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Hybrid Optimal Control for Aircraft Trajectory Design with a Variable Sequence of Modes ⋆

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Abstract: The problem of aircraft trajectory planning is formulated as a hybrid optimal control problem. The aircraft is modeled as a switched system, that is, a class of hybrid dynamical systems. The sequence of modes, the switching times, and the inputs for each mode are the control variables. An iterative bi-level optimization algorithm is employed to solve the optimal control problem. At the lower level, given a pre-defined sequence of flight modes, the optimal switching times and the input for each mode are determined. This is achieved by extending the continuous state to include the switching times and then solving a conventional optimal control problem for the extended state. At the higher level, the algorithm modifies the mode sequence in order to decrease the value of the cost function. We illustrate the utility of the problem formulation and the solution approach with two case studies in which short horizon aircraft trajectories are optimized in order to reduce fuel burn while avoiding hazardous weather.

Keywords: Optimal Control, Hybrid Systems, Aircraft Operations, Aircraft Trajectory Design

1. INTRODUCTION

A substantial change in the current Air Traffic Management (ATM) paradigm is needed in order to improve its capacity, efficiency, environmental impact, and flexibility. This need for paradigm shift is being addressed in Europe within the framework of Single European Sky ATM Research (SESAR) [1], and in the United States within the Next Generation (NextGen) of air transportation system [2]. Currently, ATM imposes certain trajectory restrictions in order to guarantee safety. Some of these restrictions result in non-minimal fuel consumptions and consequently higher operative costs and emissions. A new concept on 4D trajectory planning, referred to as Trajectory Based Operations (TBO), is being developed in order to allow optimization of individual aircraft trajectories while ensuring that the airspace is used safely and efficiently.

We present an approach to the problem of finding aircraft fuel optimal trajectories in the presence of wind and weather storms. In our approach different flight modes and operational procedures are combined in order to formulate and solve an optimal control problem. The coupling of the discrete flight modes with the continuous aircraft dynamics results in a hybrid system, Tomlin et al. (1998); Glover and Lygeros (2004); Ross and D'Souza (2005).

The flight dynamics of an aircraft intrinsically has the characteristics of a switched system, that is, a hybrid system in which there are no discontinuous state jumps at the switching times. Switches between flight modes can be autonomous or controlled. Autonomous switches take place when the continuous state hits prescribed regions of the state space. For example, when the aircraft reaches a prescribed altitude a switch from climb mode to cruise mode of flight would occur. Controlled switches occur in response to a control input. The control input for a hybrid system has both a discrete component, which is the sequence of the discrete modes, and two continuous components, which are the duration of each mode and the input for each mode. In general, finding all components of the solution to a hybrid optimal control problem is hard because it is difficult to determine the optimal sequence of discrete modes in a computationally efficient manner.

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[1] SESAR Master Plan: http://www.eurocontrol.int/sesar
[2] NextGen. Concept of operations for the next generation air transport system, 2007: http://www.jpdo.gov/library/NextGen_v2.0.pdf

Hybrid optimal control problems with a known mode sequence have been frequently formulated in aerospace engineering as multi-phase problems, Huntington and Rao (2005); Jorris and Cobb (2008). Here, a phase refers to a mode of the hybrid dynamics. The multi-phase problems are usually solved using pseudospectral methods and some have been applied to spacecraft missions, Benson (2004); Ross and Fahroo (2004); Thorvaldsen et al. (2005). However, none of the above research has focused on commercial aircraft. Another approach to solve a hybrid optimal control problem with a known mode sequence was presented in Soler et al. (2010a) and two applications to commercial aircraft trajectory optimization were derived, in which vertical profiles, Soler et al. (2010c), and 3D profiles, Soler et al. (2010b), were optimized with respect to fuel consumption. In this approach the hybrid optimal control problem was converted into an equivalent conventional optimal control problem by making the unknown switching times part of an extended continuous state using a method similar to that in Žefran (1996); Xu and Antsaklis (2004).

