Air traffic complexity in advanced automated Air Traffic Management systems

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***Abstract*—In this paper we study the issue of characterizing the complexity of air traffic to support Air Traffic Management (ATM) operations. We discuss, in particular, the relevant features of a complexity metric in advanced automated ATM systems where part of the responsibility for separation maintenance is delegated to the aircrews, and trajectory management functions are further automated and distributed.**
A probabilistic complexity metric is described and analyzed in some detail. Possible applications to conflict detection and resolution and trajectory management operations are presented through numerical examples.

I. Introduction

The growth in air traffic demand is pushing the limits of the current ground-based ATM system. For example, in 2007, there was a 5.3% growth in the air traffic over Europe over 2006, with a disproportionate increase of 17.4 % in the delay [1]. The traffic grew by 0.5% in 2008 compared to 2007 while the total delay increased by 10.4% over 2007 [2]. There is a need for the current capacity to be increased while ensuring that the safety requirements still hold. The currently adopted strategy consists in redistributing and reassigning human resources and reconfiguring sectors so as to maintain the air traffic controllers workload under sustainable levels. On a longer term perspective, the whole Air Traffic Management (ATM) system must be rethought and its degree of automation increased in order to adapt the capacity of the ATM system to the grown air traffic demand. This has fostered the development of new operational concepts in ATM, as witnessed by the SESAR (Single European Sky ATM Research, [3]) and NextGen (Next Generation Air Transportation System, [4]) projects.

In advanced automated airborne ATM, aircraft entering the self-separation airspace can modify their flight plan so as to optimize performance, while satisfying some constraint on their exit condition when synthesizing their new trajectory. This flexibility with respect to the ATC-managed airspace offers each single aircraft the possibility to improve the effectiveness of its flight. In turn, pilots will have to take over the ATC tasks for separation assurance, possibly relying on tools enabled by advanced technologies for sensing, communicating, and decision making. ASAS (Airborne Separation Assistance System) has by now become a keyword in aeronautics.

Air traffic complexity is a concept introduced to measure the difficulty and effort required to safely and efficiently managing air traffic. In the current ATM system, complexity is ultimately related to the difficulty perceived by the air traffic controllers in handling safely a certain air traffic situation (ATC workload). The idea is that assessing the impact on the ATC workload of different air traffic configurations can help to evaluate how the current ground-based ATM system is operated, and can also provide guidelines on how to obtain more manageable sectors by reconfiguring the airspace and by modifying traffic patterns, [5], [6], [7], [8]. The work [9] was perhaps the first one to systematically examine the relationship between air traffic characteristics and controller workload.

Most studies on air traffic complexity have been developed with reference to ground-based ATM, as it clearly appears from the literature reviews [10] and [11]. Among the proposed complexity measures, it is worth mentioning the *dynamic density* introduced in the pioneering work by NASA, [12], [13]. Dynamic density is a single aggregate indicator where traffic density and other controller workload contributors (such as the number of aircraft undergoing trajectory change and requiring close monitoring due to reduced separation) are combined linearly or through a neural network whose weights are tuned based on interviews to qualified air traffic controllers. The difficulty in obtaining reliable workload measures has been one of the strongest motivations for investigating complexity metrics independent of the ATC workload, such as the input-output approach in [14], [15], the fractal dimension in [16], and the intrinsic complexity measures in [17], [18], [19] and [20]. These metrics are actually those that appear more portable to advanced automated ATM.

Performance and safety of each aircraft flight is affected by the traffic present in the self-separation airspace:

- performance is deteriorated when the aircraft passes through an area with highly congested traffic, since many tactical maneuvers are required
- safety is compromised when the aircraft is involved in a multiaircraft conflict that exceeds the capabilities of the onboard conflict resolution system

Critical situations with respect to performance and safety can be timely predicted by introducing the appropriate notion of air traffic complexity, which would then play a key role in the strategic and hazards prevention phases of the ATM process. More specifically, complexity measures could be useful to predict situations that may overburden the distributed Conflict Detection and Resolution (CD&R) function, and could also benefit the strategic trajectory management operations by detecting critical areas that would require excessive tactical manoeuvring of the aircraft.

Workload-independent metrics can be classified as *control-dependent* or *control-independent*. In control-dependent metrics, knowledge of the controller is needed to compute complexity. In control-independent metrics, the actual controller in place is indirectly accounted for through its effect on the air traffic organization. In principle, control-dependent metrics could be employed in the airborne self-separation framework. In practice, however, a control-independent measure of complexity appears to be better suited for an airborne self-separation ATM system where the controller has a decentralized time-varying structure, difficult to characterize for the purpose of control effort evaluation, and involving a human-in-the-loop component represented by pilots.

The fractal dimension and the intrinsic complexity measures are control-independent metrics. The input-output approach provides a control-dependent metric, since complexity is evaluated in terms of control effort needed to accommodate an additional aircraft crossing the considered airspace region. It then presents the drawback that the algorithm for assessing complexity is tailored to the adopted conflict solver.

