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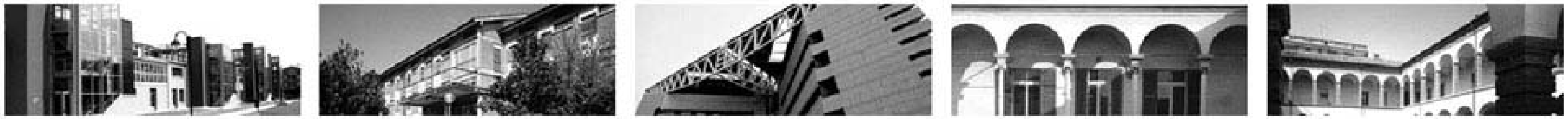

Complexity Metrics

ICRAT Tutorial on “Airborne self separation in air transportation”
Budapest, Hungary – June 1, 2010

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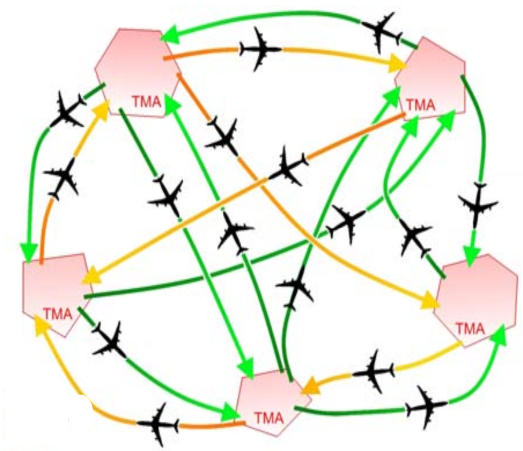
Outline

- Introduction and motivation
- The notion of air traffic complexity
- Relevant characteristics in view of the airborne self-separation application
- Brief survey of existing complexity measures:
  - aircraft density, dynamic density, fractal dimension, input-output, dynamical system, and mathematical programming approaches
  - comparative analysis
- Novel measures of complexity developed within the iFly project:
  - a probabilistic measure of complexity
  - a local flexibility measure of complexity
- References

An Air Traffic Management (ATM) system is a coordinated multi-agent system with competing agents



- each aircraft seeks to improve its own performance (passenger comfort, fuel consumption, etc), while sharing resources, namely airspace and runways, with other aircraft
- coordination is essential to prevent conflict situations were two or more aircraft come too close to each other

Introduction and motivation

The current ATM system is built around a rigid airspace structure and a centralised, mostly human-operated, system architecture

It operates on two different time scales through

- the Air Traffic Control function, guaranteeing the appropriate aircraft separation over a mid-term time horizon
- the Traffic Flow Management function, ensuring a smooth and efficient organization of the overall traffic on a long-term time horizon, possibly reducing the need for the Air Traffic Controllers (ATCs) intervention

→ capacity is limited by the sustainable ATC workload level

Introduction and motivation

Strategies for increasing the ATM system capacity:

- increasing the level of automation to support the ATC tasks

  → automated ground-based ATM

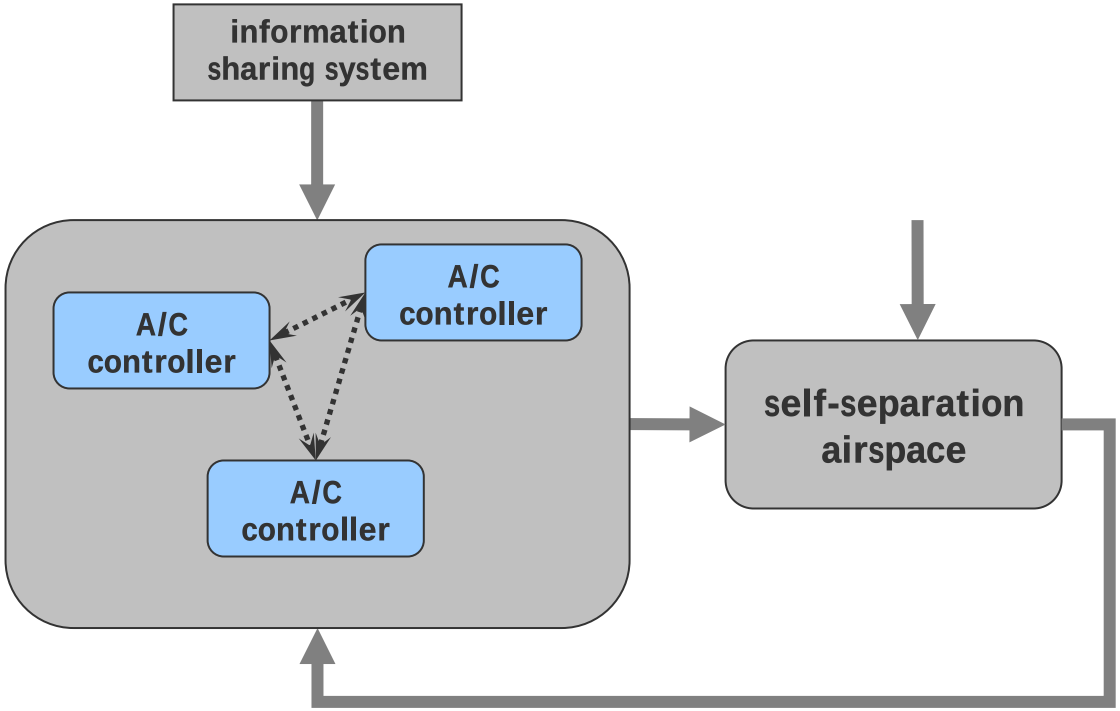
- transferring (part of) the separation responsibilities from the ground to on-board the aircraft