Although the sequence of modes in the current paradigm of flight is fixed a priori by the pilots or the air traffic controllers, variation of this sequence may improve the objective, which is for instance minimization of fuel consumption. In addition, given some stochastic phenomenon such as storms, there may be a need to update the original sequence of flight modes in order to tackle the uncertainties in a safe and optimal way. To address these problems, algorithms for hybrid optimal control with a variable mode sequence need to be used. There has been much previous work on optimal control of a hybrid system with an unknown mode sequence, for example, see Branicky et al. (1998); Shaikh and Caines (2007); Axelsson et al. (2008); Zhang et al. (2009); González et al. (2010). The previous studies do not consider the application of optimal aircraft trajectory design. Motivated by the possible gains of varying the flight mode sequence, this study applies a hybrid optimal control algorithm to address commercial aircraft trajectory optimization with a variable mode sequence.

The paper is organized as follows: In Section 2 the hybrid optimal control problem is defined and our algorithm is described. In Section 3 the equations of motion for the flight modes considered are provided. In Section 4 we apply the proposed algorithm to two aircraft trajectory design problems. Finally, in Section 5 conclusions and directions for future research are stated.

2. HYBRID OPTIMAL CONTROL APPROACH

Aircraft motion has the characteristic of a switched system due to different flight modes. A switched system is composed of a set of dynamical systems described by differential equations

$$\dot{x}(t) = f_q(x(t), u(t)),\ q \in Q = \{1, 2, \ldots, N_q\}, \qquad (1)$$

where $x \in \mathbb{R}^n$ represents the continuous state and Q represents the discrete modes of the system. The input u belongs to set of functions $\{u : [0, \infty) \to U \,|\, u \text{ is measurable}\}$, with $U \subset \mathbb{R}^m$ a compact set.

A switching sequence σ is defined as the timed sequence of active dynamical systems, referred to as modes, as follows:

$$\sigma = [(t_I, q_0), (t_1, q_1), \ldots, (t_N, q_N)], \qquad (2)$$

where $N \in \mathbb{N}_+$, $t_I \le t_1 \le \cdots \le t_N \le t_F$, and $q_i \in Q$ for $i = 0, 1, \ldots, N$. The pair (t_i, q_i) indicates that at time t_i the dynamics change from mode q_{i-1} to q_i. Consequently, in the time interval $[t_i, t_{i+1})$ the system evolution is governed by the vector field f_{q_i}.

The hybrid optimal control problem can be stated as follows: consider the switched system (1) whose state and inputs are subjected to a set of N_c constraints for $t \in [t_I, t_F]$ given as

$$h_j(x(t), u(t)) \le 0,\ j = 1, 2, \ldots, N_c. \qquad (3)$$

Find a switching sequence σ and an input u, that fulfill (1), the path constraints (3), and minimize the objective

$$J(\sigma, u) = \phi(x(t_F)) + \int_{t_I}^{t_F} L(x(t), u(t))dt. \qquad (4)$$

The term L is referred to as the Lagrangian running cost and ϕ as the final cost. The final time t_F may be fixed in advance or may be an optimization parameter. We assume that f_q, h_j, ϕ, and L are Lipschitz and differentiable and their derivatives are also Lipschitz in their arguments. To address this problem we consider the following iterative bi-level algorithm, González et al. (2010):

Bi-Level Hybrid Optimal Control Algorithm

Stage 1: Given a mode sequence, $(q_0, q_1, \ldots, q_N)$, find the optimal continuous input u, the optimal switching times $(t_1, \ldots, t_N)$, and the final time t_F. From the switching times obtain the switching sequence σ.

Stage 2: Find a new sequence $\tilde{\sigma}$ as a result of insertion of a new mode into the original sequence σ, which would decrease the cost. If such mode cannot be found, stop. Else, repeat Stage 1 using $\tilde{\sigma}$.

In general, the algorithm leads to suboptimal solutions since only certain variations of the discrete mode sequence, that is, mode insertions, are considered. However, it provides a systematic and computationally efficient manner of examining candidate mode sequences. Next, we describe our approach for solving each stage of the algorithm.

