strongest motivations for investigating complexity metrics independent of the ATC workload, such as the input-output approach in [13], [14], the fractal dimension in [15], and the intrinsic complexity measures in [16], [17], [18] and [19]. These metrics are actually those that appear more portable to an autonomous ATM context.

B. Air traffic complexity within the new generation ATM systems

In the envisioned next generation ATM systems, aircraft will be endowed with more degree of autonomy in trajectory management, while sharing with the air traffic controllers the responsibility for separation maintenance. In self-separation airspace (SSA), aircraft will be allowed to modify their flight plan so as to optimize performance, in terms, *e.g.*, of travelled distance and fuel consumption, while satisfying some constraint on their exit condition. This flexibility with respect to the ATC-managed airspace offers each single aircraft the possibility to improve the efficiency of its own flight. In turn, pilots will have to take over the ATC tasks for separation assurance with the support of the ASAS.

This shift towards a distributed ATM will be enabled by the introduction of novel information sharing systems such as the System Wide Information Management system developed within SESAR and the Net-centric Infrastructure System developed within NextGen, together with the availability of new airborne and communication, navigation, and surveillance capabilities. Aircraft will communicate with one another (air-to-air communication) and with the ground (air-to-ground communication) to get up-to-date information on the other aircraft position, velocity, and intent, on locally sensed weather data, on global weather conditions and forecast, and on areas-to-avoid.

The ATM function will then be realized by means of a (partially) decentralized control scheme (see Figure 1), where each aircraft evaluates the criticality of forthcoming encounters based on the information on the current position and intended destination of neighboring aircraft and eventually coordinates with them to avoid the actual occurrence of conflicts (intent-based conflict detection and resolution). Ground control will then assume a new role consisting in a higher level, possibly automated, supervisory function as opposed to lower level human-based control.

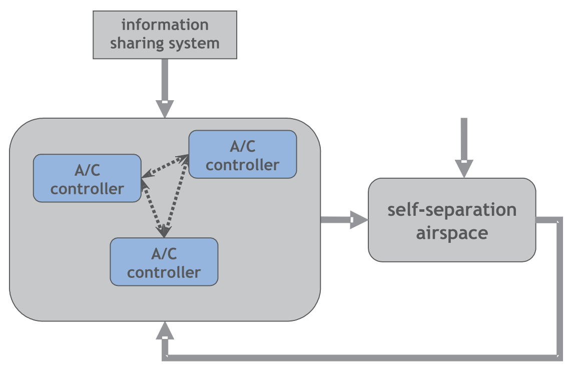

Fig. 1. Control scheme in autonomous aircraft ATM.

Performance and safety of each aircraft flight will be affected by the traffic present in the SSA. More specifically,

- Performance is deteriorated when the aircraft passes through an area with highly congested traffic, since this requires many tactical manoeuvres;

- Safety is compromised when the aircraft is involved in a multi-aircraft conflict that exceeds the capabilities of the onboard conflict resolution system.

These situations could be timely predicted by adopting the appropriate notion of air traffic complexity, which would then play an essential role within the strategic and hazard prevention phases of the ATM process. Complexity evaluation could, in particular, support the following functionalities of an airborne autonomous aircraft system:

- *Onboard trajectory management,* aiming at optimizing the effectiveness of the flight within the SSA, compatibly with the strategic flow management constraints (exit conditions from the SSA) and the presence of areas to avoid. The latter could include high "complexity" zones, potentially requiring an entering aircraft too many tactical manoeuvres to pass them through.
- *Intent-based conflict detection and resolution*, aiming at predicting and solving conflict situations on a mid term (up to $10-15$ minutes) time horizon based on the aircraft intent information. The conflict detection function could also predict those "complex" situations that are likely to overload the conflict solver. In turn, the conflict solver could favor those resolution manoeuvres with lower complexity, so as to avoid further alerting and resolution actions.