  → airborne self-separation in next generation ATM

Introduction and motivation

In airborne self-separation, each aircraft

- is allowed to modify its flight plan so as to optimize performance, while satisfying some constraints so as to match the traffic outside the self-separation airspace
- has to take over the ATCs tasks for separation assurance with the support of Airborne Separation Assistance Systems (ASAS) relying on advances in communication, navigation, and surveillance technologies



decentralized control scheme in airborne self-separation

Introduction and motivation

Performance and safety of each aircraft flight is affected by the traffic present in the self-separation airspace:

- performance is deteriorated when the aircraft passes through an area with highly congested traffic, since many tactical maneuvers are required
- safety is compromised when the aircraft is involved in a multi-aircraft conflict that exceeds the capabilities of the onboard conflict resolution system

These situations can be timely predicted by introducing the appropriate notion of air traffic complexity, which would then play a key role in the strategic and hazards prevention phases of the ATM process

The notion of air traffic complexity

In general terms, air traffic complexity is a concept introduced to measure the difficulty and effort required to safely and efficiently managing air traffic

In the current ATM system,

- complexity is ultimately related to the ATC workload
- complexity evaluation is used for
  - reconfiguring sectors,
  - redefine traffic patterns and flows
  - reassigning staff

  as a response to modified air traffic conditions, so as to reduce - or at least maintain under sustainable levels - the ATC workload

The notion of air traffic complexity

In general terms, air traffic complexity is a concept introduced to measure the difficulty and effort required to safely and efficiently managing air traffic

In the next generation ATM systems, complexity measures

- could be useful to predict situations that may overburden the distributed conflict detection and resolution function, and
- could also benefit the strategic trajectory management operations by detecting critical areas that would require many tactical maneuvers

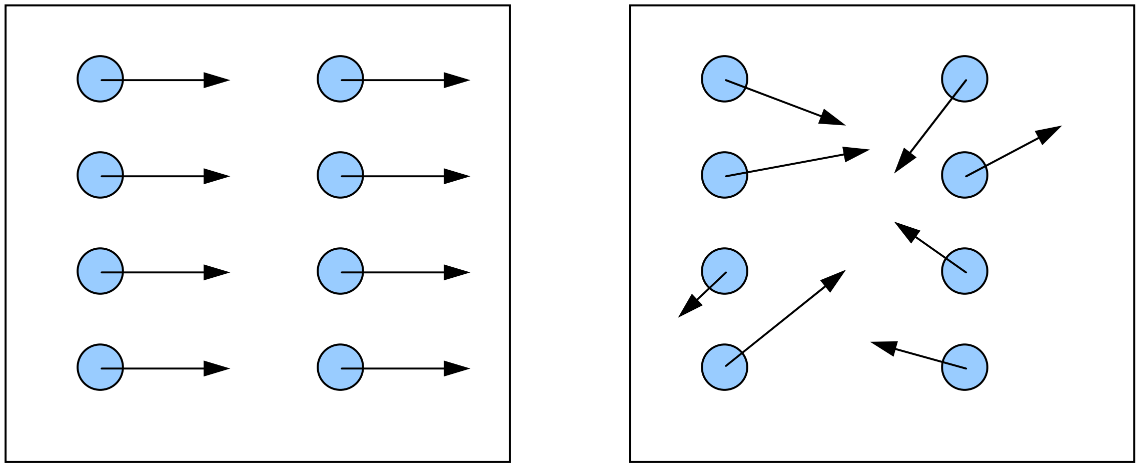
Relevant characteristics in view of the airborne self-separation application

- accounting for traffic dynamics
- independent of the airspace structure
- tailored to the look-ahead time horizon
- independent of the control effort
- with a goal-oriented output form