2.1 Solving Stage 1

In this stage of the algorithm the number of switches and the sequence of discrete modes are known. The idea is to convert the hybrid optimal control problem with known mode sequence but unknown switching times into an equivalent optimal control problem with an extended state and known switching times, Žefran (1996); Xu and Antsaklis (2004). Without loss of generality, we can assume $t_I = 0$. Define $t_0 = 0$ and $t_{N+1} = t_F$. First, we introduce new state variables x_{n+1+i} corresponding to the switching times t_i, $i = 0, \ldots, N$ and with dynamics $\dot{x}_{n+1+i} = 0$. We then introduce a new independent variable $\tau \in [0, N+1]$. The relation between τ and t is as follows:

$$t = \begin{cases} x_{n+1}\tau & \tau \in [0, 1] \\ x_{n+i+1}(\tau - i) - x_{n+i}(\tau - i - 1) & \tau \in [i, i+1], \end{cases}$$

where in the above, $1 \le i \le N$. Let $(\cdot)'$ denote the derivative of $(\cdot)$ with respect to the new independent variable τ. Next, define $\hat{f}_{q_i}$ as

$$\hat{f}_{q_i} = \begin{cases} x_{n+1} f_{q_0} & i = 0 \\ (x_{n+i+1} - x_{n+i}) f_{q_i} & i = 1, \ldots, N \end{cases}$$

The reformulated optimal control problem in the extended state is as follows:

$$\min \phi\left(x(N+1)\right) + \int_0^{N+1} L(x(\tau), u(\tau))d\tau \tag{5}$$

subject to

$$x'(\tau) = \hat{f}_{q_i}(x(\tau), u(\tau)),\ \tau \in [i, i+1],\ i = 0, \ldots, N$$
$$x'_{n+1+i}(\tau) = 0,\ i = 0, \ldots, N$$
$$h_j(x(\tau), u(\tau)) \leq 0,\ j = 1, \ldots, N_c$$

Let $\hat{x} = [x_1, \ldots, x_n, x_{n+1}, \ldots, x_{n+1+N}]^T$ denote the extended state. In the optimal solution of the above problem, $(\hat{x}^*, u^*)$, the last $N+1$ components of the state $\hat{x}^*$ are the N optimal switching times and the final time. Since the duration of each mode is constant with the introduced transformation, the new equivalent problem is a conventional optimal control problem, that is, an optimal control problem without unknown switching times. In order to solve the optimal control problem above a collocation method may be used, Hargraves and Paris (1987); Herman and Conway (1996). Collocation methods have been widely used for solving optimal control problems in the aircraft and aerospace applications due to their computational efficiency, see, for instance, Betts (1998, 2001). The main drawback on such methods is that they only ensure local optimality for the discretized problem and there is a need to verify optimality a posteriori, Yan et al. (2000).

2.2 Solving Stage 2

The difficulty with determining the discrete modes in a hybrid optimal control problem is that the trajectories obtained from variations of a given mode sequence may be far from the nominal one and hence not comparable in a computationally efficient manner. However, if one considers a variation in which the modified sequence differs from the original one by modes whose durations are sufficiently small, one can then analyze the differences in the resulting trajectory and cost function. This idea was introduced in Axelsson et al. (2008) for autonomous unconstrained switched systems. It was extended to non-autonomous constrained switched systems in González et al. (2010). We give a high-level description of this approach. For details please refer to González et al. (2010).

Consider the switched system described in Section 2. Define insertion of mode $\alpha \in Q$ at a time $\hat{t} \in [t_I, t_F]$ for a duration $\lambda > 0$ as a modification to the mode sequence σ and input u such that the subsystem $f_\alpha(x, \hat{u})$ is active in the interval $(\hat{t} - \frac{\lambda}{2}, \hat{t} + \frac{\lambda}{2})$. This mode insertion can be characterized by $\eta = (\alpha, \hat{t}, \hat{u}) \in Q \times [t_I, t_F] \times U$. The resulting discrete and continuous inputs are denoted as $(\hat{\sigma}, \hat{u})$. Let $\rho^\eta : \lambda \to (\hat{\sigma}, \hat{u})$ be a function that describes this insertion. Now, we can consider variation of the cost with respect to this mode insertion in the limit as λ approaches zero. To do this, define the directional derivative

$$\left.\frac{dJ\big(\rho^{(\eta)}(\lambda)\big)}{d\lambda}\right|_{\lambda=0} = \lim_{\lambda \downarrow 0} \frac{J\big(\rho^{(\eta)}(\lambda)\big) - J(\sigma, u)}{\lambda}.$$

