New Trends in Air Traffic Complexity

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Abstract

This paper describes a novel approach for defining an air traffic complexity indicator, aimed at improving over the current operational definition that is only the number of aircraft present in a sector at a given time. The key concept in this work is that a level of disorder or equivalently unpredictibility is the right indicator. Our approach is relevant for many applications in the air traffic management area, like sector design ([2]), traffic assignment ([1, 3]) and will be very well suited for predicting complex situations in a future autonomous aircraft environment. Moreover, since this modelling takes explicitely into account the trajectories of the aircraft, either observed or planned, it fits perfectly with 4D based ATM like SESAR or NextGen.

1. Introduction to complexity

The ATM system has to cope with an increasing number of flights, pushing the capacity to its limits. As an example, the average daily traffic above Europe was 26286 flights/day, with a peak traffic demand in excess of 31000 flights [4]. Although delays are kept low, it is expected from the same reference that capacity has to be extended in the furture. Basically, two strategies can be devised : adapt the demand to capacity (slot-route allocation, collaborative decision making, ...) or adapt the capacity to the demand (Airspace design, 4D trajectory planning, autonomous aircraft, ...). The first approach can be used in the context of current ATM system, while innovative future designs will mainly follow the second strategy.

Currently, complexity of the traffic is measured only as an operational capacity : the maximum number of aircraft that ATC controllers are willing to accept is fixed on a per-sector basis and complexity is assessed by comparing the real number of aircraft with the sector capacity. It must be noted that under some circumstances controllers will accept aircraft beyond the capacity threshold while rejecting traffic at other times although the number of aircraft is well below the maximum capacity. This simple fact clearly show that capacity as a crude complexity metric is not enough by itself to fully account for the controller's workload. In order to better quantify the complexity, geometric features of the traffic have to be included.

1.1. Complexity vs workload

As previously stated, depending on the traffic structure, ATC controllers will perceive differently situations, even if the number of aircraft present in the sector is the same. Furthermore, exogenous paramaters like the workload history can be influential on the perceived complexity at a given time (a long period of heavy load will tend to reduce the efficiency of a controller). Some reviews of complexity in ATC have been completed, mainly from the controller's workload point of view [5, 11], and have recognized that complexity is related to both the structure of the traffic and the geometry of the airspace. This tends to prove that controller's workload has two facets :

- An intrinsic complexity related to traffic structure.
- A human factor aspect related to the controller itself.

While most complexity metrics tend to capture those effects within a single aggregate indicator, the purpose of this work is to design a measure of intrinsic complexity only since it is the most relevant metric for an highly automated ATC system (no human factors).

1.2. Dynamic density

The first complexity indicator incorporating structural considerations along with the simple number of aircraft is the "Dynamic Density" of Laudeman et al. from NASA [10]. The "Dynamic Density" is a weighted sum of the traffic density (number of aircraft), the number of heading changes (> 15 degrees), the number of speed changes (>0.02 Mach), the number of altitude changes (>750 ft), the number of aircraft with 3-D Euclidean distance between 0-25 nautical miles, the number of conflicts predicted in 25-40 nautical miles. These factors are summed together using weighting factors that were determined by showing different traffic scenarios to several controllers. B.Sridhar from NASA [12], has developed a model to predict the evolution of such a metric in the near future. Efforts to define "Dynamic Density" have identified the importance of a wide range of potential complexity factors, including structural considerations. However, the instantaneous position and speeds of the traffic itself does not appear to be enough to describe the total complexity associated with an airspace. A few previous studies have attempted to include structural consideration in complexity metrics, but have done so only to a restricted degree. For example, the Wyndemere Corporation proposed a metric that included a term based on the relationship between aircraft headings and dominant geometric axis in a sector [8]. The importance of including structural consideration has been explicitly identified in work at Eurocontrol. In a study to identify complexity factors using judgment analysis, Airspace Design was identified as the second most important factor behind traffic volume [9]. Histon, et, al. [6, 7] investigated how this structure can be used to support structure-based abstractions that controllers appear to use to simplify traffic situations.