Relevant characteristics in view of the airborne self-separation application

Accounting for traffic dynamics

- aircraft density is a main factor affecting complexity, but on its own is a very coarse measure of complexity
- the evolution of the traffic must be accounted for



different air traffic situations with the same density

Relevant characteristics in view of the airborne self-separation application

Independent of the airspace structure

- sector-free context with no predefined air route structure
- possibly aircraft clustering can complement and accelerate complexity assessment

Relevant characteristics in view of the airborne self-separation application

Tailored to the look-ahead time horizon

- complexity should be assessed on different look-ahead time horizons, depending on the foreseen application:
  - mid term complexity measures for supporting distributed conflict detection and resolution operations
  - long term complexity measures for strategic trajectory management
- appropriate trajectory prediction models should be adopted

Relevant characteristics in view of the airborne self-separation application

Independent of the control effort

- control is delegated to the aircraft based on a decentralized control scheme, with pilots as human-in-the-loop component
  → difficult to compute the control effort; workload evaluation issue
- account for the controller in place indirectly, through its effect on the air traffic organization
- a possibility would be to adopt the NASA notion of “flexibility” of the aircraft trajectory (the extent to which a trajectory can be modified without causing a conflict with neighboring aircraft or entering a forbidden area) as indirect measure of the control effort

Relevant characteristics in view of the airborne self-separation application

With a goal-oriented output form

- complexity is both a time and space-dependent feature that can be expressed in an aggregate form by condensing either the space or the time information, or both of them
- spatial complexity maps appear better suited for trajectory management applications
- scalar-valued, possibly time-dependent, measures providing a concise information on the complexity encountered by the aircraft along their trajectory appear better suited for conflict detection and resolution-related applications

Brief survey of existing complexity measures

Most of the available complexity studies:

- address ground-based ATM
- aim at evaluating the ATC workload
- refer to a sector-based structure of the airspace
- overlook the look-ahead time horizon dependence aspect

We briefly review some of the approaches proposed in the literature, that is:

- aircraft density
- dynamic density
- fractal dimension
- input-output approach
- dynamical system approach
- mathematical programming approach

Aircraft density

Aircraft density, defined as the number of aircraft in a sector, is compared with the number of aircraft that ATCs are able to safely handle in that sector to assess traffic complexity.

Main characteristics:

- most frequently adopted metric (easy to measure, easy to handle)
- operational interpretation is straightforward
- instantaneous measure; can be projected in the future through aircraft trajectory prediction
- sector-dependent (compared with a sector- based threshold)
- does not account for traffic dynamics → very coarse complexity measure

Dynamic density

Dynamic density is a (linear) combination of traffic density and other static (number of airways, airway crossing, navigation aids, …) and dynamic (weather, aircraft separation and speeds, closing rates, …) air traffic factors contributing to controller workload.
Weights are set to maximize the correlation with workload.

Main characteristics:

- instantaneous measure; can be projected in the future through aircraft trajectory prediction
- over 40 possible indicators $\rightarrow$ many variants, including nonlinear and smoothed versions obtained by time averaging
- criticality of the workload measure used to optimize weights
- sector-dependent metric
- difficult operational interpretation

Fractal dimension

The fractal dimension of an air traffic pattern is a scalar measure of the geometric complexity of the trajectories observed over an infinite time horizon as a whole.

Main characteristics:

- evaluates the degrees of freedom used in the airspace by existing air routes. For example:
  - aircraft cruising on linear routes at specified altitudes, correspond to a fractal dimension of 1
  - If the airspace were covered by routes, the fractal dimension would be 3
- introduced to compare the air traffic configurations resulting from different operational concepts
- timing information is completely lost

Input-output approach

A feedback control scheme is introduced, where

- controlled system = air traffic inside a sector
- controller = automatic conflict solver
- input = additional aircraft entering the airspace
- output = aircraft deviation from their original flight plans

Complexity is measured as the control effort needed to avoid the occurrence of conflicts and computed as follows:

- introduce an additional aircraft in the sector
- launch the conflict solver
- evaluate the amount of deviation needed to recover a conflict-free situation

Input-output approach

Main characteristics:

- originally introduced with reference to a sector; extendable to a sector-free airspace
- integral measure over some look-ahead time-horizon
- dependence on the adopted conflict solver
- a map associating complexity to the entering position and heading of the additional aircraft can be built (not so easy to interpret); a scalar measures of complexity can be extracted

Dynamical system approach

A dynamical system is associated with air traffic by interpreting the aircraft trajectories along some given time horizon as integral lines of the system.