It is by now well-reckoned that the two main factors jointly affecting complexity are the *aircraft density* and the *air traffic dynamics*. Aircraft density on its own represents a very coarse measure of complexity, even of the ATC workload. The evolution of the traffic must also be accounted for when evaluating complexity.

Approaches to complexity evaluation should have a *goal-oriented output form*. Complexity is both a time and space-dependent feature that can be expressed in an aggregate form by condensing either the space or the time information, or both of them. Those approaches providing a spatial complexity map, such as the input-output and the intrinsic complexity ones, can support decision making by isolating critical areas and appear better suited for trajectory management applications. Scalar-valued, possibly time-dependent, measures providing a concise information on the complexity encountered by the aircraft along their trajectory appear better suited for CD&R-related applications. Scalar aggregate indicator such as the fractal dimension measuring the geometrical complexity of a traffic pattern based on the trajectories observed on an infinite time period can be useful as a synthetic index to compare different air traffic situations.

The *time dependence aspect* has been mostly neglected in the literature and should be better focused, introducing approaches to air traffic complexity evaluation tailored to the specific time horizon. Complexity evaluation on a long-term prediction horizon can help in identifying congested areas for strategic trajectory management, whereas complexity evaluation on a mid-term horizon can support distributed CD&R operations by timely identifying encounter situations that are critical to solve.

Uncertainty entering aircraft trajectory prediction should be possibly accounted for in the assessment of complexity. Complexity is in fact computed based on the future aircraft trajectories on the considered time-horizon, and, hence, the reliability of complexity prediction depends on that of the aircraft trajectories prediction. Despite the extensive studies on uncertainty in the modeling and analysis of ATM systems by various researchers (see e.g. [21], [22], [23], [24] and [25]) its effect on air traffic complexity evaluation has not received adequate attention.

In this paper, we focus on a novel approach to air traffic complexity evaluation proposed in [26], which explicitly accounts for the uncertainty affecting the future aircraft positions.

II. Probabilistic approach to complexity evaluation

Complexity is evaluated in terms of proximity in time and space of the aircraft present in the traffic as determined by their intent and current state, while taking into account uncertainty in the aircraft future position. Specifically, air traffic complexity at a point x in an airspace region $\mathcal{S} \subset \mathbb{R}^3$ and at time t within some look-ahead time horizon T is evaluated as the probability that a certain buffer zone in the airspace surrounding x will be "congested" within $[t, t+\Delta]$, with $\Delta > 0$ (probabilistic occupancy). By defining congestion as the simultaneous occupancy of the buffer zone by a certain number of aircraft and evaluating this complexity measure at all possible points in $\mathcal{S}$, a complexity map can be built. Forming the complexity maps associated with different consecutive time intervals allows to predict when the aircraft will enter and leave a particular zone in the airspace, and to identify regions of the airspace $\mathcal{S}$ with a limited inter-aircraft maneuverability space. This information can be used to detect critical encounter situations that would be difficult for the aircraft to solve autonomously and to provide guidance for trajectory design in conflict resolution operations.

Since the proposed complexity measure is control-independent, it can be easily applied in the current ground-based ATM system as well as in the prospective next generation ATM systems, the only requirement being that the aircraft state and intent information should be available to reconstruct the *nominal* trajectory of each aircraft over the look-ahead time-horizon T.

Analytic –though approximate– formulas have been derived in [26] under the assumption that a piecewise linear approximation of the nominal trajectory can be adopted, with the aircraft flying at constant velocity between consecutive waypoints. The reader is referred to [26] for details on the computational aspects. Here, we focus on the metric description (see Sections II-A and II-B), and on its possible applications that are illustrated through numerical examples in Section III.

A. Complexity from a global perspective

Consider N aircraft $A_i, i = 1, \ldots, N$, flying in the 3-D airspace $\mathcal{S} \subset \mathbb{R}^3$ during the look-ahead time horizon

$T = [0, t_f]$, with $t = 0$ representing the current time instant and $t_f > 0$ the time horizon length. Suppose that each aircraft is following a nominal trajectory with a velocity profile $u^{A_i} : T \to \mathbb{R}^3$, starting from the initial position $x_0^{A_i}$ at time $t = 0$. The aircraft future position during T is not known exactly, and we assume that the prediction error can be modeled through a Gaussian random perturbation whose variance grows not only linearly with time t but also faster in the along-track direction (namely the direction of u^{A_i}) than in the cross-track directions (i.e., directions orthogonal to u^{A_i}). Similar models have been proposed in [21], [22] and [25] for predicting aircraft trajectories over a mid-term look-ahead time horizon of tens of minutes.