Accordingly, long term and short/mid term horizon complexity metrics are required, respectively. Complexity evaluation on a long term horizon is based on the aircraft reference trajectories, with the understanding that each aircraft should generally conform to its own. Complexity should be recomputed from time to time to take care of possible modifications of the aircraft reference trajectories. Unexpected deviations at a finer time scale should be accounted for by the mid term complexity metric. Long term complexity metrics should serve the purpose of

- revealing the presence of repeatedly critical situations along the reference trajectory of each single aircraft that would require many tactical manoeuvres to be solved, and
- pointing out highly congested regions that will cause an entering aircraft too many adjustments of its reference trajectory to pass them through.

Given the decentralized nature of the A^{3}TM, it makes much sense to introduce complexity measures related to a single aircraft. A significant and inspiring concept in this respect is that of flexibility of a trajectory, [20], [21], [22], defined as the extent to which a trajectory can be modified without causing a conflict with neighboring aircraft or entering a forbidden airspace area. Flexibility can be further characterized in terms of robustness, *i.e.* the ability of the aircraft to keep its planned trajectory unchanged in response to the occurrence of a disturbance, and adaptability, *i.e.* the ability of the aircraft to change its planned trajectory in response to the occurrence of a disturbance that makes the current planned trajectory infeasible. Determining accurate robustness and adaptability measures is generally difficult and computationally demanding. The idea that we pursue here is to introduce some complexity indicators that represent easily computable – though rough – estimates of the flexibility of a trajectory.

III. THE PROPOSED APPROACH TO COMPLEXITY EVALUATION

Consider an airspace region $\mathcal{S}$ and a reference time horizon T. The instantaneous local density $d(x,t)$ at position $x \in \mathcal{S}$ and at time $t \in T$ can be defined as the number of aircraft present at time t in a ball of radius r centered in x.

Let $I_B(x)$ denote the indicator function of set B, *i.e.* the function that equals 1 if $x \in B$, and 0 otherwise, and A_i, $i = 1, ..., N$, be the set of aircraft in the airspace region $\mathcal{S}$ within the time horizon T. Then, the instantaneous local density function $d : \mathcal{S} \times T \to \mathbb{N}$ can be alternatively defined as follows:

$$d(x,t) = \sum_{i=1}^{N} I_{B(x_i(t),r)}(x),$$

where $B(x_i(t), r)$ denotes the ball of radius r centered in the position $x_i(t)$ of aircraft A_i at time t.

This view allows the interpretation of the ball of radius r centered on the aircraft as some sort of 'influence zone', and of the local density as an indicator of the presence of overlapping regions between the influence zones of different aircraft (see Figure 2). Accordingly, radius r is a safety parameter that should be chosen based on the aircraft manoeuvrability and on the pilot's reaction time.

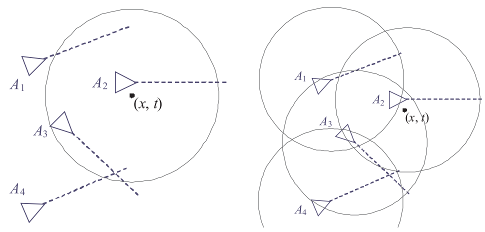

Fig. 2. Alternative views of the instantaneous local density $d(x,t)$.

Air traffic density is widely recognized as a major factor influencing complexity, irrespectively of the way traffic is managed (autonomous vs. centralized, automated vs. human-based ATM). However, it is also well known that the plain density on its own does not fully capture the concept of complexity, since it does not account for the aircraft directivity and velocity, and therefore does not take into consideration any geometrical factor. For example, a dense but geometrically ordered traffic with non interfering aircraft trajectories is actually much less complex than a less dense but unstructured traffic with multiply intersecting trajectories.

To capture notions related to traffic organization, one must also consider the directionality and velocity of the aircraft when defining its influence zone. To this purpose, attention should be restricted to that sector of the ball which includes all the trajectories obtained by locally perturbing the reference trajectory with feasible manoeuvres in some time window $[t, t+\delta]$ of length $\delta > 0$. The safety parameter is now the (local) *projection horizon* δ: the larger is δ the more is the safety margin entering the complexity evaluation.

Focusing for simplicity on the 2D case, the envelope of the possible motions of the aircraft A_i from time t to $t+\delta$ can be over-approximated by an isosceles triangle $Z_i(t)$ with vertex in the aircraft position $x_i(t)$ and base orthogonal to the aircraft velocity vector at time t (see Figure 3). The height and base length of $Z_i(t)$ are respectively related to the current velocity and acceleration rate, and the turning rate of the aircraft.