If the above directional derivative is negative, then, for sufficiently small insertion duration λ the mode insertion would decrease the cost. Additionally, we need to ensure that after the mode insertion the constraints will not be violated. Here, to ensure existence of directional derivative of the path constraints, we assume they are of the form $h_j(x(t)) \leq 0,\ j = 1, \ldots, N_c$. Define $\psi(\sigma, u) = \max h_j(x(t))$, where the maximum is with respect to $j = 1, 2, \ldots, N_c$ and $t \in [t_I, t_F]$. To ensure feasibility of constraints it is sufficient to have $\left.\frac{d\psi(\rho^{(\eta)}(\lambda))}{d\lambda}\right|_{\lambda=0} < 0$ whenever $\psi(\sigma, u) = 0$. Note that the directional derivatives are well-defined, Polak (1997), and analytical expressions for them are derived in González et al. (2010).

Based on the analysis for the variations of the cost and the constraints with respect to the mode insertion Stage 2 of the algorithm selects a mode α, an insertion time $\hat{t}$, and an input $\hat{u}$ which minimizes the directional derivative of cost with respect to mode insertion while ensuring that constraints remain feasible. The resulting mode insertion characterized by η can be stated as the solution of the following optimization problem:

$$\min_\eta \max \left\{ \left.\frac{dJ(\rho^{(\eta)}(\lambda))}{d\lambda}\right|_{\lambda=0}, \psi(\sigma, u) + \left.\frac{d\psi(\rho^{(\eta)}(\lambda))}{d\lambda}\right|_{\lambda=0} \right\}.$$

To implement Stage 2, one would solve the optimization above for every candidate mode $\alpha \in Q$ over the variables $\hat{t}$ and $\hat{u}$. If the objective function is negative for some η then a mode insertion which would decrease the cost while maintaining constraint feasibility exists and can be found as a solution of the above optimization.

Once the mode to be inserted and its associated insertion time are determined, a new mode sequence is obtained. Then, Stage 1 is used to optimize the switching times and continuous input given this new mode sequence. The iteration between Stage 1 and 2 is continued until no mode insertion can be found to decrease the cost. For detailed analysis on convergence of the algorithm please see González et al. (2010).

3. AIRCRAFT DYNAMICS

In order to design fuel optimal aircraft trajectories, it is common to consider a 3 Degree Of Freedom (DOF) dynamic model that describes the point variable-mass motion of the aircraft over a flat earth model. Wind is also included due to its considerable effects on fuel consumption. The equations of motion of the aircraft are

$$\begin{aligned} m\dot{V} &= T - D - mg\sin\gamma \\ mV(\dot{\chi}\cos\gamma\cos\mu - \dot{\gamma}) &= mg\sin\mu\cos\gamma \\ mV(\dot{\chi}\cos\gamma\sin\mu + \dot{\gamma}) &= L - mg\cos\mu\cos\gamma \\ \dot{x} &= V\cos\gamma\cos\chi + V_{windx_h} \\ \dot{y} &= V\cos\gamma\sin\chi + V_{windy_h} \\ \dot{h} &= V\sin\gamma + V_{windz_h} \\ \dot{m} &= -T\eta \end{aligned} \tag{6}$$

In the above the three kinematic equations are expressed in a ground based reference frame, while the three dynamic equations are expressed in an aircraft-attached reference frame. The states are: V, χ, γ referring to the true airspeed, heading angle, and flight path angle respectively; x, y, h referring to the aircraft 3D position; and m its mass.

V_{windx_h}, V_{windy_h}, V_{windz_h} are components of the wind, T is the thrust, and μ is the bank angle. Lift $L = C_L S\hat{q}$ and drag $D = C_D S\hat{q}$ are the components of the aerodynamic force, where S is the reference wing surface area and $\hat{q} = \frac{1}{2}\rho V^2$ is the dynamic pressure. A parabolic drag polar $C_D = C_{D0} + KC_L^2$ and a standard atmosphere are assumed. In general, the bank angle μ the engine thrust T and the coefficient of lift C_L are the inputs. The path constraints are based on aircraft's flight envelope and can be found in BADA manual, Nuic (2005). For further details on aircraft dynamics see, for instance, Hull (2007).