The predicted position $x^{A_i}(t) \in \mathbb{R}^3$ at time $t \in T$ of aircraft A_i is then given by

$$x^{A_i}(t) = x_0^{A_i} + \int_0^t u^{A_i}(s)ds + Q^{A_i}(t)\Sigma^{A_i} B^{A_i}(t),$$

where $B^{A_i}(t)$ is a standard 3-D Brownian motion starting from the origin whose variance is modulated by the matrix $Q^{A_i}(t)\Sigma^{A_i} \in \mathbb{R}^{3\times 3}$. More precisely, $\Sigma^{A_i} = \mathrm{diag}(\sigma_1^{A_i}, \sigma_2^{A_i}, \sigma_3^{A_i})$ is a diagonal matrix whose entries $\sigma_1^{A_i}$, $\sigma_2^{A_i}$, and $\sigma_3^{A_i}$ are the variance growth rates of the perturbation in the along-track direction and the two cross-track directions and satisfy $\sigma_1^{A_i} \geq \sigma_2^{A_i} = \sigma_3^{A_i} > 0$, whereas $Q^{A_i}(t) = \left[q_1^{A_i}(t) \quad q_2^{A_i}(t) \quad q_3^{A_i}(t)\right] \in \mathbb{R}^{3\times 3}$ is an orthogonal matrix whose first column $q_1^{A_i}(t)$ is aligned with $u^{A_i}(t)$: $q_1^{A_i}(t) = \frac{u^{A_i}(t)}{\|u^{A_i}(t)\|}$. Different phases of flights can be characterized through different values of the variance growth rates.

For each $x \in \mathcal{S}$, let us consider the ellipsoidal region $\mathcal{M}(x)$ centered at x and defined as:

$$\mathcal{M}(x) = \left\{\hat{x} \in \mathbb{R}^3 : (\hat{x} - x)^T M(\hat{x} - x) \leq 1\right\}, \qquad (1)$$

where $M \in \mathbb{R}^{3\times 3}$ is a diagonal matrix given by

$$M = \mathrm{diag}\left(\frac{1}{{r_h}^2}, \frac{1}{{r_h}^2}, \frac{1}{{r_v}^2}\right),$$

with $r_h \geq r_v > 0$ defining the size of the ellipsoid in the horizontal plane and in the vertical direction. If $r_h = r_v$, then the ellipsoid reduces to a sphere of radius r_h, and proximity in the horizontal plane is weighted the same as that in the vertical direction. Typically, $r_h > r_v$ since vertical proximity between aircraft is considered in ATM to be less critical than horizontal proximity.

The complexity of air traffic within the airspace region $\mathcal{S}$ can be evaluated through the following first order and second order complexity measures.

The *first order complexity* $\mathrm{c}_1(x,t)$ at position $x \in \mathcal{S}$ within the time interval $[t, t+\Delta] \subseteq T$ is defined as

$$\mathrm{c}_1(x,t) := P\big(x^{A_i}(t) \in \mathcal{M}(x), \text{ for some } t \in [t, t+\Delta] \text{ and } i \in \{1, 2, \ldots, N\}\big) \qquad (2)$$

and represents the probability of at least one aircraft entering the ellipsoid $\mathcal{M}(x)$ within the time frame $[t, t+\Delta]$.

Note that $\mathrm{c}_1(x,t) = 0$ means that none of the existing aircraft will be inside the ellipsoid $\mathcal{M}(x)$ during the time interval $[t, t+\Delta]$. On the other hand, $\mathrm{c}_1(x,t) = 1$ implies that with certainty there will be at least one aircraft within $\mathcal{M}(x)$ at some time instant belonging to $[t, t+\Delta]$.

The *second order complexity* $\mathrm{c}_2(x,t)$ at position $x \in \mathcal{S}$ within the time interval $[t, t+\Delta] \subseteq T$ is defined as

$$\mathrm{c}_2(x,t) := P\big(x^{A_i}(t) \text{ and } x^{A_j}(t') \in \mathcal{M}(x) \text{ for some } t, t' \in [t, t+\Delta] \text{ and } i \neq j \in \{1, 2, \ldots, N\}\big) \qquad (3)$$

and represents the probability of at least two aircraft entering the ellipsoid $\mathcal{M}(x)$ within the time frame $[t, t+\Delta]$.

If $\mathrm{c}_2(x,t) = 0$, then there will be at most a single aircraft inside the ellipsoid $\mathcal{M}(x)$ within the time interval $[t, t+\Delta]$. Hence, at any time $t \in [t, t+\Delta]$, an aircraft passing through $\mathcal{M}(x)$ will not be sharing $\mathcal{M}(x)$ with any of the other N aircraft. If $\mathrm{c}_2(x,t) = 1$, then with probability 1, at least two aircraft will enter the ellipsoid $\mathcal{M}(x)$ during the time interval $[t, t+\Delta]$, though possibly not at exactly the same time.

By letting x vary over $\mathcal{S}$, one can define the *first order and second order complexity maps* of the airspace region $\mathcal{S}$ within the time frame $[t, t+\Delta]$ as follows:

$$\mathbb{C}_1(\cdot, t) : x \in \mathcal{S} \to \mathrm{c}_1(x,t)$$
$$\mathbb{C}_2(\cdot, t) : x \in \mathcal{S} \to \mathrm{c}_2(x,t).$$

Evidently, at any point $x \in \mathcal{S}$, the $\mathbb{C}_2$ map has a value smaller than or equal to the $\mathbb{C}_1$ map, since the corresponding events are nested. Higher order complexity measures and maps can also be defined according to a similar procedure.