The Lyapunov exponents (LEs) of the dynamical system at some position x represent the local contraction/expansion rate:

- the larger is max(LEs(x)), the higher is the rate at which one loses the ability to predict the system evolution at x
- high air traffic complexity at x is associated with high LEs(x)

Main characteristics:

- “intrinsic complexity” metric, depends only on aircraft trajectories
- sector-independent
- integral measure over some look-ahead time horizon
- a map associating complexity to each airspace position can be built; a scalar measure of complexity can be extracted

Mathematical programming approach

An interpolating velocity vector field is determined based on a snapshot of the air traffic, and subject to constraints related to maneuvers feasibility:

- in those regions where a smooth vector field is found, aircraft can follow non intersecting trajectories (low complexity)
- separation boundaries are introduced where the vector field loses continuity; the locations of these boundaries correspond to critical areas (high complexity)

Main characteristics:

- “intrinsic complexity” metric, depends only on aircraft trajectories
- sector-independent
- integral measure over some look-ahead time horizon
- provides a map of the high complexity regions

Comparative analysis

	accounting for traffic dynamics	independent of airspace structure	look-ahead time horizon	control independent	output form
aircraft density	no	no	instantaneous, extendable with prediction	no (workload dependent)	scalar/ scalar-valued function
dynamic density	yes	no	instantaneous, extendable with prediction	no (workload dependent)	scalar/ scalar-valued function
fractal dimension	yes	yes	infinite time horizon	yes	scalar
input/output approach	yes	yes	mid term	no	map of additional a/c initialization/ scalar
dynamical system approach	yes	yes	mid/long term	yes	map of the airspace/ scalar
mathematical programming approach	yes	yes	mid/long term	yes	map of the airspace

Comparative analysis

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mathematical programming approach	yes	yes	mid/long term	yes	map of the airspace

Novel complexity measures developed within the iFly project

Two novel measures of complexity have been introduced:

- a probabilistic measure for mid term horizon applications
- a local flexibility measure for long term horizon applications

Common characteristics:

- “intrinsic complexity” measures, based on ircraft trajectory only
- independent of the airspace structure
- tailored to the look-ahead time horizon
- with goal-oriented output form

Here, we concentrate on the first one.

A probabilistic measure of complexity

Complexity is evaluated in terms of proximity in time and space of the aircraft as determined by their intent and current state, while taking into account the uncertainty in the aircraft future position

→ notion of probabilistic occupancy of the airspace

Regions with limited manoeuvrability space can be identified; this information can be used for

- detecting critical encounter situations that would be difficult for the aircraft to solve autonomously
- provide guidance for trajectory design in mid term conflict resolution operations

A probabilistic measure of complexity

Trajectory prediction model

$$x_i(t) = \underbrace{x_{i,0} + \int_0^t v_i(s)ds}_{\text{nominal trajectory based on state \& intent}} + \underbrace{\Sigma_i(t)W_i(t)}_{\text{prediction error}}$$

- the prediction error is modeled through a Brownian motion (BM) $W_i(t)$ whose variance grows in time, possibly at a different rate in the along-track and cross-track directions
- similar models adopted in the literature for (probabilistic) conflict prediction

A probabilistic measure of complexity

Consider a region S of the airspace with N aircraft $I_N = \{1,2,\ldots, N\}$.

Let E(x) be an ellipsoidal region centered at $x \in S$, and $t \in [0,t_f]$.

First order complexity at position x and time t:

$$c_1(x,t) = P(x_i(s) \in E(x) \text{ for some } s \in [t, t+\Delta] \;\&\; i \in I_N)$$

Second order complexity at position x and time t:

$$c_2(x,t) = P(x_i(s) \;\&\; x_j(s') \in E(x) \text{ for some } s, s' \in [t, t+\Delta] \;\&\; i \neq j \in I_N)$$

A probabilistic measure of complexity

First order complexity at position x and time t:

$$c_1(x,t) = P(x_i(s) \in E(x) \text{ for some } s \in [t, t+\Delta] \ \& \ i \in I_N)$$

probability that at least one aircraft will enter the buffer region E(x) centered at x within the time frame [t, t+Δ]

$c_1(x,t) = 0 \rightarrow$ no aircraft will enter E(x) within [t,t+Δ]

$c_1(x,t) = 1 \rightarrow$ at least one aircraft will enter E(x) within [t,t+Δ]

First order complexity map at time t:

$C_1(\cdot,t)$: $x \in S \rightarrow c_1(x,t)$

A probabilistic measure of complexity

Second order complexity at position x and time t:

$$c_2(x,t) = P(x_i(s)\ \&\ x_j(s') \in E(x) \text{ for some } s, s' \in [t, t+\Delta]\ \&\ i \neq j \in I_N)$$

probability that at least two aircraft will enter the buffer region E(x) centered at x within the time frame [t, t+Δ]

$c_2(x,t) = 0 \rightarrow$ at most one aircraft will enter E(x) within [t,t+Δ]