3.1 Flight Modes

A 3D flight plan can be subdivided into a sequence modes pertaining to flights in a vertical or horizontal plane. In both cases, we consider a symmetric flight, that is, we assume there is no sideslip and all forces lie in the plane of symmetry of the aircraft. Also, we neglect the vertical component of wind V_{windz_h} due to its low influence.

3.1.1 Horizontal 2D flight In the horizontal flight $\dot{h}$ and γ are set to zero. Consequently, the following algebraic constraint is now present: $L = mg\cos\mu$. We consider two modes in the horizontal flight. In mode 1, *control speed*, it is assumed that the aircraft fly with constant heading but with variable speed. The engine thrust T is the input and the bank angle μ is set to zero. In mode 2, *control heading*, the speed is set to a constant value and the input is μ.

3.1.2 Climb/Descent flight In this mode the bank angle μ is set to zero. Without loss of generality, we consider $\chi = 0$, $\dot{y} = 0$. The engine thrust and the lift coefficient are the inputs of the aircraft, that is, $u = (T, C_L)$. We refer to this mode as mode 3, the *control altitude* mode.

4. AIRCRAFT TRAJECTORY OPTIMIZATION

We consider aircraft en-route portion of the flight. In general, in this portion of the flight aircraft fly straight line segments connecting waypoints. For the purpose of avoiding hazardous weather, the aircraft may be required to deviate from their nominal paths. In terms of air traffic control, these deviations are characterized by maneuvers which may consist of heading, speed, or altitude changes. In our analysis, we consider flight maneuvers as modes of the switched system and consider maneuvers characterized by the three modes of *control speed*, *control heading*, and *control altitude* as introduced in the previous section. These types of maneuvers are routinely used in the current air traffic control practice since they are easily communicated to the pilots and are easily implemented by autopilots, Tomlin et al. (1998).

We assume a region of airspace is unsafe to fly through due to weather storms. In the weather forecast data, storms may be characterized as regions with high values of Vertically Integrated Liquid (VIL), Wolfson et al. (2004). Although the VIL forecast are provided for a gridded airspace, a minimum-volume bounding ellipsoid can be used to capture these no-fly zones as obstacles, Kamgarpour et al. (2010). Given a nominal path for the aircraft and an obstacle along the path we formulate the problem of obstacle avoidance as a hybrid optimal control problem. In this set-up, a mode (or equivalently a maneuver) needs to be inserted in the current flight plan in order to avoid the obstacle while minimizing objectives.

For the following two case studies, we solved the trajectory design problem using the bi-level algorithm of Section 2. To solve Stage 1, the transformation introduced in Section 2.1 was applied. A fixed number of sample points, $N_s = 40$, for each mode was chosen and an Euler (case study 1), or Simpson (case study 2) discretization of the dynamics was used. The equations of motion were enforced at each sample point for each mode. For example, for Euler discretization, the nonlinear equality constraint $x(k+1) - x(k) - \delta_i f_{q_i}(x(k), u(k)) = 0$ was enforced at the sampling points. The step-size δ_i was scaled based on duration of mode i, that is, $\delta_i = \frac{t_{i+1} - t_i}{N_s}$. The resulting sparse nonlinear programming problem was solved using TOMLAB SNOPT optimization software [3] . To solve Stage 2, the algorithm developed in González et al. (2010) was applied. The running time for both case studies were below 2 minutes on a 2.56 GHz laptop with 4 GB RAM. So both examples could be computed onboard.

4.1 Case 1 - Obstacle avoidance in horizontal 2D flight

We assume the aircraft is cruising at a constant altitude of 11000 meters. We used NOAA [4] wind forecast of July 6th, 2010. A 4th degree polynomial, with the appropriate study of the residual and the regression coefficients statistical significance, was fitted to the wind data.