Forming the complexity maps for different consecutive time intervals allows to detect congested areas (i.e., areas where multi-aircraft encounters with limited inter-aircraft spacing are likely to occur) in the time-space coordinates, and to identify surrounding areas where the traffic could be deviated. The presence of a region with a high value of the second order complexity implies a high likelihood that two or more aircraft will get close in time and space, hence having a conflict. Trajectories should be designed so as to reduce second order complexity.

More compact global information can be obtained according to the following procedure.

Let us parameterize the ellipsoidal region $\mathcal{M}(x)$ defined in (1) through a scaling factor $\rho > 0$ as follows:

$$\mathcal{M}_\rho(x) = \left\{\hat{x} \in \mathbb{R}^3 : (\hat{x} - x)^T M(\hat{x} - x) \leq \rho^2\right\}, \qquad (4)$$

so that, by varying ρ, the ellipsoidal region can be either squeezed ($\rho < 1$) or enlarged ($\rho > 1$). Denote the complexity measures associated with region $\mathcal{M}_\rho$ and parameterized by ρ as $\mathrm{c}_1^\rho(x,t)$ and $\mathrm{c}_2^\rho(x,t)$. Both $\mathrm{c}_1^\rho(x,t)$ and $\mathrm{c}_2^\rho(x,t)$ are increasing as a function of ρ.

Let

$$\rho_{\max}(t) := \sup\{\rho \geq 0 : \sup_{x \in \mathcal{S}} \mathrm{c}_2^\rho(x,t) \leq p_T\},$$

where p_T is some threshold value for the probability that two aircraft come close one to the other, and define

$$\rho^\star_{\max} := \sup_{t \in T} \rho_{\max}(t).$$

Then, one can take

$$\xi := \frac{1}{\rho^\star_{\max}}$$

as a synthetic indicator of complexity of the traffic during the time horizon T. Note that the extent of the available maneuverability space as measured by ξ will depend on both the local aircraft density and the traffic dynamic through the aircraft intent. Since uncertainty in the predicted aircraft position models possible deviations of the aircraft from their intended trajectory, ξ can be interpreted as a measure of robustness of air traffic to perturbations of the nominal situation.

B. *Complexity from a single aircraft perspective*

According to Definitions 2 and 3, complexity is evaluated from a global perspective as the probability of occupancy of a buffer zone surrounding a point by a certain number of the aircraft A_i, $i = 1, 2, \ldots, N$, that are present in the airspace region $\mathcal{S}$ (at least one aircraft for the first order complexity and at least two for the second order complexity). These complexity measures can be easily adapted to provide a measure of complexity from the perspective of an additional aircraft that is entering the airspace region $\mathcal{S}$ following some nominal trajectory. The resulting single-aircraft complexity measure can be interpreted as an indicator of the robustness of the aircraft nominal trajectory with respect to possible deviations of the other aircraft from their intended trajectory.

Suppose that an additional aircraft, say aircraft B, is entering $\mathcal{S}$ at time 0 following a nominal trajectory $\bar{x}^B : T \to \mathbb{R}^3$. The idea is to evaluate the complexity encountered by aircraft B along its nominal trajectory by making the buffer zone move along the trajectory of aircraft B and computing the probability that some of the other aircraft A_i, $i = 1, 2, \ldots, N$, will enter such moving zone. This leads to the following definition of single-aircraft complexity.

The complexity experienced by aircraft B along its nominal trajectory $\bar{x}^B : T \to \mathcal{S}$ within the time interval $[t, t + \Delta]$ is defined as:

$$\begin{aligned} \mathrm{c}_B(t) := P\big(&x^{A_i}(t) \in \mathcal{M}(\bar{x}^B(t)) \text{ for some } t \in [t, t+\Delta] \\ &\text{and } i \in \{1, 2, \ldots, N\}\big) \end{aligned} \tag{5}$$

Interestingly, if the time window $[t, t + \Delta]$ extends to the whole look-ahead time horizon T and the buffer zone reproduces the protection zone surrounding each aircraft, the single-aircraft complexity measure can as well be interpreted as the probability of aircraft B getting in conflict with another aircraft A_i within T. CD&R then becomes an integrable task in complexity evaluation.

According to a reasoning similar to that in Section II-A, based on the re-scaled ellipsoidal region (4) and the corresponding single-aircraft complexity function $\mathrm{c}^\rho_B : T \to [0, 1]$, we can introduce function $\rho_{\max,B} : T \to \mathbb{R}_+$ given by

$$\rho_{\max,B}(t) := \sup\{\rho \geq 0 : \mathrm{c}^\rho_B(t) \leq p_T\},$$

and define

$$\rho^\star_{\max,B} := \sup_{t \in T} \rho_{\max,B}(t).$$

$\rho^\star_{\max,B}$ is an index of robustness of the nominal trajectory of aircraft B. The larger is $\rho^\star_{\max,B}$, the more aircraft B is far from the other aircraft, both in time and in space, with high $(> 1 - p_T)$ probability, and, hence, the larger is the robustness of its trajectory to possible deviations of the other aircraft from their intent.