$c_2(x,t) = 1 \rightarrow$ at least two aircraft will enter E(x) within [t,t+Δ], though not exactly at the same time instant

Second order complexity map at time t:

$C_2(\cdot,t)$: $x \in S \rightarrow c_2(x,t)$

Remarks:

- $c_2(x,t) \leq c_1(x,t)$
- higher order complexity measures and maps can be defined

A probabilistic measure of complexity

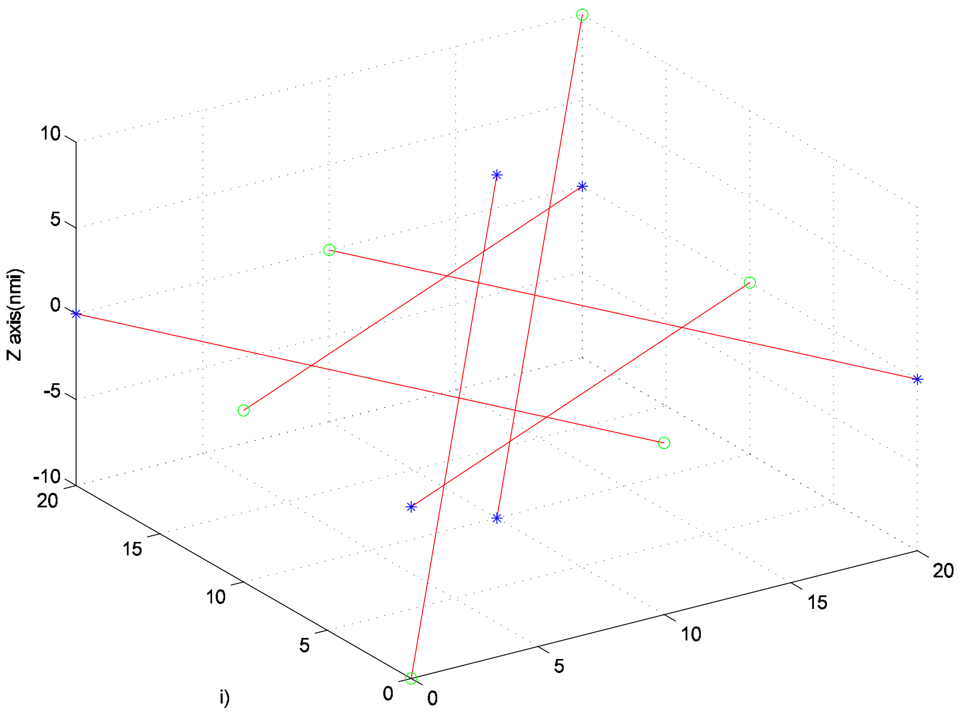
From an operational perspective,

- forming the complexity maps for different consecutive time intervals allows to predict when the aircraft enter and leave a certain zone of the airspace and to define the occupancy of the airspace region S

→ congested areas (i.e. areas where multi-aircraft encounters with limited inter-aircraft spacing are likely to occur) can be detected in the time-space coordinates

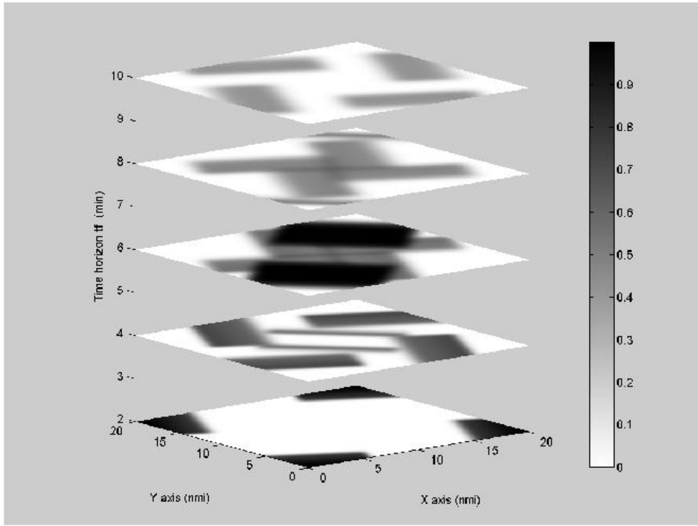
Example 1 (evaluating the airspace occupancy):