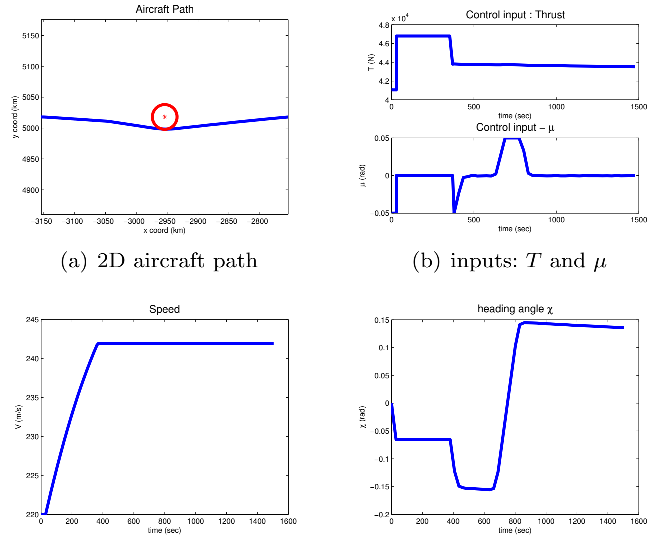
The equations of motion are presented in (6) with the hypothesis of Subsection 3.1.1. There are two modes for the horizontal flight. In mode 1, *control speed*, the aircraft is flying with constant heading angle and hence the input μ is set to zero and the only control input is the thrust T. For this mode the states with dynamics are V, x, y, and m. In mode 2, *control heading*, the speed is held constant by setting the thrust equal to the drag, $T = D$, and the input is μ. The state with dynamics are χ, x, y, m. The aircraft needs to reach a target point $z_d \in \mathbb{R}^2$ while avoiding the hazardous weather obstacle. Let x_{pos} be the 2D position of the aircraft. The objective consists of a final cost term which is a weighted sum of the distance from the target point, the cost of fuel consumption, and the final time to reach the target point and is given as

$$J(\sigma, u) = K_d||x_{pos}(t_F) - z_d||^2 - K_m m(t_F) + K_t t_F.$$

The weights were set to $K_d = 10$, $K_m = 0.5$, and $K_t = 0.1$. The aircraft path was initialized as a straight line segment connecting the initial position of $(-3154, 5018)$ km to the final desired position of $(-2754, 5018)$ km. The weather obstacle was centered at $(-3054, 5018)$ with a radius of 20 km. The algorithm was initialized in mode 2. In the first iteration, Stage 1 of the algorithm returned an optimal path in which the obstacle was avoided by flying around it. Next, Stage 2 of the algorithm determined that an insertion of mode 1 at time 121 seconds would result in reduction of cost while ensuring feasibility of the path. The second iteration of Stage 1 of the algorithm, now initialized with mode sequence $(2, 1, 2)$ resulted in a reduced cost and

[3] SNOPT: an SQP algorithm for Large-Scale Constrained Optimization, www.sbsi-sol-optimize.com
[4] http://www.noaa.gov/

a modified path. Figure 1 shows the aircraft path and the inputs. The numerical results are summarized in Table 4.



(a) 2D aircraft path (b) inputs: T and μ

(c) speed (d) heading angle

Fig. 1. In the optimal 2D aircraft trajectory, the speed is increased immediately in the *control speed* mode, and then the turn maneuver around the obstacle is carried out in the *control heading* mode to avoid the obstacle.

	iteration 1	iteration 2
mode sequence	(2)	(2,1,2)
switching times	(1622)	(29.52, 379.68, 1504.2)
cost	214.23	202.97

Table 1. Optimization results for Case 1

This case study indicates that given a pre-defined path of aircraft that is designed to avoid the obstacle using only a turn maneuver, the objective function can be reduced by including a straight flight maneuver, through the application of control speed mode at an appropriate time, and by increasing the speed to an optimal value for an optimal duration of time.