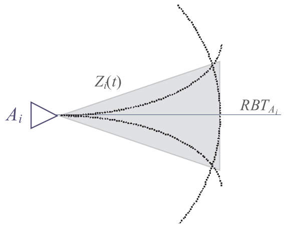

Fig. 3. Envelope of the possible motions of aircraft A_i from time t to $t+\delta$, given its position and velocity at time t.

The presence of a non empty intersection between the influence zones of different aircraft relates to the possibility that two aircraft occupy the same region of the airspace in the same time frame, thus reducing their manoeuvrability spaces and possibly getting in conflict.

According to this observation, an aircraft A that has to enter some airspace region should better design its trajectory so as to avoid to intersect at any time instant t the triangular influence zones $Z_j(t)$ of the other aircraft A_i's present in that region. This would in fact make aircraft A preserve its trajectory flexibility and avoid conflicts despite of the possible local deviation of the other aircraft from their intended trajectory.

As the projection horizon δ grows, the size of the influence zone of each aircraft increases and the influence zones of different aircraft are more likely to intersect. In the sequel, we shall denote by $Z_{\delta,i}(t)$ the influence zone of aircraft A_i associated with the projection horizon δ, and by $\delta_{\min}$ and $\delta_{\max}$, with $\delta_{\min} < \delta_{\max}$, the values defining the admissible range for δ.

Motivated by the discussion above, we next introduce a measure of the complexity encountered by aircraft A along its nominal path $x(t)$, $t \in T$, when flying in a region where other aircraft A_i, $i = 1, 2, \ldots, N$, are present.

Define the *limit projection horizon* at time $t \in T$ as

$$\delta(t) = \begin{cases} \delta_{\min}, & \text{if } \Delta Z_{\delta_{\min}}(t) \neq \emptyset \\ \max\{\delta \in [\delta_{\min}, \delta_{\max}] : \Delta Z_\delta(t) = \emptyset\}, & \text{otherwise} \end{cases}$$

where $\Delta Z_\delta(t) = \bigcup_{i=1}^N Z_\delta(t) \cap Z_{\delta,i}(t)$. Then, the instantaneous complexity function $c : T \to [0,1]$ of aircraft A is defined as:

$$c(t) = \frac{\delta_{\max} - \delta(t)}{\delta_{\max} - \delta_{\min}}. \tag{1}$$

Note that, according to equation (1), the complexity encountered by aircraft A at time t is maximal and equal to 1 if $\delta(t) = \delta_{\min}$, *i.e.*, aircraft A has a local flexibility that is limited to a projection horizon smaller than or equal to $\delta_{\min}$ at time t. If $\delta_{\min}$ is chosen appropriately small, the complexity measure (1) can then identify short-term conflict situations along the reference trajectory of aircraft A. This information can be mapped into the spatial coordinates by recovering the position $x(t)$ of aircraft A at time t along its reference trajectory. Complexity is instead equal to 0 when flexibility is guaranteed on the whole projection horizon $\delta_{\max}$.

Remark 1 *It is worth noticing that the proposed measure of complexity can easily account for the presence of temporarily forbidden areas due,* e.g.*, to bad weather conditions. It in fact suffices to describe those areas as fictitious time-dependent influence zones, and take into consideration also these additional influence zones when determining the limit projection horizon.*

Regarding the properties identified in [23] as relevant for a complexity metric in an ATC context, *i.e.*,

1. Adding an aircraft should not reduce complexity
2. The metric should be independent of the orientation and origin of the coordinate system
3. Shrinking the geometry of the airspace, or increasing the speeds of all aircraft in the airspace, should not reduce complexity
4. Repositioning one aircraft so that it is now farther from every other aircraft should not increase complexity,

we can observe that property 1 is satisfied because the proposed measure of complexity is density-based. Property 2 holds because it depends on the relative position and direction of motion of the aircraft, whereas property 3 holds because the size of the influence zone increases as the velocity grows. Property 4 instead should be formulated more precisely, since complexity can vary even for constant aircraft distance, depending on their flight direction. If we interpret this property as that one aircraft is moved far enough not to interfere with the other aircraft within the reference time horizon T, then, property 4 holds in our case. Indeed, applying a conflict resolution strategy that guarantees a certain pair-wise distance R among the aircraft all over the time-horizon T reduces the complexity function to zero if the aircraft influence zone is strictly contained within the ball of radius R.