The quantity $\xi_B := \frac{1}{\rho^\star_{\max,B}}$ can then be taken as synthetic indicator of the air traffic complexity from the perspective of aircraft B during the time horizon T. Let ρ_{safe} denote the value of ρ such that $\mathcal{M}_\rho(x)$ represents the protection zone surrounding an aircraft positioned at x. If $\xi_B > \frac{1}{\rho_{\text{safe}}}$, then, some conflict can occur with probability $\geq p_T$ and the criticality of this conflict can be better assessed by computing, for instance, the earliest *conflict time*: $t^\star_B = \min\{t \geq 0 : \rho_{\max,B}(t) < \rho_{\text{safe}}\}$.

The introduced single-aircraft complexity measure (5) can be used by aircraft B to evaluate the maneuverability space surrounding its nominal trajectory and to eventually redesign its trajectory so as to improve its robustness. According to a similar perspective, in the works on trajectory flexibility [27], [28] it is suggested that, to achieve the aggregate objective of avoiding excessive ‘air traffic complexity’ in autonomous aircraft ATM, aircraft should plan their trajectory so as to preserve maneuvering flexibility to accommodate possible disturbances stemming, for example, from other traffic.

C. *Computational aspects*

If the errors affecting the prediction of the position of different aircraft are independent, each single aircraft contribution to complexity can be computed in isolation and then incorporated in the overall complexity measure. This means that the computational effort scales linearly with the number of aircraft, and that the impact of potential trajectory changes can be easily evaluated by removing the contribution of the original trajectory and introducing the one based on the new trajectory.

Although evaluating complexity does not require to analyze the interactions of the aircraft, the interaction between aircraft will still affect the complexity measure: if two aircraft are converging, complexity will be high in the area they are converging to; whereas if they are diverging, they will not cause complexity to be high in any position of the airspace.

III. Numerical Examples

A. *A 2D numerical example*

Consider a rectangular airspace region $\mathcal{S}$ where 6 aircraft are following a one-leg nominal trajectory from some starting to some destination position during the look-ahead time horizon $T = [0, t_f]$ with $t_f = 15$ minutes (min), while trying to keep at a minimum safe distance of 3 nautical miles (nmi).

The configuration of the aircraft nominal trajectories is shown in Figure 1, where starting positions are marked with $*$ and destination positions with $\diamond$.

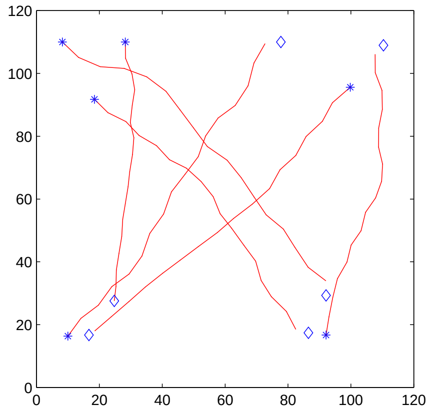


Fig. 1. Sample paths of 6 aircraft moving from starting position ($*$) to destination position ($\diamond$), while trying to keep at a distance 3 nmi.

The trajectories in this figure are obtained by implementing the decentralized resolution strategy introduced in [25], which accounts for the uncertainty affecting the aircraft motion according to a similar model for the aircraft predicted motion. According to this strategy, resolution maneuvers involve only heading changes.

In the 2D level-flight case, the ellipsoidal region $\mathcal{M}(x)$ in (1) for complexity computation becomes a circle of radius r_x. In this example we set $r_x = 1$ so that the scaling factor ρ becomes the actual radius of the re-scaled ellipsoidal region $\mathcal{M}_\rho(x)$ in (4) and $\rho_{\text{safe}} = 3$.

The global complexity of the considered air traffic system obtained with $p_T = 0.2$ is $\xi \simeq 3$, which means that aircraft are only guaranteed to keep at a distance of about 0.33 nmi, with probability greater than 0.8.

The complexity map $\Xi_2 : \mathcal{S} \to [0, 1]$ plotted in Figure 2 is obtained by condensing the timing information as follows:

$$\Xi_2(x) = \frac{1}{t_f} \int_0^{t_f} c_2^{\rho_{\text{safe}}}(x, t) dt. \qquad (6)$$

This map reveals that there are two main regions with some significant percentage of occupancy (larger than 10%): one in the upper left-hand-side, and the other close to the center of the airspace area $\mathcal{S}$.

$\Xi_2(x) = 0$ means that there will be at most a single aircraft within the ball of radius 3 nmi centered at x during the whole interval T. Aircraft passing through x such that $\Xi_2(x) > 0$ will be possibly involved in a conflict and the likelihood of this event grows with $\Xi_2(x)$. If $\Xi_2(x) = 1$, in particular, there will be more than 2 aircraft within the ball of radius 3 nmi centered at x during the whole interval T.