2. Dynamical System Modeling

The main idea behind this new complexity indicator is to consider that observed traffic is a sample of an underlying flow. Within this framework, aircraft can be thought as particles moving in a stream, the geometry of the corresponding system being the most influencial factor for complexity. While this modelling is not directly related to a physical reality it can be noted however that in an highly automated context, the manoeuvres generated by an automatic conflict solver will adhere to this principle quite accurately. The first step for complexity computation is thus to find an interpolating vector field based on the past aircraft positions and intents. Given a set of N observations :

$$(t_i, x_i, v_i)_{i=1}^N$$

with t_i, x_i, v_i respectively the measurement time, the position and the velocity of the i-th aircraft, the interpolation problem is to find a smooth enough vector field : $X : \mathbb{R} \times \mathbb{R}^3 \to \mathbb{R}^3$ such that :

$$\forall i = 1 \ldots N, X(t_i, x_i) = v_i$$

Unfortunately, this problem is ill-posed since this requirement will not uniquely define X. A classical way to deal with this problem is to add an extra requirement on X, namely that it realizes the minimum of a so-called energy functional. Most of the time, this functional is obtained as a spatial L^2-norm of the form :

$$\int_{\mathbb{R}^3} \|LX(t,x)\|^2 dx$$

with L a differential operator. It has to be noted that variable t is not taken into account, so that the solution is expected to be a stationary vector field. A lot of works have been devoted to this kind of problem that falls in the category of interpolating vector splines ([14],[13]). It is well known that the optimal X satisfying the interpolation condition is of the form :

$$X(t,x) = \sum_{i=1}^{N} \lambda_i^t G(x, x_i)$$

Where $G(x,y)$ is the Green's function associated with the differential operator $L^t L$ and λ_i is a 3-dimensional vector coefficient. A convenient L for many applications in physics is the div-curl operator :

$$L = \alpha \nabla div + \beta \nabla curl$$

with α, β positive real numbers tuning the respective contributions of the div and the curl part : a high α will yield to a nearly constant divergence while a high β will enforce constant curl. It has to be noted that if $\alpha = \beta$, the criterion degenerates to a square laplacian operator, which has the nice property of being diagonal, so that the vector problem reduces to 3 one dimensional problems. While this particular approach has been exploited in some of our past works, it is more relevant for our application to allow time-varying interpolating vector fields : in the following $X : \mathbb{R} \times \mathbb{R}^3$ will be a time dependent vector field with time as first coordinate. The energy functional is chosen to be :

$$E(X) = \int_{\mathbb{R}} \int_{\mathbb{R}^3} \left\| \frac{\partial X}{\partial t}^2 \right\| + \mu \|\nabla(\mathbf{div} + \mathbf{curl})X\|^2 \, dt dx$$

where $\mu > 0$ tunes the relative part of time variation and div-curl criterion. Intuitively, X will be the most stationary field with minimal div-curl variation. The differential operator associated with this variational problem is given by :

$$P = \frac{\partial^2}{\partial t^2} - \mu\Delta^2$$

An elementary solution can be found in $\mathscr{S}'$ as :

$$\mathbf{pf}\left(\frac{1}{\sqrt{\mu}\|\xi\|^2}\right) exp(-|t|\sqrt{\mu}|\xi|^2)$$

where the previous expression is the fourier transform in $\mathscr{S}'$, the space a rapidly decreasing smooth functions, with respect to the spatial coordinates of the elementary solution. It turns out that this distribution is indeed obtained from a L^1_{loc} (integrable on compact sets) mapping so that the inverse Fourier transform can be obtained readily by integration as :