As for the behavior of the complexity function in time, it is apparent that if the complexity of aircraft A presents multiple isolated maxima close to 1, then aircraft A will be repeatedly involved in critical situations along its reference trajectory, so that many tactical manoeuvres would be required to avoid possible conflicts. If the complexity function remains consistently low (but not zero) for some extended time interval, then aircraft A will possibly have just to limit its own manoeuvrability space in order to avoid to get close to the other aircraft, without the need to coordinate with them.

This situation can be detected by evaluating the behavior of the complexity function as the size of the influence zone of aircraft A is decreased (*i.e.*, angular deviation and velocity are decreased), with the size of the influence zone of the other aircraft constant and maximal.

IV. CLASSIFICATION OF PAIR-WISE AIRCRAFT ENCOUNTERS

The suggested air complexity metric provides a rich information that can be put in direct relation with the geometry of the air traffic pattern responsible for the complexity increase. To see this, we shall analyze the behavior of the complexity function for different type of pair-wise aircraft encounters, examining separately the effect of unilateral reduction of the velocity and the angular deviation.

Consider for this purpose the following three types of encounter geometries: *head-on*, *catch-up*, and *crossing*. For illustrative purposes, in each of these three geometries, we consider both the cases when the reference trajectories lead to a loss of separation (LOS) and to a collision. In particular, in the catch-up scenario, a collision will occur if the two aircraft fly on the same track at different speeds so that the one behind will catch up the one in front. The considered 6 scenarios are schematically represented in the first column of Figure 4, assuming different aircraft velocities for more generality. The aircraft with higher speed can be easily identified as the aircraft with the larger influence zone.

Figures 4-5 show the evolution in time of the complexity measure (1) experienced by each aircraft for all the 6 listed cases, analyzing its sensitivity to an unilateral variation of the velocity range and heading, respectively. In the simulations, the other aircraft corresponding parameters are kept at the maximum values, in the understanding that it represents a disturbance to the free navigation of the ownship. This analysis can give indirect information as to the possible local improvement of complexity that each aircraft can achieve by limiting its own range of possible deviations from the current nominal velocity and heading, thus reducing the size of its influence zone. In other words, it reveals conditions in which the lateral or speed manoeuvrability are (partially or totally) compromised.

In the head-on case with LOS, the complexity increases linearly for both aircraft in the first portion of the transient, up to a maximum value, and then falls abruptly to 0 when the two influence zones disengage. Figure 4 points out a similar dependence on the velocity factor for the two aircraft: an increase in velocity reduces for both the maximum time horizon without intersection of the influence zones. Increasing the angle parameter instead increases the maximum complexity value reached. For sufficiently restricted lateral manoeuvrability the complexity would be zeroed out. In the corresponding case with collision the angle effect is null and the complexity saturates to the maximum value. There is no heading variation that robustly guarantees that the other aircraft will be avoided in the absence of coordination, and velocity reductions can only slightly delay the problem.