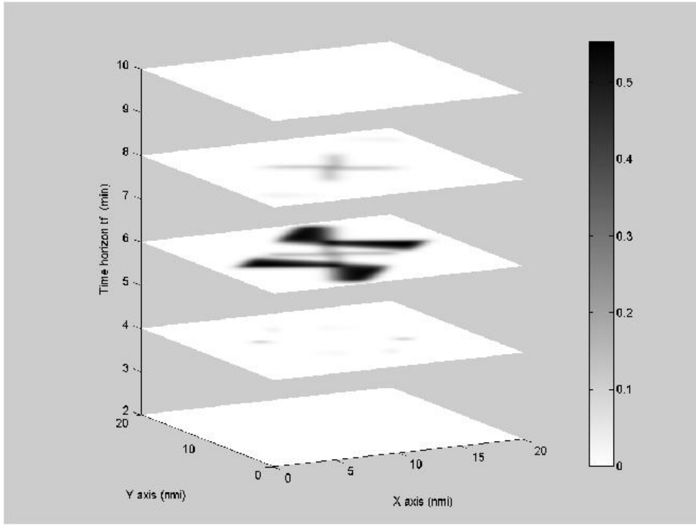
3D airspace region with 6 aircraft flying at constant velocity along a straight line trajectory from starting (‘$*$’ blue) to destination (‘o’ green) positions during a 10 minutes time horizon



A probabilistic measure of complexity

Example 1 (evaluating the airspace occupancy):





first and second order complexity maps on the horizontal plane XY for 5 consecutive intervals [t, t+Δ] with Δ = 2 min and t = 0,2,4,6,8

A probabilistic measure of complexity

Suppose that some additional aircraft A enters the airspace region S following a nominal trajectory x_A: $[0,t_f] \rightarrow S$.

Single aircraft complexity:

The complexity encountered by aircraft A along its nominal trajectory $x_A(\cdot)$ within the time interval $[t,t+\Delta]$ can be defined as

$$c_A(t) = P(x_i(s) \in E(x_A(s)) \text{ for some } s \in [t, t+\Delta] \ \& \ i \in I_N)$$

A probabilistic measure of complexity

Suppose that some additional aircraft A enters the airspace region S following a nominal trajectory x_A: $[0,t_f] \rightarrow S$.

Single aircraft complexity:

The complexity encountered by aircraft A along its nominal trajectory $x_A(\cdot)$ within the time interval $[t,t+\Delta]$ can be defined as

$$c_A(t) = P(x_i(s) \in \boxed{E(x_A(s))} \text{ for some } s \in [t, t+\Delta] \ \& \ i \in I_N)$$

the ellipsoidal buffer zone moves along the trajectory of aircraft A

- $c_A(t)$ is the probability that at least one aircraft enters the buffer zone centered at the nominal position of aircraft A within $[t,t+\Delta]$

A probabilistic measure of complexity

From an operational perspective,

- the introduced single-aircraft complexity measure $c_A(t)$ can be used by aircraft A to evaluate the maneuverability space surrounding its nominal trajectory and eventually redesign it
- if $[t, t+\Delta] = [0,t_f]$ and buffer zone = protection zone, then, $c_A(t)$ is the probability of aircraft A getting in conflict with another aircraft → conflict detection is an integrable task in complexity evaluation