The earliest conflict time for both the two aircraft in the upper left-hand-side of the airspace area $\mathcal{S}$ is $t_B^\star = 2$ min. Indeed, the snapshot of the resolution maneuvers taken at time

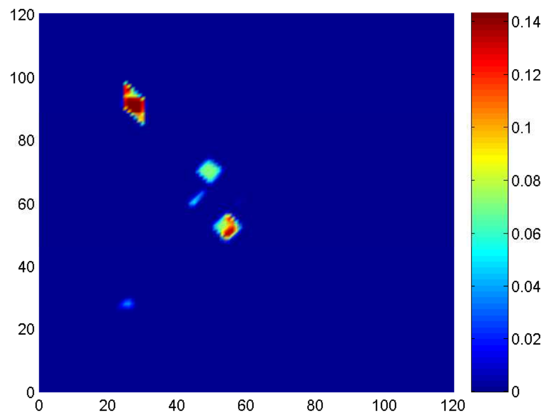


Fig. 2. Complexity map $\Xi_2 : \mathcal{S} \to [0, 1]$ obtained for $\rho_{\text{safe}} = 3$ nmi.

$t = 2$ min shows that this is the earliest time that a significant deviation action is taken by the decentralized solver and that it involves the two aircraft in the upper left-hand-side (Figure 3).

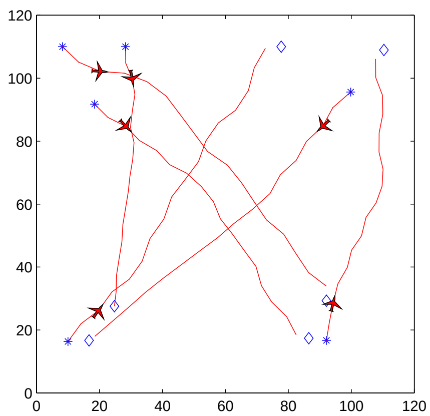


Fig. 3. Snapshot of the resolution maneuvers for the 6 aircraft system in Figure 1 at time $t = 2$ min.

In this example, the complexity map Ξ_2 has been evaluated at uniformly sampled grid points $x \in \mathcal{S} = [0, 120] \times [0, 120]$ with a grid size $\delta_{x_1} = \delta_{x_2} = 1$. In the numerical evaluation of the integral over T involved in (6), T has been uniformly sampled with $\delta_t = 1$. The short term look-ahead time horizon Δ has been set equal to 2 min, and the spectral densities $\sigma_1^{A_i} = 0.25$ nmi $\cdot$ (min)$^{-1/2}$ in the along track direction, and $\sigma_2^{A_i} = 0.2$ nmi $\cdot$ (min)$^{-1/2}$ in the cross track directions.

To reduce the computational load, one could adopt a variable spatial grid resolution, with a larger grid size far from the aircraft and a finer one close to the aircraft.

B. 3D numerical examples

In all the examples to follow, the uncertainty affecting the aircraft future positions is characterized through the spectral densities $\sigma_1^{A_i} = 0.25$ nmi $\cdot$ (min)$^{-1/2}$ in the along track direction, and $\sigma_2^{A_i} = \sigma_3^{A_i} = 0.2$ nmi $\cdot$ (min)$^{-1/2}$ in the cross track directions. The parameters r_h and r_v defining the ellipsoidal buffer region $\mathcal{M}(x)$ in (1) are set equal to $r_h =$ 5 nmi and r_v = 2000 feet (0.3291 nmi), and the look-ahead time horizon is $T = [0, 10]$ min.

a) Evaluating the airspace occupancy: Consider a 3-D airspace region with six aircraft. Each aircraft is moving at constant velocity along a straight line during the time interval T. The nominal trajectories of the aircraft are shown in Figure 4. Figure 5(a) shows the first order complexity map

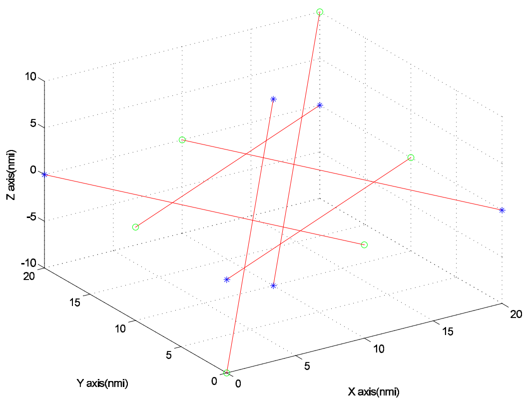
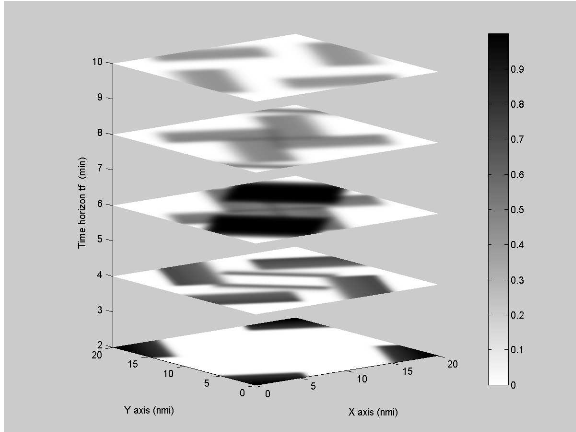


Fig. 4. Initial positions and nominal trajectories of the aircraft. '*' denote starting points, and 'o' denote the nominal position of the aircraft at time t= 10 min.