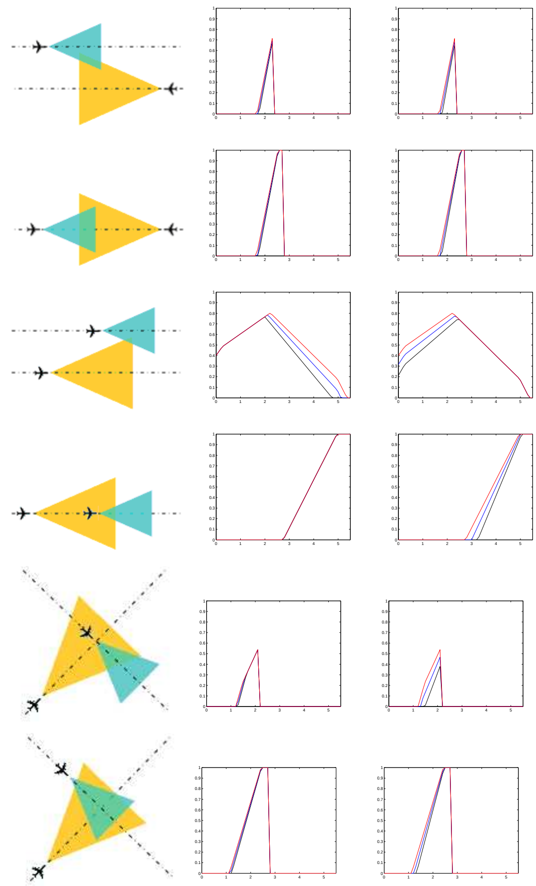

Fig. 4. Complexity measure as a function of time during a pair-wise aircraft encounter for unilateral variation of the aircraft velocity. Left column: encounter configuration; Central and right columns: corresponding complexity curves from the perspective of the slower aircraft (central column) and of the faster aircraft (right column) when its speed is decreased (black curve), kept constant (blue curve), and increased (red curve) with respect to its nominal value.

In the catch-up pattern complexity rises and falls with milder ramps, due the smaller relative velocity. The limit projection horizon is intuitively determined by the aircraft that is behind (the faster plane at the beginning, the slower one in the end). For that aircraft a reduction of the velocity is always beneficial in terms of complexity, whereas it does not have any effect for the aircraft that is in front. A reduction of the lateral manoeuvrability on the other hand decreases the complexity for both the aircraft. In the limit case when the two aircraft have identical velocity, the complexity is constant for given velocity and angle. In the catch-up configuration with collision, complexity reaches 1, and only the complexity of the aircraft that is behind is sensitive to a variation of a parameter, *i.e.* the velocity, indicating that the only robust manoeuver in this case is a deceleration of the follower aircraft.

Finally, the crossing case has a complexity pattern that resembles that of the head-on case, with a (less than) linear increase followed by an abrupt fall to 0. However, the dependence of the angle parameter is completely different from the head-on case. In addition, the complexity of the aircraft that passes behind the other is extremely sensitive to both angle and velocity variations. Compared to the corresponding head-on case, the crossing with collision situation displays a very similar behavior in terms of velocity dependence, but is now sensitive to angle variations.

In summary, the complexity behavior and sensitivity patterns can be mapped to different encounter conditions, and at the same time give insight as to the more critical flight parameters.

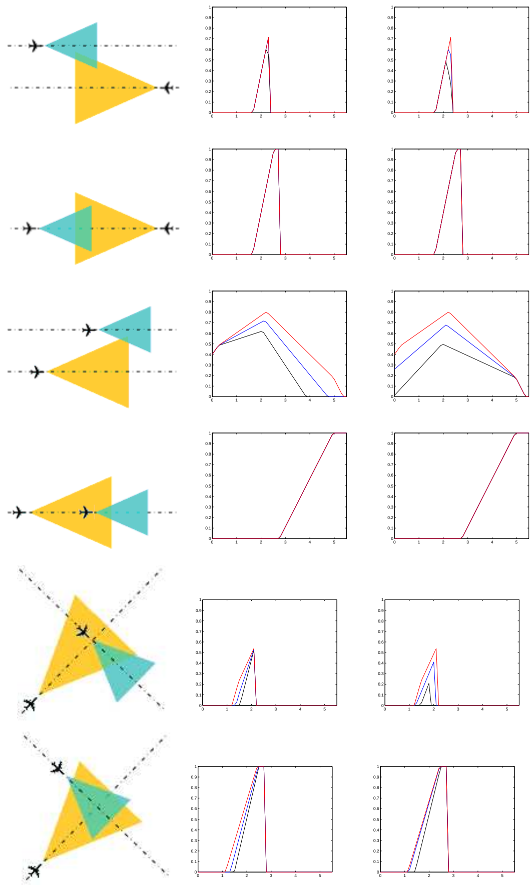

Fig. 5. Complexity measure as a function of time during a pair-wise aircraft encounter for unilateral variation of the admissible range of the aircraft heading. Left column: encounter configuration; Central and right columns: corresponding complexity curves from the perspective of the slower aircraft (central column) and of the faster aircraft (right column) when its admissible heading angle variation is small (black curve), medium (blue curve), and large (red curve).