$$p(t,x) = \frac{1}{8\pi^3}\int_{\mathbb{R}^3} \frac{1}{\sqrt{\mu}\|\xi\|^2} exp(-|t|\sqrt{\mu}\|\xi\|^2) exp(i\langle x,\xi\rangle)d\xi$$

by Fubini's theorem and polar change of variables it becomes :

$$p(t,x) = \frac{1}{8\pi^3\sqrt{\mu}}\int_{\mathbb{R}} exp(-|t|\sqrt{\mu}r^2) \int_{\mathscr{S}^2} exp(i\langle x, rs\rangle)d\sigma(s)dr$$

with $d\sigma$ the solid angle measure. Using a polar parametrization of the unit sphere :

$$p(t,x) = \frac{1}{8\pi^3\sqrt{\mu}}\int_{\mathbb{R}} exp(-|t|\sqrt{\mu}r^2) \int_0^{2\pi}\int_0^{\pi} exp(i\|x\|r\cos\theta)\sin\theta d\theta d\phi dr$$

and finally :

$$p(t,x) = \frac{1}{2\pi^2\sqrt{\mu}} \int_{\mathbb{R}} exp(-|t|\sqrt{\mu}r^2)\frac{\sin\|x\|r}{\|x\|r}dr$$

by Parseval equality :

$$p(t,x) = \frac{1}{4\|x\|\pi^2}\sqrt{\frac{\pi}{|t|\sqrt{\mu}}} \int_{-\frac{\|x\|}{2\pi}}^{\frac{\|x\|}{2\pi}} exp\left(-\frac{\pi^2\omega^2}{|t|\sqrt{\mu}}\right)d\omega$$

so in terms of error function **erf** :

$$p(t,x) = \frac{1}{4\pi^2\|x\|}\mathbf{erf}\left(\frac{\|x\|}{2\sqrt{|t|\sqrt{\mu}}}\right)$$

Like the heat kernel, the fundamental solution obtained is singular for $x = 0, t = 0$. Before introducing a way of obtaining computable kernels, it is interesting to look at some properties of $p(t,x)$:

- Error function is rapidly converging to 1 when its argument goes to $+\infty$. In fact, $erf(4)$ is equal to 1 at machine precision when computing with single precision float numbers. It thus means that contributions in the interpolating field coming from aircraft far from the evaluation point are very close to being proportional to a standard $1/\|x\|$ potential function.

- Unlike classical $div - curl$ splines obtained from the first non time-varying functional, these new splines are decreasing at infinity, allowing to drop contributions of far enough aircraft (strictly speaking, this is the case even for the div-curl splines, but decrease is due to cancellations between contributions and is slower).

The main issue in using the kernel p is that the reconstructed field :

$$X(t,x) = \sum_{i=1}^{N}\lambda_i p(t-t_i, x-x_i)$$

is singular at observation points (t_i, x_i, v_i). A simple way of avoiding this is to change the interpolation criterion : instead of enforcing that $X(t_i,x_i) = v_i, i = 1\ldots N$, we impose that the mean value of the field in a neighborood of x_i has to be equal to v_i. In order to allow simple computation of the interpolating splines in closed-form, it has been chosen to compute the mean of X with respect to a gaussian density, so that the interpolation condition becomes :

$$\frac{1}{(2\pi\sigma)^{3/2}}\int_{\mathbb{R}^3} e^{-\frac{\|x-x_i\|^2}{2\sigma^2}} X(t,x)dx = v_i$$

σ is the standard deviation for the gaussian density and is a tuning parameter for the shape of the field. Solving the functional equation with this new interpolation criterion yields a non singular kernel :

$$p(t,x) = \frac{\sigma}{\|x\|} erf\left(\frac{\|x\|}{\sigma\sqrt{2+|t|}}\right)$$

Here again, it worth notice that the asymptotic behaviour of p is to revert to a classical $1/\|x\|$ potential.