4.2 Case 2 - Obstacle avoidance in variable altitude flight

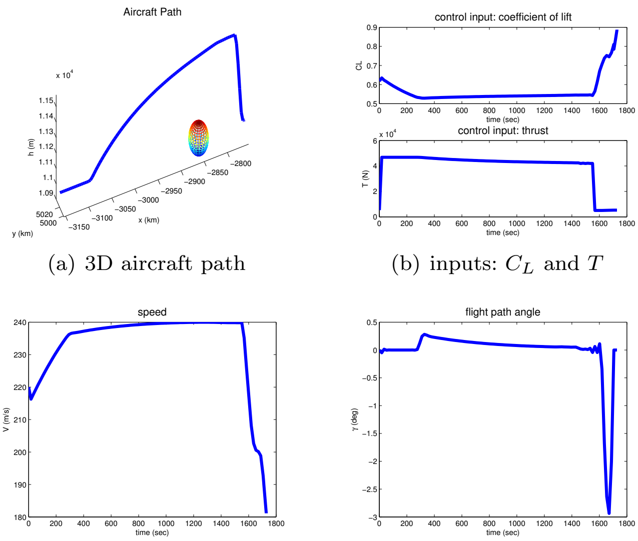
It is assumed that the aircraft can be in three possible modes of 1 *control speed*, 2 *control heading*, 3 *control altitude* defined previously. In the first two modes where the altitude is held constant, the hypothesis of Subsection 3.1.1 hold. In *control altitude* mode, the inputs and equations of motion are modified based on the hypothesis in Section 3.1.2. For this mode, the states with dynamics are V, χ, γ, x, h, and m. In this case study, for the sake of simplicity in optimization, wind is not taken into account. Let $x_{pos} = (x, y, h)$ denote the aircraft position in 3D and $z_d \in \mathbb{R}^3$ denote the desired aircraft position. The cost function is

$$J(\sigma, u) = K_d||x_{pos}(t_F) - z_d||^2 - K_m m(t_F) + K_t t_F.$$

The weights in the objective, and initial and final state of the aircraft were set to that of the previous case study. The weather obstacle was set to an ellipsoid in 3 dimensions, centered at $(-2854, 5018, 11000)$, with an axis length of 20 km in the horizontal plane and 100 meters in the vertical plane.

In this case study, Euler integration did not provide good results due to nonlinearities in the Climb/Descent flight dynamics. Consequently, a Simpson collocation method, as described in Hargraves and Paris (1987), was used to solve Stage 1. The mode sequence was initialized at the *control heading* mode. In the first iteration of Stage 1, the algorithm resulted in an optimal solution in which the aircraft avoided the obstacle by flying around it, similar to the maneuver in the previous case study. Stage 2 of the algorithm found that an insertion of mode 3 at time index of 36 seconds would reduce the cost while maintaining feasibility. Then, in the second iteration of Stage 1, initialized with mode sequence $(2, 3, 2)$, the aircraft gradually climbed to the maximum allowable altitude of 11500 meters and avoided the obstacle by remaining at the high altitude. At the very last portion of flight, it quickly descended to the desired final point. Figure 4.2 shows the aircraft path and the inputs. The inputs for mode 2 are not shown due to the small duration of this mode. The numerical results are summarized in Table 4.2.

This case study indicates that it is optimal to avoid the obstacle by flying at a higher altitude. This is consistent with the knowledge that there is less drag at higher altitudes due to reduced air density. Also, the comparison of the two case studies indicates that inclusion of wind in Case 1 results in reduced cost compared to Case 2 since the optimization was able to find a wind optimal path.



(a) 3D aircraft path (b) inputs: C_L and T

(c) speed (d) flight path angle

Fig. 2. In the optimal 3D aircraft trajectory, the aircraft remains at high altitude to avoid the obstacle.

	iteration 1	iteration 2
mode sequence	(2)	(3,2)
switching times	(1828)	(1705, 1728)
cost	241.61	227.09

Table 2. Optimization results for Case 2

5. CONCLUSIONS

We motivated the problem of hybrid optimal control for aircraft trajectory design and described our algorithm for addressing this problem. Two applications on aircraft trajectory optimization were formulated in this framework and successfully solved. Based on the case studies, we propose several possible applications for the hybrid optimal control formulation and the bi-level algorithm. At the strategic level, given a predefined sequence of modes that define the flight plan, the algorithm can be utilized to provide modifications to the mode sequence such that gate to gate 4D trajectory is optimized. At the operational level, the modification of planned trajectories due to appearance of stochastic phenomenon such us storms, potential collision, or the appropriate sequencing of aircraft at top of descent for starting a Continuous Descent Approach, is currently addressed by an ad-hoc redefinition of the flight plan. This algorithm will be able to tackle such modifications through optimal maneuver insertions. However, to address the complexity in such realistic problems due to presence of multiple aircraft, it is necessary to further explore different integration schemes, Non Linear Programming (NLP) solvers, and programing languages.

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