Notice that the sensitivity analysis with respect to angle and speed is essential to reveal the characteristic of the aircraft encounter and also for distinguishing the roles of the two aircraft when they are not symmetric.

V. APPLICATION TO TRAJECTORY MANAGEMENT

In this section, we show that the introduced air traffic complexity measure can be used for trajectory planning. Suppose that an aircraft, say A, has to enter some airspace region $\mathcal{S}$ at time 0, flying from some starting position P_0 to some destination position D_0 within the time horizon $T = [0, t_f]$.

The minimum-energy trajectory $x : T \to \mathcal{S}$ of aircraft A, *i.e.*, the trajectory minimizing

$$\mathcal{E} = \frac{1}{2}\int_0^{t_f} \|\dot{x}(t)\|^2 dt$$

is given by a straight line traveled at constant velocity between P_0 and D_0, [24]. However, due to the presence of other aircraft, aircraft A could be involved in critical encounter situations along this trajectory.

We propose to formulate the problem of finding a suitable reference trajectory as that of determining the position of an intermediate way-point Z at time $t_f/2$ such that the weighted sum of the energy of the resulting trajectory and of the complexity associated with it is minimized.

If we assume that the two-legged trajectory to be designed is traveled at constant velocity in each linear segment, then its energy can be expressed as

$$\mathcal{E}_Z = \frac{(\|Z - P_0\|^2 + \|D_0 - Z\|^2)}{t_f},$$

and when the way-point is positioned at $Z_c := \frac{D_0 + P_0}{2}$, we obtain the one-legged trajectory with minimum energy.

We next report the results obtained in a numerical example, where 3 aircraft are present in the region where aircraft A has to enter, and are flying at the same constant altitude along straight line paths.

We take as cost function to be minimized

$$J(Z) = \frac{\mathcal{E}_Z - \mathcal{E}_{Z_c}}{\mathcal{E}_{Z_c}} + \lambda \max_{[0,t_f]} c_Z(t), \qquad (2)$$

where $c_Z(t)$ is the complexity encountered by aircraft A at time t along the two-legged trajectory with the intermediate way-point positioned at Z, and $\lambda > 0$ is a weighting factor.

The trajectory of aircraft A obtained for $\lambda = 1$ is plotted in red in Figure 6.

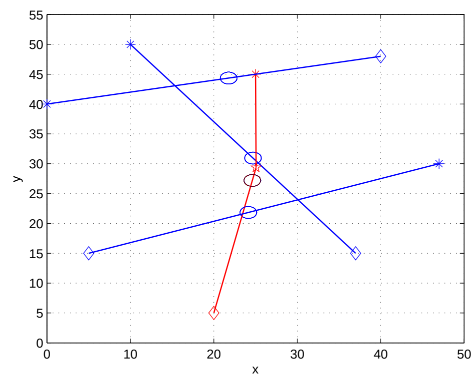

Fig. 6. Optimal trajectory design ($\lambda = 1$): the resulting trajectory of aircraft A is plotted in red with the intermediate way-point marked with '$\star$'. Starting and destination positions of all the aircraft are marked with '*' and '$\diamond$'. The circles represent the aircraft position at some time $t \in T$.

The complexity experienced by aircraft A along the designed trajectory is plotted in Figure 7 together with the complexity curve associated with the minimum-energy trajectory. This latter curve clearly reveals the presence of a collision

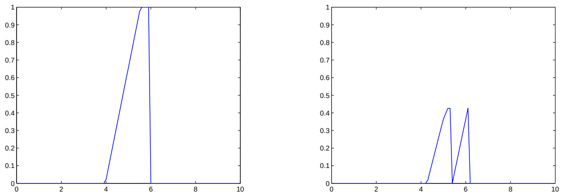

Fig. 7. Optimal trajectory design ($\lambda = 1$): complexity function of aircraft A along the minimum-energy trajectory (on the left) and along the trajectory obtained by minimizing the cost (2) (on the right).