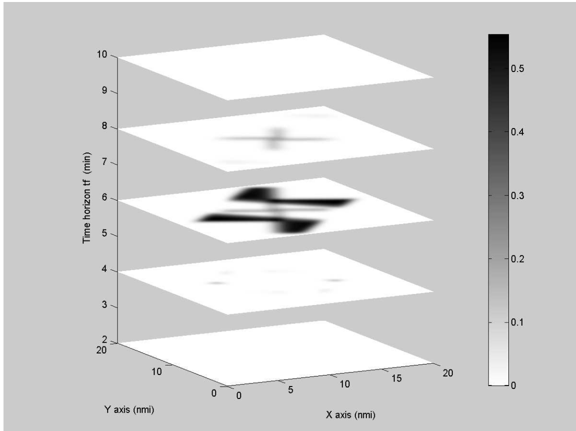
$\mathbb{C}_1(\cdot, t)$ for five different consecutive time frames $[t, t+\Delta]$ of length $\Delta = 2$ min, covering the whole time horizon T. For each time frame, the complexity map is evaluated at uniformly sampled points in the horizontal plane XY with an uniform gridding of size $\delta_x = \delta_y = 0.2$ nmi. Similarly, the complexity maps $\mathbb{C}_2(\cdot, t)$, $t = 0, 2, 4, 6, 8$, are plotted in Figure 5(b).

Figure 5(a) shows that the first order complexity is high initially in those zones that the aircraft are most likely to occupy in the XY plane. However, it can be seen from the second order complexity map that no two aircraft come close to each other in the first two time frames. During the time frame $[4, 6]$, there is a zone of high $\mathbb{C}_1$ and $\mathbb{C}_2$ complexity in the airspace. From the $\mathbb{C}_2$ map, we can deduce that there will be more than one aircraft during this interval in that zone. This is to be expected considering that the nominal trajectories take the aircraft close to each other around this time. Also, the drastic decrease in the $\mathbb{C}_1$ complexity map in successive subintervals indicates that the aircraft then move away from each other. Additional traffic entering the airspace should then better avoid crossing the XY plane in the time frame $[2, 4]$.

b) Evaluation of the maneuverability space: Suppose that an additional aircraft B is introduced at time $t = 0$ at the point $[8, 8, -2]^T$ nmi in the airspace where the six aircraft are flying. Aircraft B is following a straight line trajectory at the constant velocity $u^B = [2, 2, 2]^T$ nmi/min. Due to the presence of the six aircraft, aircraft B is not free to change its heading



(a) $\mathbb{C}_1$ complexity maps



(b) $\mathbb{C}_2$ complexity maps

Fig. 5. Complexity maps over the XY plane corresponding to different time frames $[t_s, t_f]$ of length 2 min in the time horizon $[0, 10]$ min.

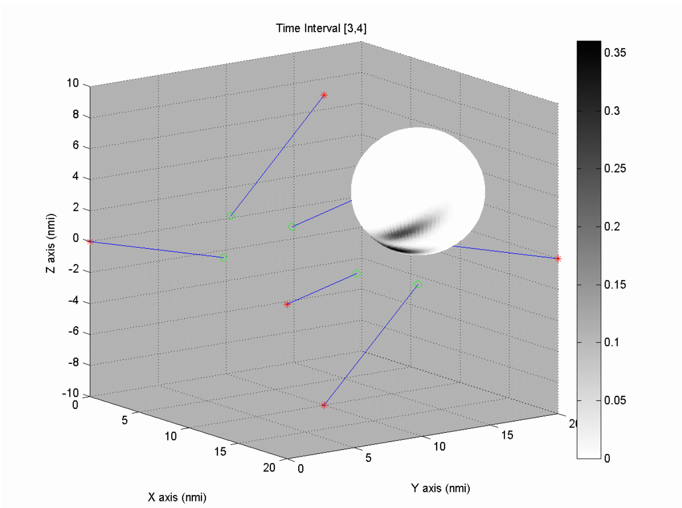


Fig. 6. Complexity experienced by aircraft B entering an airspace region with other six aircraft as a function of its heading at a point along its straight line trajectory.

arbitrarily during the flight. In Figure 6 we represent the complexity $c_B(t)$ defined in (5) as a function of the heading of aircraft B over a time frame of length $\Delta = 1$ minute at a sampled-point ($t = 3$) along the nominal trajectory of aircraft B. It can be observed that aircraft B faces a decrease in the amount of low-complexity prospective headings at some of these points, indicating that the airspace surrounding them is congested. This information might be used by aircraft B to find a minimal-complexity trajectory through the airspace.

c) Trajectory design: Suppose that an aircraft B has to enter the airspace region $\mathcal{S}$ at time 0 and reach some destination position at time t_f. The intended trajectory of the aircraft is a straight line traveled at constant velocity between its entry point and destination. However, this trajectory is not guaranteed to be of low-complexity due to the presence of other aircraft. Aircraft B can then choose a fixed number m of velocity changes at specified points in time $0 < t_1 < t_2 < \ldots < t_m < t_f$ to reduce the complexity along its trajectory.