3. Lyapunov exponents

The metric chosen for complexity computation relies on a measure of sensitivity to initial conditions of the underlying dynamical system called Lyapunov exponents. In order to figure out what Lyapunov exponents are, let consider a point and look at its evolution when transported by the dynamical system.

Let x be fixed (initial point) and let ϕ be a point trajectory of the dynamical system associated to the vector field X given by :

$$\phi(t,x) = x + \int_0^t X(s, \phi(s,x))ds \tag{3.1}$$

Assume now that trajectory is disturbed by a small perturbation ε, we have :

$$\phi(t, x+\varepsilon) = \phi(t,x) + \mathbf{D}_x\phi(t,x)\cdot\varepsilon + o(\|\varepsilon\|)$$

where $\mathbf{D}_x\phi(t,x)$ is the differential of the vector field at x that satisfies :

$$\frac{\mathbf{D}_x\phi(t,x)}{dt} = \mathbf{D}_xX(\phi(t,x))\cdot\mathbf{D}_x\phi(t,x) \tag{3.2}$$

The Lyapunov exponents are closely related to the singular values of the matrix $\mathbf{D}_x\phi(t,x)$ and can be thought as local shear values for the dynamical system.

When Lyapunov exponents are high, the trajectory of a point under the action of the dynamical system is very sensitive to initial conditions (or to the parameters on which the vector field may depend), so that situation in the future is unpredictable. On the other hand, small values of the Lyapunov exponents mean that the future is highly predictable (expected to be comfortable for a controller). *So, the Lyapunov exponent map determines the area where the underlying dynamical system is organized. It identifies the places where the relative distances between aircraft do not change with time (low real value) and the ones where such distance change a lot (hight real value).*

Let us now describe the practical procedure for computing complexity maps.

First of all, the optimal dynamic div-curl approximation for the observed trajectories is computed, based on the defining equations. That step requires a linear system solving.

The second step computes the second derivatives matrix $\mathbf{D}_x\phi$ at each point of the grid for ϕ trajectory starting at x. This is done by solving the differential equation 3.2 with a Runge-Kutta integrator. The complexity value at point x is then obtained by averaging Lyapunov exponents over the time (assuming sampling times $(t_1,\ldots,t_n)$:

$$\kappa(x) = \frac{1}{n}\sum_{i=1}^{i=n}\|\mathbf{D}_xX(\phi(t_i,x))\|_2 \tag{3.3}$$

The figure 1 shows an example of Lyapunov exponents map for which full organized miles in trail trajectories (from south west to north east) cross two random traffic situations. This figure shows clearly a complexity valley on the miles in trail direction. This organization may have been detected even if the miles in trail trajectories would have been structured on a curve trajectory. That is the strong point of this metric: **Lyapunov exponents are able to identify any kind of trajectory organization**. The next figure is an example of a complexity map computed on real traffic over France : Major airports are clearly visible as high complexity areas. It has to be noted that on this picture, organized traffic is classified as low complexity as expected.

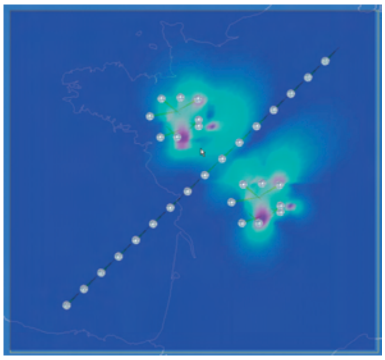

Figure 1. Miles in trail trafic between disordered areas

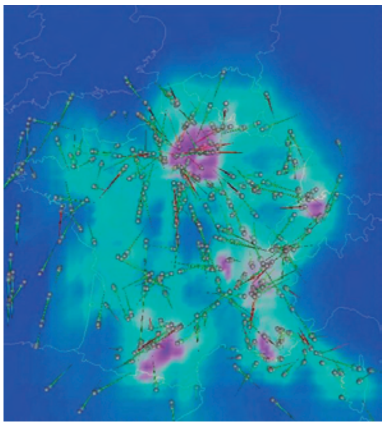

Figure 2. Complexity over France

4. Computational issues

Computing Lyapunov exponents amounts to integrate a differential equation. Under standard smoothness assumptions on X :