(aircraft A would collide with the aircraft flying from right to left in the lower part of Figure 6 if it would follow the straight line from starting position to destination). The trajectory determined by minimizing the cost (2) eliminates the peak with value 1 from the complexity function and generates two peaks of smaller amplitude. As it can be seen from the snapshot of the aircraft positions in Figure 6, these peaks are due to consecutive crossings of aircraft A with two different aircraft.

VI. A LARGE SCALE AIR TRAFFIC EXAMPLE

A dataset containing simulated air traffic data over France on a typical day (courtesy of ENAC) was studied to correlate the complexity analysis with traffic density information and conflict detection and resolution. Simulations were run by ENAC on real flight plans over France, based on the BADA (Base of Aircraft DAta) aircraft model developed by Eurocontrol. The data provided refer to both uncontrolled and controlled traffic. Controlled trajectories were generated using a genetic algorithm for conflict resolution. Elementary control actions were allowed (offset or turning point) and optimization was carried out until the elimination of all (original and control-induced) conflicts, minimizing the length increase of the trajectories.

Trajectories are sampled every 15 seconds, and information is given at every sample instant concerning each aircraft position (horizontal coordinates are given in $1/64$ nautical miles (nmi), and the aircraft altitude is measured in flight levels, *i.e.* in hundreds of feet) and velocity (horizontal plane velocity in knots, the rate of climb/descent in feet/minute). For simplicity reasons attention has been restricted to an hour-long time history selected in the midday rush hour period, which involves 450 to 500 aircraft flying over the considered region.

To approximately account for the different flight levels where the aircraft are located, the triangular influence zones have been projected in the vertical direction to obtain triangular prisms extending 1000 feets above and below the current aircraft position. Then, three dimensional intersections between the influence zones of the various aircraft involved have been considered, in nmi^3. To evaluate the complexity of the air traffic, the integral of the pair-wise intersection volumes has been computed over a 60 seconds projection horizon (see Figure 8, top) both for the unresolved and resolved trajectories. As expected, the complexity is highly correlated with the number of detected conflicts (see Figure 8, bottom), at least regarding the low frequency tendency. On the contrary, it is much less correlated with the instantaneous traffic density (see Figure 8, middle), indicating that this figure is insufficient to reveal complexity, especially if evaluated in a very wide region, where the majority of the aircraft fly without generating any security concern. Finally, the complexity is consistently much lower for the resolved trajectories, confirming the intuition that complexity should always be reduced by conflict resolution. Both shorter and longer projection horizons have been tested as well, showing the same qualitative behavior. Clearly, the projection horizon is a crucial design parameter, since it determines the extension of the influence zones, and consequently the number of pair-wise intersections as well as the resulting overall intersection volume.

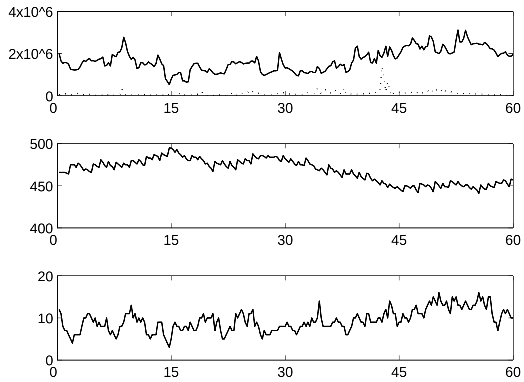

Fig. 8. Traffic complexity evaluation during one hour over France. From top to bottom: sum of volume intersections (uncontrolled traffic = solid line, controlled traffic = dotted line), instantaneous traffic density, number of conflicts in the uncontrolled traffic.

VII. CONCLUSIONS AND FUTURE WORKS

In this paper, we introduced a new measure for evaluating long term complexity and discussed its properties and application to an autonomous aircraft ATM context. The proposed complexity measure is based on the concept of influence zone of an aircraft and serves the purpose of identifying critical situations with limited manoeuvrability along the aircraft intended trajectory. Computationally, this involves evaluating the intersection between polyhedral approximations of the influence zones.

Future directions of research include developing a methodology for the design of optimal aircraft trajectories with guaranteed low complexity. In this respect, it would be interesting to explore the possibility of adopting the shortest path planning algorithms developed in robotics for moving an object from a starting to a destination position while avoiding polyhedral obstacles.

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