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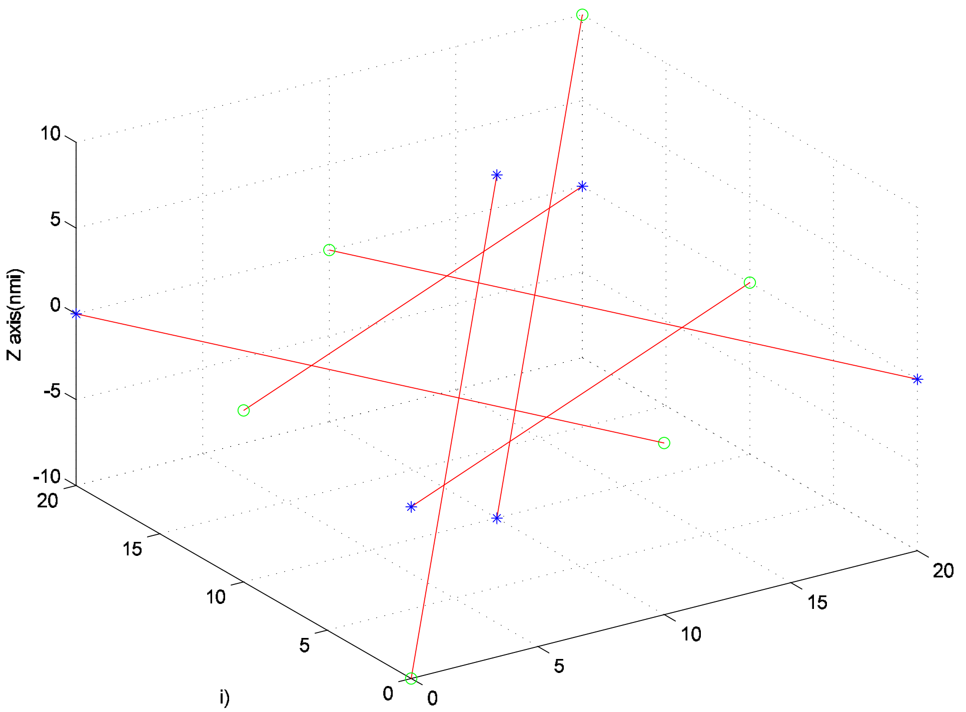
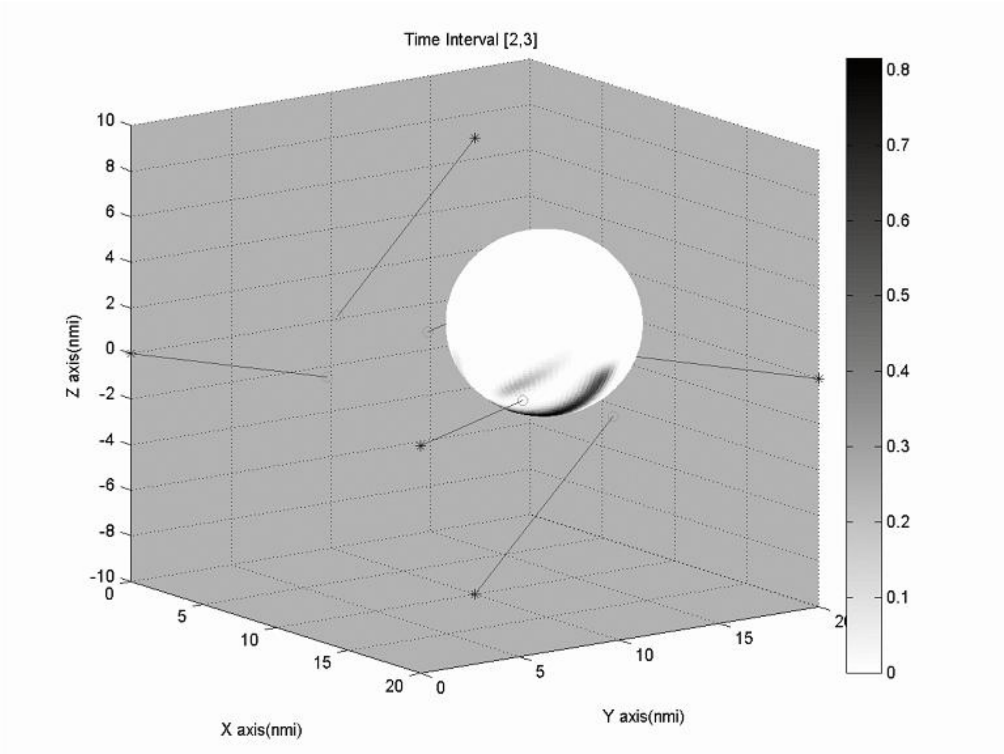
Example 2 (evaluating the maneuverability space):

Aircraft A enters the 3D airspace region flying at some constant velocity. Due to the presence of the 6 aircraft, aircraft A is not free to change its heading arbitrarily.



A probabilistic measure of complexity

Example 2 (evaluating the maneuverability space):



$c_A(t)$ within $[t, t+\Delta]$ with $\Delta = 1$ min as a function of the heading at a sampled point along the nominal trajectory of aircraft A

A probabilistic measure of complexity

Example 3 (trajectory design):