$$\frac{d}{dt}D_x\phi(t,x) = D_xX(t,\phi(t,x))D_x\phi(t,x)$$

giving a linear cocyle equation. Putting

$$A(t,x) = D_x\phi(t,x)$$

the cocyle equation is :

$$\frac{d}{dt}A(t,x) = D_xX(t,\phi(t,x))A(t,x)$$

which is a linear differential equation in matrix function $A(t,x)$. Nearly all standard algorithms can be used for that purpose but one quickly realize that the problem is far from being well conditionned. In fact, most of the time it is easy to obtain the flow $\phi(t,x)$ with good accuracy but by construction $A(t,x)$ tends to grow exponentially fast in some directions (corresponding to positive Lyapunov exponents) and to decay exponentially fast in others (corresponding to negative Lyapunov exponents) : the condition number is thus increasing again exponentially. Then it will be extremely inaccurate to compute Lyapunov exponent merely by integrating the linear cocyle equation : one needs a kind of rescaling to recover good numerical properties. There is a abundant literature on the subject, however all methods fall into one of two categories : spatial integration or temporal integration. Direct application of the definition gives the second approach while ergodic theorem gives the first one. Both have advantages and drawbacks :

- Spatial integration is efficient and free from slow convergence phenomenon occuring sometimes in temporal integration. However, in order to use the ergodic theorem, one must find an invariant measure. Most of the time, it has to be done by covering algorithms.

- Temporal integration can suffer from slow convergence. Moreover, it requires periodic rescaling to avoid numerical problems.

In our complexity application, only the temporal approach has been tested.

4.1. Algorithms based on differential geometry

All the machinery used in this part is borrowed from a paper from T.J. Bridges and S.Reich. Basically, the trick is to use the Perron lemma to compute the k largest Lyapunov exponents : the matrix $A(t,x)$ is factored as $A(t,x) = Q(t)R(t)$ with $Q(t)$ orthogonal and $R(t)$ upper triangular, both matrices beeing smooth in t. The Lyapunov exponents are directely related to the diagonal elements of $R(t)$. The problem is then to update continuously a QR factorisation (or a polar decomposition which is very similar except that the right hand side matrix is symetric instead of being upper triangular). The differentiel equation satisfied by Q can be established first by noticing that the matrix :

$$M(t) = Q^h(t)A(t)Q(t) - Q^h(t)\dot{Q}(t)$$

is upper triangular so adding its conjugate yields a symetric matrix with the same coefficients except on the diagonal where there are doubled. However :

$$M^h(t) + M(t) = Q^h(t)(A(t) + A^h(t))Q(t)$$

since $Q^h(t)\dot{Q}(t) + \dot{Q}^h(t)Q(t) = 0$ (Q is unitary so the derivative of $Q^h(t)Q(t) = Id$ vanishes). So $M(t)$ can be otained without $\dot{Q}(t)$. Now, simply use the relation :

$$\dot{Q}(t) = A(t)Q(t) - Q(t)M(t)$$

This differential equation can be solved readily by standard runge-kutta integrator. Orthogonality of Q is nevertheless hard to preserve, so that efficient implementations require specfic algorithms. A natural approach is to use integrators working on the Stiefel manifold on which Q lives. Another one is to use a parametrization of Q using elementary rotations (we need 6 such rotations for a 3x3 matrix). Regardless of the method used, computing a complexity map on large airspaces is still a challenge. Many improvements have been made that allow now to produce country-sized complexity time in about 10 minutes with several thousands aircraft involved. Investigation on paralell computing and more specifically on GPU computing is ongoing : the expected increase in computation speed will probably allow real-time computations even for relatively large airspaces.

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