Note that the way points $X_1, X_2, \ldots, X_m$ at which aircraft B changes its velocity completely specify its nominal multi-legged trajectory. Since the flight time between successive way points is given, the velocity of aircraft B within each interval can be determined from the way points $X_1, X_2, \ldots, X_m$ and the starting and destination positions.

We seek to find an optimal trajectory in the sense that both the deviation from the intended trajectory and the complexity $c_B(0)$ experienced by aircraft B within the flight time $[0, t_f]$ ($\Delta = t_f$) are minimized. We take the sum of the distances of the way points $X_1, X_2, \ldots, X_m$ from the intended trajectory as measure of the deviation d.

The complexity experienced by aircraft B along a multi-legged trajectory is not easy to compute since aircraft B does not have a constant velocity through out its flight, but only keeps its velocity constant during each interval $[t_i, t_{i+1}], i = 0, 1, \ldots, m$. However, we can over-approximate it by the sum of the complexities evaluated along the time intervals $[t_i, t_{i+1}]$ where aircraft B is flying at constant velocity v_i from X_i to X_{i+1}.

The problem of finding a suitable trajectory is then formulated as that of minimizing the cost:

$$J := d + \lambda \hat{c}_B(0), \quad (7)$$

which is a weighted sum of the deviation measure d and the over-approximation of the complexity measure $\hat{c}_B(0)$. A higher value of the weighting coefficient $\lambda > 0$ attributes a greater priority to the low-complexity requirement, and results in a less conflict-prone trajectory for appropriately chosen size of the buffer zone.

In Figure 7, an encounter situation is shown, where some aircraft B enters an airspace region at time 0 and aims at reaching a destination position at time $t_f = 10$, while keeping at some constant altitude. Four aircraft are already present in that region. Assume that aircraft B follows a level flight trajectory with one possible velocity change ($m = 1$) at $t_1 = 5$ out of a total flight time $t_f = 10$.

Figure 7 shows the optimal trajectory of aircraft B obtained by minimizing the cost function (7) with $\lambda = 1500$. The minimization was done using the MATLAB function *fmincon* and with the intended straight trajectory as the initial guess for the solution. The color map in Figure 7 represents the sum of the complexity measures within the time intervals $[0, 5]$ and $[5, 10]$ evaluated for different choices of the intermediate way point. The original intended trajectory is also plotted for comparison. It can be observed that the sum of the complexity measures along this trajectory is greater than that of the optimal one. A larger value of λ places more emphasis on the low-complexity requirement and thus leads to more aggressive maneuvering.

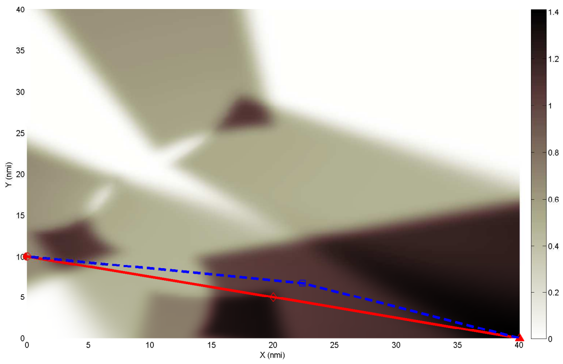


Fig. 7. Originally intended trajectory (solid line) and optimal trajectory (dashed line) of aircraft B flying from the starting position on the left to the destination position on the right ($\lambda = 1500$). The color map in the background represents the complexity along aircraft B trajectory as a function of the intermediate way point position.

IV. Conclusions

New generation ATM systems will have a decentralized and distributed control structure, with separation and management tasks shared between the ground and the flight-deck. This poses new and formidable challenges in the ATM system design. This work has addressed in particular the problem of assessing air traffic complexity in an autonomous aircraft context. Perspective applications have been described, such as onboard trajectory management and CD&R. The corresponding requirements on complexity metrics have been discussed.

A method to evaluate air traffic complexity on a mid-term horizon from a global perspective and from the perspective of a single aircraft has been described. A key feature of the method is that it accounts for the uncertainty in the prediction of the aircraft future positions. Possible applications have been illustrated through some simple numerical examples, which include using complexity maps and measures for detecting congested airspace areas and critical situations from the CD&R perspective, and the design of a trajectory for an additional aircraft crossing some airspace region.

Acknowledgment

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