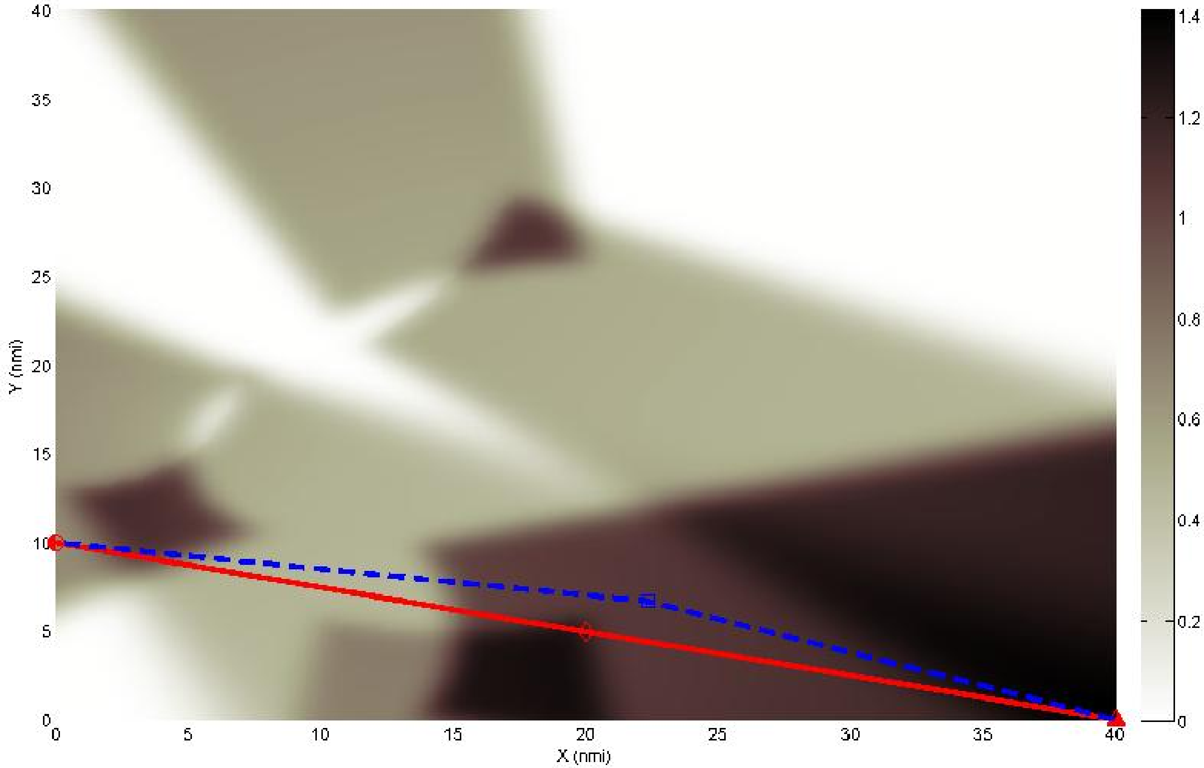
Aircraft A enters a 2D airspace region (flight-level case) with 4 aircraft, and is supposed to fly from a starting to a destination position within the time interval [0,10] min

- the straight-line trajectory from starting to destination position is not guaranteed to be low-complexity
- aircraft A selects an intermediate way-point at $t^* = 5$ min so as to minimize the weighted cost

$$J = d + \lambda\,(c_A(0) + c_A(5))$$

where
d = deviation of the two-legged trajectory from the straight one
$c_A(0)$ = single-aircraft complexity within [0,5] (first leg)
$c_A(5)$ = single-aircraft complexity within [5,10] (second leg)

Example 3 (trajectory design):



straight-line trajectory and 2-legged solution. The color map represents the complexity term as a function of the intermediate way-point position

A probabilistic measure of complexity

Main characteristics:

- “intrinsic complexity” metric, depends only on aircraft trajectories
- independent of the airspace structure
- evaluated based on a mid term trajectory prediction model
- output form:
  - 4D (space cross time) global map of complexity
  - scalar-valued function along the aircraft nominal trajectory
  - scalar integral measure providing the probability of conflict

	accounting for traffic dynamics	**independent of airspace structure**	**look-ahead time horizon**	**control independent**	**output form**
probabilistic measure	yes	yes	mid term	yes	4D map/ scalar-valued function/ scalar

A probabilistic measure of complexity

Computational aspects:

- analytical – though approximated – expressions of the air traffic complexity have been derived under the assumptions that
  - the BMs affecting the future position of different aircraft are independent
  - the nominal trajectories of the aircraft are 3D straight-line or multi-legged trajectories
- adaptive gridding scheme should be adopted to reduce the computational load involved in complexity map building (finer grid close to the nominal trajectories)

References

Survey on existing complexity studies, containing the relevant references:

[1] M. Prandini, L. Piroddi, S. Puechmorel, S.L. Brázdilová. *Complexity metrics applicable to autonomous aircraft.* Deliverable 3.1 of the iFly project.

[2] M. Prandini, L. Piroddi, S. Puechmorel, S.L. Brázdilová. *Towards air traffic complexity assessment in new generation air traffic management systems.* Journal paper. Submitted, 2009

Novel metrics:

[3] M. Prandini and J. Hu. *A probabilistic approach to air traffic complexity evaluation.* 48th Conf. on Decision and Control, Shanghai, China, Dec. 2009

[4] M. Prandini, V. Putta, J. Hu. *A probabilistic measure of air traffic complexity in three-dimensional airspace.* Int. Journal of Adaptive Control and Signal Processing, special issue on Air Traffic Management: Challenges and opportunities for advanced control. Accepted, 2010

[5] L. Piroddi and M. Prandini. *A geometric approach to air traffic complexity evaluation for strategic trajectory management.* Conference paper. Submitted, 2010

[1] can be downloaded from the iFly project website: http://ifly.nlr.nl/
[2]-[5] are available upon request (please, send an e-mail to prandini@elet.polimi.it)